Dynamical Effects on the $^9$Li-$n$ Interaction Induced by Pauli Blocking of the $J^\pi = 0^+$ Pairing Correlation

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The $^9$Li-$n$ interaction is discussed based on the idea of the Pauli blocking for the $J^\pi = 0^+$ neutron pairing correlation in the $^9$Li core due to the presence of the last valence neutron. The present studies are developed in a framework of the microscopic coupled-channel model for $\{^9$Li$((0p_{3/2})^4_0 + n)\} + \{^9$Li$((0p_{3/2})^2_0(0p_{1/2})^2_{1/2} + n)\} + \{^9$Li$((0p_{3/2})^2_0(1s0d)^2_{1/2} + n)\}$. The channel of the present model consists of the last valence neutron coupled with a $^9$Li configurational state connected by the $J^\pi = 0^+$ pairing interaction. Due to the presence of the valence neutron in the $0p_{1/2}$ orbital, the $(0p_{3/2})^2 \rightarrow (0p_{1/2})^2$ jump in the $^9$Li core is prevented. Thus, the $p_{1/2}$-orbital neutron states experience a strong repulsive effect due to the lack of the configuration mixing between $(0p_{3/2})^4_0$ and $(0p_{3/2})^2_0(0p_{1/2})^2_{1/2}$, and its energy rises up significantly. In contrast to this, the $sd$-orbital neutron imparts a very weak effect on the configuration mixing. As a result, the $s_{1/2}$-orbital state is obtained in the vicinity of the threshold as a bound state or virtual resonance for the $^9$Li-$n$ interaction which reproduces $1^+(S_n = -0.42 \text{ MeV})$ and $2^+(S_n = -0.82 \text{ MeV})$ states consistently with experimental data observed by Bohlen et al. The effective $^9$Li-$n$ potential which has the strong parity dependence deduced from the present coupled-channel calculation is discussed to be very promising for studying the halo structure of $^{11}\text{Li}$ within the $^9\text{Li} + n + n$ model.

§1. Introduction

Recent developments in experiments using radioactive nuclear beams have provided us with very interesting discoveries of nuclear properties of neutron dripline nuclei.1) Elucidating the halo structures in $^6\text{He}$, $^{11}\text{Li}$, $^{11}\text{Be}$, $^{14}\text{Be}$ and $^{17}\text{B}$ is one of the most interesting problems to be tackled. However, we have not yet reached a satisfactorily proper understanding even for specific problems involving $^{10}\text{Li}$ and $^{11}\text{Li}$, that is, the lowering of $s$-orbital states in $^{10}\text{Li}$ and $^{11}\text{Li}$ and the large amount of the mixing with $s$-orbital configurations in the $^{11}\text{Li}$ ground states, together with the underbinding of the three-body system $^{11}\text{Li}$ ($= ^9\text{Li} + n + n$).1, 2) In this paper, we focus the theoretical study on the $^9\text{Li}-n$ interaction with the aid of experimental results accumulated recently, and we make a successful attempt at solving the question of the lowering problem of $s$-orbital states in $^{10}\text{Li}$.

Knowledge concerning the $^9\text{Li}-n$ interaction has been extracted through microscopic model studies from the experimental properties of the low-lying resonance states in $^{10}\text{Li}$, where there are no bound states below the lowest open channel of $^{10}\text{Li}$. Therefore, the $^9\text{Li}-n$ potential is studied by using the results of the microscopic model studies. The present coupled-channel calculations are developed in a framework of the microscopic coupled-channel model for $\{^{10}\text{Li}((0p_{3/2})^4_0 + n)\} + \{^{10}\text{Li}((0p_{3/2})^2_0(0p_{1/2})^2_{1/2} + n)\} + \{^{10}\text{Li}((0p_{3/2})^2_0(1s0d)^2_{1/2} + n)\}$. The channel of the present model consists of the last valence neutron coupled with a $^{10}\text{Li}$ configurational state connected by the $J^\pi = 0^+$ pairing interaction. Due to the presence of the valence neutron in the $0p_{1/2}$ orbital, the $(0p_{3/2})^2 \rightarrow (0p_{1/2})^2$ jump in the $^{10}\text{Li}$ core is prevented. Thus, the $p_{1/2}$-orbital neutron states experience a strong repulsive effect due to the lack of the configuration mixing between $(0p_{3/2})^4_0$ and $(0p_{3/2})^2_0(0p_{1/2})^2_{1/2}$, and its energy rises up significantly. In contrast to this, the $sd$-orbital neutron imparts a very weak effect on the configuration mixing. As a result, the $s_{1/2}$-orbital state is obtained in the vicinity of the threshold as a bound state or virtual resonance for the $^9\text{Li}-n$ interaction which reproduces $1^+(S_n = -0.42 \text{ MeV})$ and $2^+(S_n = -0.82 \text{ MeV})$ states consistently with experimental data observed by Bohlen et al. The effective $^9\text{Li}-n$ potential which has the strong parity dependence deduced from the present coupled-channel calculation is discussed to be very promising for studying the halo structure of $^{11}\text{Li}$ within the $^9\text{Li} + n + n$ model.
Although the ground state energy is still not well understood experimentally, several candidates for the $^9\text{Li}^+$ ground state energy have been reported in the region of $0 \text{ MeV} \leq E < 1.0 \text{ MeV}$ above the $^9\text{Li} + n$ threshold. Experimental information regarding $^{10}\text{Li}$ resonances was limited to that gained from a single result until the 1980’s. This was an observation of the resonance at 0.8 MeV with a width of about 1 MeV. This resonance has been understood to come from the $p_{1/2}$ orbit of the last neutron. At the beginning of the 1990’s, Amelin et al. reported the observation of a peak at 0.15 MeV in $\pi^-$ capture in $^{11}\text{Be}$, which was interpreted as the ground state of $^{10}\text{Li}$ being analogous to $^{10}\text{Be}$ ($T = 2$, $J = 2^-$) with a $(0s)^4(0p)^4(1s0d)^2$ configuration. However, the existence of the barely-bound $sd$-orbital valence neutron states has not yet been confirmed, although several experiments have been reported.

We studied the interaction between a neutron and $^9\text{Li}$ in the framework of a microscopic two-body model, assuming the $(0p_{3/2})^4$-closed neutron configuration and the single $0p_{3/2}$-proton configuration for the $^9\text{Li}$-core nucleus. It was shown that the low-lying $1^+$ and $2^+$ doublet resonances observed by Bohlen et al. could be understood as $p_{1/2}$-neutron orbital states coupled with the $J^\pi = 3/2^-$ $^9\text{Li}$ core. The resonance energies calculated using a $^9\text{Li}-n$ folding potential obtained from the effective $N-N$ interaction (modified Hasegawa-Nagata force) are 0.42 MeV ($J^\pi = 1^+$) and 0.86 MeV ($J^\pi = 2^+$), which are in good agreement with the experiments, but their decay widths are obtained to be 2–3 times larger than the observed ones.

Applying the obtained $^9\text{Li}-n$ interaction to the microscopic $^9\text{Li} + n + n$ model, we calculated the binding energy of $^{11}\text{Li}$. The underbinding was improved slightly from the previous calculations because the $^9\text{Li}-n$ interaction was strengthened to fit the new data (0.42 MeV) of the ground resonance state of $^{10}\text{Li}$. However, the result was still too small for the observation by about 1 MeV. We argued that this difficulty is caused by the simple assumption that the $^9\text{Li}$-core nucleus is described by a single configuration of the neutron $p_{3/2}$-closed shell.

