The effective potential in the MSSM at the one-loop level is used to evaluate masses of the neutral Higgs scalars and to study a finite-temperature phase transition. $CP$ violation in the Higgs sector, which is induced by a spontaneous mechanism or by the complex parameters in the MSSM through radiative corrections, is determined at zero temperature and finite temperatures.

§1. Introduction

The scenario of electroweak baryogenesis requires the electroweak phase transition (EWPT) to be of first order and $CP$ violation to be effective at that transition temperature. The EWPT is of first order only for the case in which Higgs bosons are too light in the minimal standard model, and it is argued that $CP$ violation in the CKM matrix cannot generate sufficient baryon asymmetry. Both requirements, however, will be met by some extension of the standard model. The minimal supersymmetric standard model (MSSM) is one promising candidate. When some of the scalar partners of the quarks and leptons are sufficiently light, the EWPT becomes so strongly first order that the sphaleron process decouples just after it with acceptable mass of the lightest Higgs scalar. Although the masses of Higgs bosons in the MSSM are constrained by some tree-level relations, they receive large radiative corrections from the top quark and squarks. This may broaden the window for successful baryogenesis within the MSSM.

The MSSM has many sources of $CP$ violation, in addition to the KM phase, such as the relative phases of the complex parameters $\mu$, the gaugino mass parameters and scalar trilinear couplings, which are effective in generating baryon asymmetry. In addition to these complex phases, the relative phase of the expectation values of the two Higgs doublets could be the source of baryon number. This phase might be induced by radiative and finite-temperature effects near the transition temperature or it may be dynamically generated near the bubble wall created at the EWPT, which we call transitional $CP$ violation. This mechanism has been examined dynamically by solving equations of motion for the classical Higgs fields connecting the broken and symmetric phases. Then the potential for the fields is given by the effective potential at the transition temperature, which is approximated by a gauge-invariant polynomial whose coefficients are given by the effective parameters at that temperature. It was shown that the contributions from charginos and neutralinos are important to trigger $CP$ violation in the intermediate region. These analyses

* E-mail: funakubo@cc.saga-u.ac.jp
contain undetermined parameters such as the transition temperature, thickness and velocity of the bubble wall, expectation values of the Higgs fields and the magnitude of explicit $CP$ violation at the transition temperature. These quantities should be determined by the parameters in the MSSM.

One of our main concerns presently is to determine the EWPT in the MSSM is sufficiently strongly first order for the sphaleron process to decouple after its appearance, with acceptable Higgs boson masses. Recent studies of the two-loop resummed effective potential at finite temperature suggest that the EWPT is strong enough for $2 \leq \tan \beta \leq 4$, $m_A \gtrsim 120 \text{ GeV}$ and a light Higgs with $m_h \lesssim 85 \text{ GeV}$. This and the previous analyses based on the one-loop resummed potential did not include the contributions from the charginos and neutralinos. For parameters which result in a strongly first-order EWPT, we must examine whether efficient $CP$ violation exists at the transition temperature. The effective potentials used in previous studies of the phase transition are functions of only the $CP$-conserving order parameters, so that they