In connection to the binding problem of the $^{11}\text{Li}$ ground state, there is an experimentally important problem of whether the $s_{1/2}$-orbital state in $^{10}\text{Li}$ exists or not in the low energy region. If $s$- and $d$-orbits exist in the low energy region around the $0p$-orbital states, the $J^\pi = 0^+$ pair excitation could easily occur, since the matrix elements of the pairing interaction are sufficiently large for two neutrons to rise from $0p$- to $sd$-orbits. Therefore, the energy gain from the pairing correlation is expected to be responsible for solving the binding energy problem of $^{11}\text{Li}$. However, it is not necessarily clear from the side of experiments where the $s_{1/2}$-orbital states do exist. Furthermore, there is no known theoretical reasoning why $s$- and $d$-orbits come down at low energies in $^{10}\text{Li}$. Nevertheless, many discussions have been made under the assumption of the probable existence of such $s$-orbits in order to obtain an understanding of the phenomena related to the $^{11}\text{Li}$ nucleus.
Sagawa, Brown and Esbensen\textsuperscript{21,22} discussed the idea that Pauli blocking of the pairing correlation and the quadrupole excitation play equally important roles in causing the parity inversion in $^{11}\text{Be}$ based on the shell model analysis. They also showed that Pauli blocking imparts an appreciable effect on the spectra of the $N=7$ nuclei. In this paper, we develop similar considerations of Pauli blocking for the problem of the interaction between the $^9\text{Li}$-core nucleus and the last neutron. Since the quadrupole moment of the $^9\text{Li}$ nucleus is on the order of the single particle value, in contrast to the large value in the case of $^{10}\text{Be}$, the quadrupole excitation is thought to play a minor role in the nuclear system of $^{10}\text{Li}$. Therefore, we shall single out the effect of Pauli blocking for the study on the $p_{1/2}$-orbital resonance state in $^{10}\text{Li}$.

The main purpose of this paper is to give a theoretical reason explaining why the positive-parity states come near the low-lying negative-parity states in $^{10}\text{Li}$ through the study of the $^9\text{Li}-n$ interaction by taking into account the Pauli-blocking effect. For this purpose, we use a microscopic coupled-channel formalism where the $^9\text{Li}$-core states are expressed as a superposition of $[j^2]_{J=0^+}$ configurations, for example, the $^9\text{Li}$ ground state is able to have the leading configurations of \{$(p_{3/2})^4(0p_{3/2})^4\nu,J=0$\}, \{$(p_{3/2})^2(0p_{3/2})^2(0p_{1/2})^2\nu,J=0$\}, and \{$(p_{1/2})^2(0p_{1/2})^2(0p_{1/2})^2\nu,J=0$\}. In an isolated system of $^9\text{Li}$, a large energy gain is obtained as the result of coupling between these configurations that is realized in the system of $^{10}\text{Li}$ in the asymptotic region of $^9\text{Li} + n$. However, when the last neutron occupies the $p_{1/2}$ orbit in the $^{10}\text{Li}$ system, the latter configurational state is diminished by the Pauli-exclusion principle. Thus, the pairing coupling among $[j^2]_{J=0^+}$ configurations is broken in the interaction region of the $^{10}\text{Li}$ ($= ^9\text{Li} + n$) system. The microscopic coupled-channel formalism presented here can describe the above-mentioned dynamical behavior of the $^{10}\text{Li}$ system. As a result of these effects, the levels of the $p_{1/2}$-orbital states will be shown to rise, roughly speaking, by an amount equal to the energy loss of the pairing correlation. Therefore, we can expect the positive-parity state to be close to the low-lying negative-parity states in $^{10}\text{Li}$.

In the next section, we explain the microscopic coupled-channel formalism of $^9\text{Li} + n$ for $^{10}\text{Li}$. The numerical results are shown in §3. The interaction between the $^9\text{Li}$-core nucleus and neutrons is discussed in detail in §4. The summary and concluding remarks are given in §5.

\section{A microscopic coupled-channel model for $^{10}\text{Li}$}

We formulate a simple microscopic coupled-channel model of $^9\text{Li} + n$ for $^{10}\text{Li}$. In the asymptotic region between $^9\text{Li}$ and a neutron, the $^9\text{Li}$-core part is described by the wave function of the $^9\text{Li}$ ground state in an isolated system. The dominant configuration of the $^9\text{Li}$ ground state is considered to be the $p_{3/2}$-neutron sub-closed shell, $(0p_{3/2})^4$, which has been assumed in many studies of $^{10}\text{Li} = ^9\text{Li} + n$ and $^{11}\text{Li} = ^9\text{Li} + n + n$ performed so far.\textsuperscript{9,10,17,19} However, such a simple assumption of the sub-closed shell might necessarily be invalid due to the residual interaction providing deformation and/or pairing correlations. In fact the shell model calculation for $^9\text{Li}$ predicts a strong mixing with the $(0p_{3/2})^2(0p_{1/2})^2$ configuration.\textsuperscript{24} In the present
model, we assume that the pairing interaction causes the configuration mixing in $^9\text{Li}$.

To take into account the neutron pairing correlation in $^9\text{Li}$, we describe the ground state wave function by a superposition of the following configurations with the seniority $v = 1$:

$$\Psi_{\text{gr}}(^9\text{Li}) = \sum_{i=0}^{\infty} \alpha_i \Phi(ci),$$

(2.1)

where

$$\Phi(c0) : \left[ (0s)^4(0p_{3/2})^1_\pi(0p_{3/2})^4_{\nu,J=0} \right],$$

$$\Phi(c1) : \left[ (0s)^4(0p_{3/2})^1_\pi(0p_{3/2})^2_{\nu,J_1=0}(0p_{1/2})^2_{\nu,J_2=0} \right],$$

$$\Phi(c2) : \left[ (0s)^4(0p_{3/2})^1_\pi(0p_{3/2})^2_{\nu,J_1=0}(1s_{1/2})^2_{\nu,J_2=0} \right],$$

$$\Phi(ci) : \left[ (0s)^4(0p_{3/2})^1_\pi(0p_{3/2})^2_{\nu,J_1=0}(n\ell j)_{\nu,J_2=0} \right],$$

(2.2)

When a valence neutron approaches to the $^9\text{Li}$-core nucleus in the $^{10}\text{Li}$ system, the $^9\text{Li}$-core configuration must be changed due to both the interaction and the Pauli principle between $^9\text{Li}$ and the valence neutron. Especially, we should note that the $p_{1/2}$-orbital valence neutron which has, in the short distant region, a large overlap with the occupied neutron orbit of $0p_{1/2}$ in the case of the channel configuration $\Phi(c1)$, whose component is excluded by the Pauli principle. Therefore, when we consider the relative motion between $^9\text{Li}$ and a neutron, we must treat the dynamical change of the $^9\text{Li}$-core configuration, which depends on the valence neutron orbit and also on the relative distance between them.

The microscopic coupled-channel formalism is appropriate to deal with the above-mentioned dynamics. The model wave function is given as

$$\Psi(^{10}\text{Li}) = \sum_{i=0}^{\infty} A' \{ \Phi(ci) \chi_i(r) \},$$

(2.3)

where $A'$ is the antisymmetrization between $^9\text{Li}$ and valence neutron, $\Phi(ci)$ is the configurational state defined by Eq. (2.2), and the channel amplitude $\chi_i(r)$ is the valence neutron wave function which depends on the relative coordinate between $^9\text{Li}$ and valence neutron. Here, it should be noted that the last neutron does not disturb the $^9\text{Li}$-core nucleus at a sufficiently large separation distance. Naturally, the wave function of (2.3) has the following asymptotic form:

$$\Psi(^{10}\text{Li}) \longrightarrow \{ \Psi_{\text{gr}}(^9\text{Li}) \chi(r) \}.$$  

(2.4)

The relative wave functions of the channel amplitudes ($\chi_0$, $\chi_1$, ...) are determined by considering the Schrödinger equation

$$H\Psi(^{10}\text{Li}) = E\Psi(^{10}\text{Li}),$$

(2.5a)
where the Hamiltonian is given as a sum of the intrinsic part \(H^{(9\text{Li})}\) of \(^9\text{Li}\) and the part \((T_n + V_n)\) of the relative motion of the last neutron with respect to the center of mass of \(^9\text{Li}\):

\[
H = \sum_{i=1}^{10} -\frac{\hbar^2}{2M} \Delta_i + \sum_{i>j} v_{ij}, \quad (2.5b)
\]

\[
= T_n + V_n + H^{(9\text{Li})}. \quad (2.5c)
\]

Multiplying both sides of the Schrödinger equation (2.5a) by \(\Phi(ci)^*\), and integrating over the internal degree of freedom of \(^9\text{Li}\), we have the equations of the coupled-channel resonating group:

\[
\sum_{j=0}^{\infty} \langle \Phi(ci)| T_n + V_n + H^{(9\text{Li})} | A' \{ \Phi(cj) \chi_j \} \rangle = E \sum_{j=0}^{\infty} \langle \Phi(ci)| A' \{ \Phi(cj) \chi_j \} \rangle \quad (2.6)
\]

for \(i = 0, 1, \cdots\). To solve these equations for the relative wave functions \((\chi_0, \chi_1, \cdots)\), we introduce the approximations

\[
\langle \Phi(ci)|V_n| A' \{ \Phi(cj) \chi_j \} \rangle \approx \delta_{ij} V_F N_{ij} \chi_j, \quad (2.7)
\]

\[
\langle \Phi(ci)|H^{(9\text{Li})}| A' \{ \Phi(cj) \chi_j \} \rangle \approx h_{ij}^{(9\text{Li})} N_{ij} \chi_j, \quad (2.8)
\]

where \(\langle \Phi(ci)| A' \{ \Phi(cj) \chi_j \} \rangle = N_{ij} \chi_j\), \(V_F = \langle \Phi(cj)|V_n| \Phi(cj) \rangle\) and \(h_{ij}^{(9\text{Li})} = \langle \Phi(ci)| H^{(9\text{Li})}| \Phi(cj) \rangle\). Then, the above equations of the coupled-channel resonating group method are written as

\[
\sum_{j=0}^{\infty} [(T_n + V_F) \delta_{ij} + h_{ij}^{(9\text{Li})}] N_{ij} \chi_j = E \sum_{j=0}^{\infty} N_{ij} \chi_j. \quad (2.9)
\]

Here, we note that the potential \(V_F\) goes to zero and the norm kernels \(N_{ij}\) become \(\delta_{ij}\) at a large distance of \(r\). Then, the asymptotic form of Eq. (2.9) is given by

\[
\sum_{j=0}^{\infty} [T_n + h_{ij}^{(9\text{Li})}] \chi_j = E \chi_i. \quad (2.10)
\]

Apart from the kinetic energy term \(T_n\), the matrix \(\{ h_{ij}^{(9\text{Li})} \}\) properly gives the amplitudes \((\chi_i = \alpha_i; i = 0, 1, \cdots)\) of the wave function \(\psi_{gr}^{(9\text{Li})}\) in Eq. (2.1) and the energy \(E_{gr}^{(9\text{Li})}\) of the \(^9\text{Li}\) ground state by diagonalization of \(\{ h_{ij}^{(9\text{Li})} \}\). In other words, we must chose the matrix elements \(h_{ij}^{(9\text{Li})}\) so as to reproduce the properties of the ground state of \(^9\text{Li}\). Then, we subtract the ground state energy \(E_{gr}^{(9\text{Li})}\) from both sides of Eq. (2.9) and replace \(E\) on the right-hand side by \(E = E - E_{gr}^{(9\text{Li})}\) being the energy with respect to the threshold of \(^9\text{Li} + n\).

Here, we should also note that the norm kernels \(N_{ij}\) work as a kind of projection operators which eliminate the Pauli-forbidden states. In the present model, \(0s_{1/2}\) and \(0p_{3/2}\) states are forbidden; the relative wave function \(\chi_0\) cannot have these forbidden state components. For the cases of \(ci\)-channels \((i \neq 0)\) with \((n, \ell = 0\ or \ 1, j = 1/2)\)-orbital neutrons, we treat this \((ntj)\) orbit as a kind of the Pauli-forbidden state. In addition to the orbital states of \(0s_{1/2}\) and \(0p_{3/2}\), \(0p_{1/2}\) is a kind of forbidden state...
in the c1 channel, because the $0p_{1/2}$-orbits are already occupied by the $(0p_{1/2})_{j_2=0}^2$ pair in the $^9$Li-core configuration of the c1 channel. In general, such a treatment is not correct, except for the $j = 1/2$ case, because the $(n\ell j)$-orbital states for the case of $j \neq 1/2$ are not completely filled by the one pair occupation of $(n\ell j)_{j_2=0}^2$.

In the present model, we assume that the $(n\ell j \neq 1/2)$-orbital valence neutron moves freely without any constrain by orthogonality to the $0s_{1/2}$ and $0p_{3/2}$ forbidden states for the following reasons: As for the $p_{3/2}$ valence neutron case, the $(0p_{3/2})$ orbit is partially forbidden, except for the c0 channel which has the closed $(0p_{3/2})^4$ configuration. The corresponding allowed states are of the $(0p_{3/2})_{\nu}^2(n\ell j)^2_{\nu}$ configuration for $^9$Li + $n$, but this state represents the $1p-1h$ excitation for the $p$-shell neutron of the $^9$Li core. However, we omit these kinds of ($p$-$h$) channels and adopt only the channels with the $J^\pi = 0^+$ pairing-type configurations, because the latter configurational states have been known to play an important role in the present problem. As for cases of the $d_{5/2}$ and other orbital neutrons, our assumption is justified for the reason that the channels with the $(0d_{5/2})_{j_2=0}^2$ and other pair configurations are minor components in describing the ground state of $^9$Li.

We introduce the following projection operators for the valence neutron:

$$A_0 = |0s_{1/2}\rangle \langle 0s_{1/2}| + |0p_{3/2}\rangle \langle 0p_{3/2}|,$$

$$A_1 = A_0 + |0p_{1/2}\rangle \langle 0p_{1/2}|,$$

$$A_2 = A_0 + |1s_{1/2}\rangle \langle 1s_{1/2}|,$$

$$\ldots$$

$$A_j = \begin{cases} A_0 + |n\ell j\rangle \langle n\ell j| & \text{for } j = 1/2, \\ A_0 & \text{for } j \neq 1/2, \end{cases}$$

\hspace{2cm} (2.11)

Furthermore, replacing the norm kernel in Eq. (2.9) by the above projection operators, we obtain the coupled-channel equation of the orthogonality condition model, \footnote{Further details on the orthogonality condition model can be found in Ref. 25.}

$$\sum_{j=0} \left[ (T_n + V_F + \lambda A_j)\delta_{ij} + h'_{ij}^{(9Li)} \right] x_j = E x_i,$$

\hspace{2cm} (2.12)

where $h'_{ij} = h_{ij} - \delta_{ij} E_{gr}$ and $E = E - E_{gr}$ as the matrix elements for every configuration of the $^9$Li ground state are measured from the ground state energy. For the strength parameter $\lambda$, we chose a value large (for instance, $\lambda \sim 10^8$ MeV) enough to separate the Pauli-forbidden states into an energy region far from the region under consideration. \footnote{Further details on the choice of $\lambda$ can be found in Ref. 26.}

As $V_F$, we use the following folding-type potential with the central term and the density-derivative spin-orbit term in the manner discussed in Ref. 17:

$$V_F(\vec{r}) = V_c(\vec{r}) + V_0^{ls} f^{ls}(r) \hat{l} \cdot \hat{s}.$$  \hspace{2cm} (2.13)

Here $\hat{l}$ and $\hat{s}$ are the orbital angular momentum and spin operators of the valence neutron, respectively. The radial form $f^{ls}(r)$ of the $l \cdot s$ potential is approximated by the density-derivative form \footnote{Further details on the approximation of $f^{ls}(r)$ can be found in Ref. 19.} of $^9$Li, $f^{ls}(r) = dp(r; ^9Li)/dr$, where the density
distribution $\rho(r; ^9\text{Li})$ is calculated by assuming the neutron closed configuration of
\[\{(0s_{1/2})^4(0p_{1/2})^4(0p_{3/2})_s\}_{I=3/2}\] for the $^9\text{Li}$-core with the size parameter $b_c$:
\[
f^{ls}(r) = -\frac{27}{56\sqrt{2\pi}b_c^3}\exp\left\{-\frac{9}{8}\left(\frac{r}{b_c}\right)^2\right\}\left[1 - \frac{45}{4}\left(\frac{r}{b_c}\right)^2\right].
\]
(2.14)

The central part $V_c(\vec{r})$ of the folding potential is given in operator form as
\[
V_c(\vec{r}) = g^C_c(\vec{r}) + g^T(\vec{r})P^s(\rho n) + \{g^T_c(\vec{r}) + g^T_\sigma(\vec{r})P^s(\rho n)\}[T_2 Y_2(\hat{r})]_0,
\]
where $P^s(\rho n)$ and $P^\sigma(\rho n)$ are the spin and isospin exchange operator, respectively. ($P^s(\rho n)$ in the folding potential $V_c(\vec{r})$ is the spin exchange operator between a valence proton in $^9\text{Li}$ and a neutron outside $^9\text{Li}$.) The quadrupole moment operator $T_2$ of the $^9\text{Li}$-core is defined by
\[
T_2 = \sum_{i=1}^9 \xi_i^2 Y_2(\hat{\xi}_i),
\]
(2.16)

where $\hat{\xi}_i$ ($i = 1, 9$) are the internal coordinates of $^9\text{Li}$. The resultant quadrupole moment of $^9\text{Li}$ results from the single proton in the $p$-shell. The radial dependent terms in $V_c(\vec{r})$ are
\[
g^C_c = \sum_{n=1}^{N_\text{p}} \left(\frac{\eta_n}{2b_c^2}r_n\right)^{3/2} v_n^0 \exp\left(-\frac{\eta_n}{2b_c^2}r_n^2\right)\left\{(9W_n + 4B_n - 6H_n - 3M_n) - \frac{1}{2}(5W_n + 2B_n - 4H_n - 2M_n)\right\},
\]
(2.17a)
\[
g^\sigma_c = \sum_{n=1}^{N_\text{p}} \left(\frac{\eta_n}{2b_c^2}r_n\right)^{3/2} v_n^0 \exp\left(-\frac{\eta_n}{2b_c^2}r_n^2\right) B_n \left(1 - \frac{1}{2}\eta_n + \frac{1}{6b_c^2}\eta_n r_n^2\right),
\]
(2.17b)
\[
g^T_c = \sum_{n=1}^{N_\text{p}} \left(\frac{\eta_n}{2b_c^2}r_n\right)^{3/2} v_n^0 \exp\left(-\frac{\eta_n}{2b_c^2}r_n^2\right) W_n \frac{16\pi}{6\sqrt{5}b_c^2}\eta_n r_n^2,
\]
(2.17c)
\[
g^\sigma_T = \sum_{n=1}^{N_\text{p}} \left(\frac{\eta_n}{2b_c^2}r_n\right)^{3/2} v_n^0 \exp\left(-\frac{\eta_n}{2b_c^2}r_n^2\right) B_n \frac{16\pi}{6\sqrt{5}b_c^2}\eta_n r_n^2,
\]
(2.17d)

where $\eta_n = 18\rho_n b_c^2/(9 + 8\rho_n b_c^2)$ and $(v_n^0, \rho_n, W_n, B_n, H_n, \text{and} M_n)$ are a parameter set of the two-nucleon force
\[
v(\vec{r}_i, \vec{r}_j) = \sum_{n=1}^{N_\text{p}} v_n^0\left[W_n + B_n P^s(\rho n) - H_n P^\sigma(\rho n) - M_n P^\sigma(\rho n)\right] \exp\{-\rho_n(\vec{r}_i - \vec{r}_j)^2\}.
\]
(2.18)

To solve the resonance states of the coupled-channel equations (2.12), we apply the complex scaling method.\(^{29}\)-\(^{31}\) This method starts by applying the transformation
\[
U(\theta); \vec{r} \rightarrow \vec{r}e^{i\theta}
\]
(2.19)

to Eq. (2.12), where $\theta$ is a scaling parameter taking real values satisfying $0 \leq \theta < \pi/4$. According to the ABC-theorem,\(^{29}\) we can obtain the resonance states, whose
resonance energies $E_r$ and widths $\Gamma$ are solved as complex eigenvalues $W = E_r - i\Gamma/2$ of the complex-scaled coupled-channel Schrödinger equation, in addition to bound state solutions corresponding to the negative real eigenvalue.

By the complex scaling method, we can obtain solutions even with large resonance widths which are located in the fourth quadrant of the complex energy plane, but we cannot obtain the so-called anti-bound resonances (virtual resonances) corresponding to the $S$-matrix poles on the negative imaginary momentum axis in the complex momentum plane; the energies $E_a = k_a^2 \hbar^2 / (2\mu)$ are negative, but the momenta are negative imaginary values $k_a = -i\gamma_a$ ($\gamma_a > 0$). As will be discussed later, the $s$-wave neutron resonance is such a case. The characteristic property of the anti-bound resonances is that they become bound states when the attractive potential is increased slightly. In the present work, we identify the $s$-wave anti-bound resonances by using such a characteristic property.

§3. Numerical results of coupled-channel calculations

3.1. Two-channel model

To easily see the Pauli-blocking effect of the $p_{1/2}$-orbital valence neutron, we solve Eqs. (2.10) and (2.12) assuming two channels of the $c0$ and $c1$ configurations.

Equation (2.10) in the asymptotic region gives the following solution for the $^9\text{Li}$ ground state:

$$
\Psi_{as}(^9\text{Li}) = \alpha_0 \Phi(c0) + \alpha_1 \Phi(c1).
$$

By considering the strength of the pairing interaction and the $\ell \cdot s$-splitting between $p_{1/2}$ and $p_{3/2}$, the matrix elements $h'_{ij}(^9\text{Li})$ ($h'_{00}(^9\text{Li}) = 0 \text{ MeV}$) in Eq. (2.12) are taken as

$$
h'_{01}(^9\text{Li}) = h'_{10}(^9\text{Li}) = -G_{\text{pair}} = -5.62 \text{ MeV}, \quad h'_{11}(^9\text{Li}) = \Delta \epsilon = 6.46 \text{ MeV}. \quad (3.2)
$$

(Here, we should note that $G_{\text{pair}}$ includes the statistical factor $\langle \gamma_a \rangle^2$, $J = 0 \text{ } H_{\text{pair}}(\langle \gamma_b \rangle^2, J = 0) = -\sqrt{(2J_a + 1)(2J_b + 1)}G_{\text{pair}}^0$.) These matrix elements cause a strong mixture of the $(0p_{3/2})^2\gamma_{1\gamma} = 0$ $(0p_{1/2})^2\gamma_{1\gamma} = 0$ configuration; $\alpha_0^2 = 75\%$ and $\alpha_1^2 = 25\%$.

Since the Cohen-Kurath shell model calculation of $(0p_{3/2}, 0p_{1/2})^5$ for $^9\text{Li}$ indicates $G_{\text{pair}} \approx 5 \text{ MeV}$ and a smaller value of $\Delta \epsilon$, the admixture due to the pairing interaction is larger ($27\%–35\%$). Therefore, the pairing correlation may appear in a moderate manner in the present case. The energy gain due to the present pairing correlation is calculated to be $E_{as} - h'_{00} = -3.25 \text{ MeV}$, which is appreciably large in view of the binding ability for the neutron resonance in $^{10}\text{Li}$.

For calculations of the $^{9}\text{Li}-n$ folding potential $V_{fr}$, we use the Hasegawa-Nagata (HN) no. 1 and no. 2$^{28}$ and the modified Hasegawa-Nagata (MHN)$^{18}$ forces, where the strength $v_0^2$ of the second range term in these forces is tuned to the $1^+\text{MeV}$ of $^{10}\text{Li}$ by multiplying by $(1 + \delta)$. The strength of the spin-orbit potential is fixed, as discussed in Ref. 17, so that it reproduces the spin-orbit energy difference 3.23 MeV of $(p_{3/2})^4(p_{1/2})$ and $(p_{3/2})^3(p_{1/2})^2$ states in $^{11}\text{Be}$. For the harmonic oscillator size parameter of the $^9\text{Li}$ core, we use $b_c = 1.69 \text{ fm}$, as discussed in Ref. 19).
Dynamical Effects on the $^9\text{Li}$-n Interaction

Table I. Results of resonance energies and widths calculated using the coupled-channel model in comparison with those of the $[^9\text{Li}]_{c1} + n$ single-channel model$^{17}$ and experimental observations.$^{16}$ (a) and (b) are results using HN no. 1 and MHN effective nuclear forces. The dashes which appear in the entries for the $s_{1/2}$-neutron orbit indicate that in these cases resonance solutions cannot be obtained in the present calculation.

(a) HN no. 1

<table>
<thead>
<tr>
<th>State</th>
<th>Exp.</th>
<th>Coupled-Channel Model $\delta = 0.245$</th>
<th>Single-Channel Model $\delta = 0.0288$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$E_r$ MeV $\Gamma$ MeV</td>
<td>$E_r$ MeV $\Gamma$ MeV</td>
</tr>
<tr>
<td>$p_{1/2}$</td>
<td>1$^+$</td>
<td>0.42 0.15</td>
<td>0.42 0.19</td>
</tr>
<tr>
<td></td>
<td>2$^+$</td>
<td>0.80 0.30</td>
<td>0.71 0.40</td>
</tr>
<tr>
<td>$s_{1/2}$</td>
<td>1$^-$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>2$^-$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>$d_{5/2}$</td>
<td>1$^-$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>2$^-$</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>3$^-$</td>
<td>---</td>
<td>---</td>
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<tr>
<td></td>
<td>4$^-$</td>
<td>4.47 0.7</td>
<td>4.13 3.12</td>
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</tbody>
</table>

(b) MHN

<table>
<thead>
<tr>
<th>State</th>
<th>Exp.</th>
<th>Coupled-Channel Model $\delta = 0.157$</th>
<th>Single-Channel Model $\delta = 0.0442$</th>
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</thead>
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<tr>
<td></td>
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<td>$E_r$ MeV $\Gamma$ MeV</td>
<td>$E_r$ MeV $\Gamma$ MeV</td>
</tr>
<tr>
<td>$p_{1/2}$</td>
<td>1$^+$</td>
<td>0.42 0.15</td>
<td>0.420 0.166</td>
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<tr>
<td></td>
<td>2$^+$</td>
<td>0.80 0.30</td>
<td>1.31 0.822</td>
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<tr>
<td>$s_{1/2}$</td>
<td>1$^-$</td>
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</tr>
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<td>2$^-$</td>
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<td>$d_{5/2}$</td>
<td>1$^-$</td>
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<tr>
<td></td>
<td>4$^-$</td>
<td>4.47 0.7</td>
<td>5.40 3.75</td>
</tr>
</tbody>
</table>

The results obtained by solving the coupled-channel orthogonality condition model (Eq. (2.12)) are given for the NH no. 1 and MHN forces in Tables I(a) and (b), respectively. They are compared with those of the $[^9\text{Li}]_{c1} + n$ single-channel calculation and also the experimental result of Bohlen et al. As for the results of HN no. 2, we only report here that they are very similar to those for the MHN force. The results for the energy spectra are exhibited in Fig. 1 for the HN no. 1 force case, which are also compared with the experimental results,$^{16}$ and the single-channel calculation result of the MHN force.

We obtain the 2$^+$ state as a partner of the 1$^+$ state with the same $p_{1/2}$-orbital neutron at an energy slightly higher than that of the 1$^+$ state. This result is consistent with the observation of Bohlen et al.$^{16}$ In the single-channel model, the MHN force fully explains the energy difference between the 1$^+$ and 2$^+$ states. However, when we treat the $p_{1/2}$ neutron in the coupled-channel model, there arises a kind of strong repulsive effect on these 1$^+$ and 2$^+$ states due to Pauli blocking for the c1 channel state, because the valence neutron in the forbidden $p_{1/2}$-orbit breaks the pairing correlation in the $^9\text{Li}$ core and provides a large energy loss equal to the pairing energy (~3.25 MeV) estimated above. Therefore, to keep the energy of the
Fig. 1. Levels of $^{10}\text{Li}$. The experimental results are taken from 16). The coupled-channel results for HN no. 1 and the single-channel results for MHN are shown. The hatched lines indicate widths of resonance states.

$1^+$ state at 0.42 MeV, we must make the folding potential more attractive so as to recover the energy loss caused by blocking of the pairing correlation. This is done by increasing the values of $\delta$. As seen in Table I, the strength of the second range in the effective nuclear forces is increased only by 3–4% in the single-channel model, but contrastingly by 15–25% in the coupled-channel model. These large enhancements of $\delta$ in the coupled-channel model imply that the repulsive effects on the $p_{1/2}$ neutron is very strong. The HN no. 1 and MHN forces strengthened in the coupled-channel model cause a large energy difference between the $1^+$ and $2^+$ states, which results from the spin coupling of the valence neutron with a $p_{3/2}$ proton in the $^9\text{Li}$ core. The MHN force has an odd component, while the HN no. 1 force does not. Therefore, the energy difference between the $1^+$ and $2^+$ states contributed from the odd force term in the MHN case is added to that in the HN no. 1 case. This $1^+$ and $2^+$ energy difference in the single-channel model with the MHN force is in good agreement with experiment, but the coupled-channel calculation gives a difference which is too large. The main reason for this can be understood from the fact that the $^9\text{Li}-n$ folding potential in the coupled-channel model is strengthened much more than in the single-channel case. In the coupled-channel calculation, it is found that
Dynamical Effects on the $^9$Li-n Interaction

Fig. 2. The $\delta$-parameter dependence of the positive- and negative-parity neutron orbits obtained in the coupled-channel model of HN no. 1. For the $p_{1/2}$-orbital neutron including the coupling with the $f_{5/2}$-orbit, the binding and resonance energy (open circle) and the width (dashed lines) of the $1^+$ state are shown. For the $s_{1/2}$-orbital neutron, the binding energies of $2^-$ (black circles) and $1^-$ (black triangles) states are shown, where the calculated results are obtained by including the coupling with the $d_{5/2}$ ($2^-$) and $d_{3/2}$-orbits ($1^-$) in the two-channel model for the $^9$Li core.

the energy difference between the $1^+$ and $2^+$ states is satisfactorily explained by the HN no. 1 force, although the MHN force predicts a comparatively large difference. (We have not obtained a clear explanation for these results.)

Next, we discuss the calculated results of the resonance width. Our remark here is that the coupled-channel model reduces the decay widths of the $1^+$ and $2^+$ states in comparison with the single-channel results, and thus the resultant decay widths agree nicely with the experimental results, as mentioned below. In the single-channel model, both the HN no. 1 and MHN forces predict decay widths that are about twice as large as those observed in experiments. In a previous paper, $^{17}$ we argued that these decay widths in the single-channel calculation would be improved by taking into account the activation of the degree of freedom in the $^9$Li core. The present coupled-channel calculation confirms this speculation. As shown in Table I, the decay width of the $1^+$ state is calculated to be 0.19 MeV and 0.166 MeV in the coupled-channel model for the HN no. 1 and the MHN force cases, respectively. The experimental value is 0.15 MeV. As for the $2^+$ state, the HN no. 1 force calculation is in satisfactory agreement with the observed resonance energy and also the decay width. The MHN force calculation predicts a large width of the $2^+$ state due to the resonance energy which is higher than that seen in experiments.

In the single-channel model, it has been shown that the $s$-orbital valence neutron has no bound and no resonance states. However, in the coupled-channel model using the HN no. 1 force, as shown in Fig. 1, we obtain the bound $2^-$ state which is one of the doublet states with $s_{1/2}$ (valence neutron) $\otimes 3/2^-$ ($^9$Li) just below the threshold. A doublet partner, $1^-$, is also expected to exist around the threshold. In fact, when we strengthen the $^9$Li-n interaction by increasing $\delta$, we can find a bound state solution of $1^-$, as shown in Fig. 2. Therefore, it is confirmed that in the case of HN no. 1 the $s_{1/2}$-orbital neutron is obtained as a barely bound state. On the other hand, we obtain no
bound states in the case of the MHN force whose \( \delta \) parameter has been fitted so as to reproduce the \( 1^+ \) energy at 0.42 MeV. As in the case of \( 1^- \) with the HN no. 1 force, a bound \( 2^- \) solution appears also in the MHN force case when we make the folding potential more attractive by changing \( \delta \) from 0.157 to 0.162. These results indicate that the \( s \)-wave neutron has solutions near the threshold independently of the details of the effective interaction as long as the \( ^9\text{Li}-n \) potential is adjusted to obtain the resonance energy of the lowest \( 1^+ \) state. Furthermore, the present coupled-channel results indicate that the \( ^9\text{Li}-n \) potential obtained by treating the pairing correlation in \( ^9\text{Li} \) is strong enough to bring about the \( s \)-orbital states near the threshold as virtual resonances or barely bound states.

As shown in Fig. 1, the \( d \)-orbital neutron resonance states with total spins \( J^\pi = 1^- , 2^- , 3^- \) and \( 4^- \) are obtained in the energy region higher than 4 MeV with respect to the threshold. The dominant component of these states is the \( d_{5/2} \)-neutron orbital configuration coupled with the \( 3/2^- \) ground state of \( ^9\text{Li} \). We clearly see that the resonance energies come down by about 3 MeV in comparison with the single-channel ones. This is caused by the increase of the strength of the \( ^9\text{Li}-n \) folding potential in the coupled-channel model and also by the fact that there are no Pauli-blocking effects on the \( d \)-orbital neutron. In the \( p_{1/2} \)-orbital case, the increase of the potential strength compensates the repulsive effects due to Pauli blocking, and resultantly the coupled-channel model reproduces \( 1^+ \) and \( 2^+ \) states at resonance energies similar to those of the single-channel model. In contrast to this, in the \( d \)-orbital case, there remains an increase in the potential strength because there are no Pauli-blocking repulsive effects on the \( d \)-orbital neutron. In other words, the \( d \)-wave resonances do not strongly disturb the pairing correlation, and then their resonance energies result in a decrease by about 3 MeV, which just corresponds to the energy gain of the pairing correlation in \( ^9\text{Li} \). Since the resonance energies (4–6 MeV) of the coupled-channel model become smaller than the single-channel model results (7–9 MeV), the resonance widths are obtained to be about half in comparison with the single-channel case. In a previous paper,\(^{17}\) we argued that the \( d \)-wave solutions of the single-channel model do not explain the resonance observed at 4.47 MeV.\(^{16}\) The present coupled-channel results suggest, as seen from Fig. 1, the \( d \)-wave resonances as a candidate for the observed resonance, though the calculated width is larger than that seen in experiments.

Finally, we study the dependence of the results on the choice of matrix elements \( h'_{ij}(^9\text{Li}) \). As mentioned above, the matrix elements \( h'_{ij}(^9\text{Li}) \) determine the properties of the ground state of \( ^9\text{Li} \). For the present study on the Pauli-blocking effect of the neutron pairing correlation in the \( ^9\text{Li} \) core of the \( ^9\text{Li} + n \) system, the most important factors are the correlation energy of the \( ^9\text{Li} \) ground state and the mixing probability due to the pairing interaction, which are expressed by \( E_{gr} \) and \( \alpha_1^2 \), respectively, in the present model. When we change the pairing strength \( |G_{pair}| \) by fixing the correlation energy \( E_{gr} \), we obtain the \( 1^+ \) resonance state of \( ^{10}\text{Li} \), whose energy and width exhibits a weak dependence on the pairing strength, as shown in Fig. 3. This result indicates that the Pauli-blocking effect on the \( ^9\text{Li}-n \) interaction causing the \( 1^+ \) resonance depends dominantly on the pairing correlation energy of the \( ^9\text{Li} \) core. On
and\( \rho \{0p_{3/2}\} \) in the ground state wave function of \( ^9\text{Li} \) strongly influences the resonance width of the \( 1^+ \) state. From Fig. 3, we see that the width reduces as the mixing amplitude increases. This property of the decay width can be understood from the fact that the Pauli-blocking effect gives a smaller overlap of the \( ^9\text{Li} \) core wave functions at the interaction distance and the asymptotic distance between \( ^9\text{Li} \) and the neutron because of the decrease of the \( \{0p_{3/2}\}^2 \{0p_{1/2}\} \) amplitude in the interaction region of \( ^9\text{Li} \) and the neutron.

3.2. Pauli-blocking effects of the \( (1s_{1/2})^2 \)-pairing amplitude on \( s_{1/2}-\)orbital neutron states

Here, we investigate the Pauli-blocking effects on the \( s_{1/2} \)-neutron states in more detail. Within the two-channel approximation, the presence of the \( s_{1/2} \)-orbital neutron does not disturb the pairing correlation in the \( ^9\text{Li} \) ground state, since the ground state is described as the superposition of the two configurations \( \{0p_{3/2}\}_\nu \{0p_{3/2}\}_{\nu',J_1=0} \) and \( \{0p_{3/2}\}_\nu \{0p_{3/2}\}_{\nu',J_1=0} \{0p_{1/2}\}_{\nu',J_2=0} \}. \) However, if the pairing component of \( \{0p_{3/2}\}_\nu \{0p_{3/2}\}_{\nu',J_1=0} \{1s_{1/2}\}_{\nu',J_2=0} \) were mixed in the \( ^9\text{Li} \)-core ground state, the Pauli-blocking arguments are needed for study of the \( s_{1/2} \)-orbital resonance. This should be checked more carefully through calculations of the coupled-channel model, including the component of the \( (1s_{1/2})^2 \)-pairing configuration. Therefore, we express the \( ^9\text{Li} \) core in \(^{10}\text{Li}\) by three components as

\[
\Psi_{\text{gr}}(\text{Li}) = \alpha_0 \Phi(c0) + \alpha_1 \Phi(c1) + \alpha_2 \Phi(c2),
\]

(3-3)

where \( \Phi(c0) \), \( \Phi(c1) \) and \( \Phi(c2) \) are given in Eq. (2-2). Then the \(^{10}\text{Li}\) nuclear states are described by the three corresponding channels. The \(^9\text{Li}\) ground state of Eq. (3-3) is obtained by solving the asymptotic equation of Eq. (2-10). Here, we take the following intrinsic Hamiltonian matrix of \(^9\text{Li}\):
\[
\left( h'_{ij}(^9\text{Li}) \right) = \begin{pmatrix}
0 & -G_{\text{pair}} & -G_{\text{pair}}/2 \\
\Delta \epsilon & 0 & -G_{\text{pair}}/2 \sqrt{2}
\end{pmatrix}, \tag{3.4}
\]

where \(-G_{\text{pair}} = 6.9 \text{ MeV}, \Delta \epsilon = 6.46 \text{ MeV} \) and \(\Delta \epsilon' = 6.5 \text{ MeV} \) are assumed. These matrix elements reproduce the \(^9\text{Li} \) ground state energy \(E_{\text{gr}} - h'_{00} = -4.64 \text{ MeV} \) and the wave function with

\[
\alpha_0^2 = 72.9\%, \quad \alpha_1^2 = 24.7\% \quad \text{and} \quad \alpha_2^2 = 2.4\%. \tag{3.5}
\]

Here, we chose a slightly larger pairing-interaction strength \(|G_{\text{pair}}| \) in order to keep a \((0p_{1/2})^2\)-amplitude \((\alpha_1) \) of \(\sim 25\% \), similar to that obtained in the two-channel case. The matrix elements \(h'_{02} \) and \(h'_{12} \) between \((1s_1/2)^2\) of the halo neutrons and \((0p_{3/2,1/2})^2\) of the core neutrons are assumed to reduce to approximately half of the ordinal matrix elements between the core neutrons. The unperturbed energy \(\Delta \epsilon \) corresponding to the \(\ell \cdot s \) splitting is taken to be the same as the two-channel case. For \(\Delta \epsilon' \), we have assumed that the unperturbed \(s\)-orbital energy is very similar to the \(p_{1/2}\)-orbital energy.

The result obtained by using the HN no. 1 force with \(\delta = 0.2415 \) is shown in Table II. We can see that the three-channel calculation gives a result which is very similar to that of the two-channel model calculation without the \((1s_1/2)^2\) configuration. The value of \(\delta = 0.245 \) is close to \(\delta = 0.245 \) for the two-channel case. This result indicates that the repulsive effect of Pauli blocking does not change significantly in the \(p_{1/2}\)-orbital solutions (1\(^+ \) and 2\(^+ \)). On the other hand, we can find no solutions of the \(s_{1/2}\)-orbital valence neutron in the present three-channel calculation. This result can be understood from the repulsive effect of the \(s_{1/2}\)-orbital valence neutron due to Pauli blocking on the \((1s_1/2)^2\)-pairing component in \(^9\text{Li} \). In fact, when we increase the \(\delta \) value to adjust the strength of the folding potential, we obtain the 2\(^- \) bound solution at \(\delta = 0.25 \), as shown in Fig. 4, whereas the 2\(^- \) bound state appears at \(\delta = 0.23 \) in Fig. 2 of the two-channel calculation result. This means that the repulsive effect on the \(s_{1/2}\)-orbital neutron due to Pauli blocking is compensated by increasing \(\delta \) from 0.23 to 0.25. However, this increase is very small in comparison with the case of \(p_{1/2}\)-orbital solutions, as discussed above.

Therefore, we can conclude that the Pauli-blocking effect on the state of the \(s\)-orbital neutron is much smaller than that in the \(p\)-orbital neutron. This small Pauli-blocking effect on the \(s\)-orbital neutron is understood from the strikingly spatially-
extended property of the \( s_{1/2} \)-neutron wave function and the small overlap between such a valence neutron wave function and the rather compact wave functions of the \( ^9\text{Li} \)-core neutrons.

§4. \(^9\text{Li}-n\) interactions for \( p-, s- \) and \( d-\)waves

In this section, we discuss the \(^9\text{Li}-n\) interaction in the coupled-channel model, where the Pauli-blocking effects on the valence neutron orbit due to the presence of the \( J^\pi = 0^+ \) neutron pairing correlation in the \(^9\text{Li} \) core are properly taken into account. As mentioned in the previous section, the present coupled-channel model reproduces the \( p-\)wave resonances observed in the low energy region of 0.4–0.8 MeV, and the \( s-\)wave coupled-channel solutions are obtained near the threshold as barely-bound or anti-bound states (i.e. virtual resonances). The \( d-\)wave solutions under the same conditions are obtained in the energy region of 4–6 MeV, lower than the single-channel results by \( \sim 3 \) MeV. Although the effects of the Pauli blocking on the \(^9\text{Li}-n\) interaction have been explained in the previous sections, here we demonstrate them more explicitly in the potential form for \( p-, s- \) and \( d-\)wave neutrons.

The lowering of the \( s- \) and \( d-\)wave solutions in the coupled-channel model implies that the effective \(^9\text{Li}-n\) potential for \( s- \) and \( d-\)waves is more attractive in comparison with the effective potential for the \( p_{1/2} \)-wave neutron. As explained above, Pauli blocking ascribed to the appearance of the \( p_{1/2} \)-orbital neutron breaks the pairing correlation in the \(^9\text{Li} \) core in the interaction region of the \(^{10}\text{Li} \) (= \(^9\text{Li} \) + \( n \) ) system and makes the internal energy increase. Since the energy gain of the \(^9\text{Li} \) ground state due to the pairing correlation is estimated to be about 3 MeV, the energy loss of the pairing correlation due to Pauli blocking is thought to be the almost same amount. We attempt to express the effects of Pauli blocking as a sort (\( \Delta V_{\text{Pauli}} \)) of
repulsive potentials.

To estimate this repulsive potential, we perform the coupled-channel calculations for the \(p\)-wave resonances in cases with and without Pauli blocking, which have been performed by solving Eq. (2.12) with and without the additional projection operator of \(|0p_{1/2}\rangle\langle0p_{1/2}|\), respectively. Of course, the former calculation was done in previous sections. The latter solutions in cases without Pauli blocking have been found to be the same as those found in our previous paper,\(^{17}\) because the folding potential for the \(^9\text{Li}\)-core configuration of \((0p_{3/2})\pi(0p_{3/2})^4\nu,J_1=0\) is the same as that for the pairing configuration \((0p_{3/2})\pi(0p_{3/2})^2\nu,J_1=0(0p_{1/2})^2\nu,J_2=0\).

The two calculations have been done so as to fit the same experimental data of the \(p_{1/2}\)-wave resonance with \(J^z=1^+\) at 0.42 MeV by adjusting the folding potential through changing the parameter \(\delta\). In the case without Pauli blocking, the effective potential is nothing but the folding potential \(V_S\) of the single-channel model. In a similar way, we adjust \(V_S\) for the \(s\) - and \(d\)-orbital neutrons so as to reproduce the energies of the \(s\) - and \(d\)-orbital \(2^-\) states, respectively, obtained by the coupled-channel calculation with Pauli blocking. Therefore, the effective \(^9\text{Li}-n\) potential in the case without Pauli blocking is obtained state-dependently as \(V_p^p\), \(V_S^s\) and \(V_d^d\). On the other hand, in the case with Pauli blocking, the effective \(^9\text{Li}-n\) potential \(V_{\text{eff}}\) consists of two components of the folding potential \(V_C\) of the coupled-channel model.

Fig. 5. (a) Effective \(^9\text{Li}-n\) potentials for the \(p_{1/2}^-\), \(s_{1/2}^-\) and \(d\)-orbital neutrons in the three-channel model. \(\Delta V_{\text{Pauli}}^p\) and \(\Delta V_{\text{Pauli}}^s\) are the repulsive potentials due to the Pauli-blocking effect on \(p_{1/2}^-\) and \(s_{1/2}^-\)-orbital neutrons, respectively. (b) The dependence of the Pauli-blocking repulsive potential \(\Delta V_{\text{Pauli}}\) on the pairing correlation energy of \(^9\text{Li}\). The effective \(^9\text{Li}-n\) potential was obtained by the \(E_{\text{gr}}\)-fixed two-channel calculation for the \(1^+\) state.
and the Pauli-blocking repulsive one, $\Delta V_{\text{Pauli}}$; that is, $V_{\text{eff}} = V_C + \Delta V_{\text{Pauli}}$. In the sense that the two cases reproduce the same energy of the corresponding solutions, we regard the effective potentials to be approximately equal: $V_S \approx V_{\text{eff}}$. Then, we obtain the Pauli-blocking repulsive potential by using the following relation:

$$\Delta V_{\text{Pauli}} \approx V_S - V_C. \quad (4.1)$$

In Fig. 5, we display the obtained $\Delta V_{\text{Pauli}}$ and the effective $^9\text{Li}-n$ potentials $V_{\text{eff}}$ for the $p_{1/2}$-, $s_{1/2}$- and $d$-wave neutrons. We see that the Pauli-blocking repulsive potential has a very large strength in the interior region between $^9\text{Li}$ and the $p_{1/2}$-wave neutron, but a very small and negligible contribution to the $s$- and $d$-wave neutrons, respectively. This result is consistent with the discussion regarding the Pauli-blocking effects in the previous sections. Therefore, we can summarize the effective potential for the low-lying resonances in $^{10}\text{Li} (= ^9\text{Li} + n)$ as

$$V_{\text{eff}}^p \approx V_C + \Delta V_{\text{Pauli}}^p \quad \text{for } p\text{-orbit}, \quad (4.2)$$

$$V_{\text{eff}}^{s,d} \approx V_C \quad \text{for } s\text{- and } d\text{-orbits}. \quad (4.3)$$

This is the new view extracted from the coupled-channel model studies; that is, the $p_{1/2}$-orbital resonances are pushed up due to the Pauli-blocking repulsive effects by an amount equal to the energy loss due to the breaking of the pairing correlation. Therefore, for the case of larger correlation energy, a larger energy loss results and a stronger repulsive effect is produced. We can see this situation from Fig. 5 (b), where the pairing correlation energy dependence of the Pauli-blocking repulsive potentials $\Delta V_{\text{Pauli}}$ is shown for the $1^+$ states calculated by the two-channel model with fixed energies $E_{\text{gr}} = -2.25$, $-3.25$ and $-4.25$ MeV of the $^9\text{Li}$ ground state. These just correspond to the pairing correlation energies.

As for the lowering of $s$- and $d$-waves, many phenomenological discussions have been made. In such studies, the parity-dependent $^9\text{Li}-n$ interaction is assumed to be a more attractive for $s$-wave (positive-parity) states than for $p$-wave (negative-parity) states. Aoyama et al.,\cite{32,33} including two of the present authors (Katô and Ikeda), discussed such parity-dependent potentials for $^9\text{Li}-n$ and its mirror $^9\text{C}$-$p$, and also $^{10}\text{Be}$-$n$ and $^{10}\text{C}$-$p$. For the former, the parity-dependent $^9\text{Li}-n$ potential yields results for $p$- and $s$-wave states in $^{10}\text{Li}$ very similar to those of the present microscopic coupled-channel model. It was also shown\cite{32} that the anti-bound $s$-wave state in the $^9\text{Li}-n$ potential appears as a resonance in the $^9\text{C}$-$p$ state due to the additional Coulomb potential barrier. These discussions indicate that the property of the $^9\text{Li}-n$ interaction can also be provided from an experimental study of the $^{10}\text{C}$ nucleus. The present study gives a theoretical background basis for such a parity dependence in the $^9\text{Li}$-$n$ interaction, as shown by Eq. (4.2).

On the other hand, the inversion problem of $p$- and $s$-orbits in the Be-isotope region has been studied from the point of view that the $s$-$d$ coupling with the deformation of the core nucleus plays the most important role for parity inversion.\cite{21,34} However, the deformation in $^9\text{Li}$ is thought (even if existing) to be very small, as seen in the experimental quadrupole moment. In the present study, the $s$-$d$ coupling term due to the quadrupole moment of the valence proton in $^9\text{Li}$ is involved in the folding
potential, as shown by Eq. (2.15). We have treated these couplings throughout the calculations of the present paper. Here, it is enough to say that the contributions of the quadrupole tensor coupling to the energies of the neutron s-orbital states are negligibly small, for example, 0.01 MeV for the lowest 2− state. This quadrupole tensor term brings about the p-f coupling for 1− and 2− states. The contributions to the energy are estimated to be again the order of 0.01 MeV.

§5. Summary and conclusion

We here discussed the problem why the positive-parity orbital states appear in the low energy region where the low-lying negative parity orbital resonance states exist in 10Li. A new idea for solving this problem was proposed on the basis of the Pauli-blocking effects on the $J^\pi=0^+$ pairing correlation in the 9Li core nucleus. To this time, in many theoretical studies of 10Li and 11Li employing the microscopic models of 9Li + n and 9Li + n + n, the ground state of the 9Li core nucleus has been assumed to be described by a simple $(0p_{3/2})^4_s$-closed shell configuration. Although this simple assumption is attractive for practical studies of these microscopic few-body problems, we should note that such studies inevitably suffer the strong restriction that the important $J^\pi=0^+$ pairing correlation due to the residual interaction is disregarded. In the present paper, we withdraw this restriction and treat properly not only the pairing interaction effects but also the Pauli-blocking effects on the 9Li-n interaction by adopting the microscopic frame of the coupled-channel orthogonality condition model developed in §2.

Our coupled-channel formalism gives the asymptotic wave function of 9Li as a superposition of $\{(0s)^4(0p_{3/2})^1_s(0p_{3/2})^1_{s,J=0}\} + \{(0s)^4(0p_{3/2})^1_s(0p_{3/2})^2_{s,J=0}(0p_{1/2})^2_{p,J=0}\} + \{(0s)^4(0p_{3/2})^1_s(0p_{3/2})^2_{s,J=0}(1s_{1/2})^2_{s,J=0}\} + \cdots$, by which the $J^\pi=0^+$ pairing correlation is taken into account properly. The mixing amplitude of these configurations in the asymptotic region is determined so as to be the same as the isolated 9Li nucleus mixing amplitudes. In this paper, we parameterized the mixing amplitudes of the $(0p_{1/2})^2_p$ and $(1s_{1/2})^2_p$ occupied configurations which are coupled with the dominant $(0p_{3/2})^4_s$ configuration, referring to the results of the Cohen-Kurath shell model calculations for 9Li. First, assuming the two-channel model for 9Li + n, we studied the spectroscopy of 10Li. The results reproduce the p-orbital neutron resonances (1+ and 2+) in the low energy region of 0.4–0.8 MeV corresponding to the observation of Bohlen et al.,\(^9\) and the neutron s-wave solutions near the threshold as virtual resonances or barely-bound states. Furthermore, the d-wave resonance states are obtained in the energy region of 4–6 MeV, lower than the single-channel results by $\sim 3$ MeV. These results do not change essentially even if we take into account many other pairing configurations in addition to the $(0p_{3/2})^4_s$ plus $(0p_{3/2})^2_s(0p_{1/2})^2_p$ of the two-channel model for the 9Li core. We checked this by performing the three-channel calculation including the $(0p_{3/2})^2_s(1s_{1/2})^2_p$ configuration.

In comparison with the single-channel model without consideration of the pairing correlation in the 9Li core, the coupled-channel calculations show that the energies of s- and d-wave neutron states decrease relative to the p-wave states. We argued
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that this lowering of the positive-parity orbital states of the valence neutron in $^{10}\text{Li}$ can be ascribed to the Pauli-blocking effects on the $(0p_{1/2})^2_0$ pairing configuration in the $^9\text{Li}$-core wave function. The pairing correlation in $^9\text{Li}$ strongly mixes the $(0p_{3/2})^2_{j_1=0}(0p_{1/2})^2_{j_2=0}$ configuration with the neutron sub-closed $(0p_{3/2})^4$ configuration. Such a pairing correlation experiences Pauli blocking by the appearance of the $p_{1/2}$-orbital neutron in the $^9\text{Li}+n$ system, whereas the neutron of other orbits such as $1s_{1/2}$ and $0d_{5/2}$ experiences very weak or negligible repulsive effects. Through this discussion, we can conclude that the Pauli-blocking effect on the $p_{1/2}$-orbital valence neutron is caused by the loss of the pairing correlation energy in the $^9\text{Li}$ ground state.

Such effects of Pauli blocking can also be seen in the calculational results of resonance widths. The coupled-channel model improves the decay widths of the $p_{1/2}$-orbital neutron $1^+$ and $2^+$ states calculated in the single-channel model, and the resultant decay widths fully explain the experimental data.\footnote{16} Although the $d$-wave resonances calculated in the coupled-channel model still have large decay widths in comparison with experiments, the $4^-$ state seems to correspond to the observed resonance at 4.47 MeV.\footnote{16} These results indicate that the present coupled-channel model is highly reliable for description of the pairing correlation in the $^9\text{Li}$ core and the Pauli-blocking effect in the $^{10}\text{Li}$ (= $^9\text{Li}+n$) system.

Furthermore, we have investigated such effects of Pauli blocking on the pairing correlation in the $^9\text{Li}$ core in the form of the $^9\text{Li}$-$n$ potential. The obtained effective $^9\text{Li}$-$n$ potential has a strong state dependence which is caused by the state-dependent repulsive property due to Pauli blocking for the pairing correlation. These repulsive potential terms in the effective $^9\text{Li}$-$n$ interaction caused by the Pauli-blocking effect raise the energies of the $p$-wave states around the energy of the $s$-orbital states, which comes down due to the extremely small binding energy. Although, in order to explain the parity-inversion phenomenologically, the parity-dependent $^9\text{Li}$-$n$ potential has been used for $^9\text{Li}+n$\footnote{32,33} and $^9\text{Li}+n+n$\footnote{35} models, it is very interesting to study the $^9\text{Li}+n+n$ system by using the present $^9\text{Li}$-$n$ potential which has the strong parity-dependence caused by the Pauli-blocking effects on the pairing correlation in the $^9\text{Li}$ core.

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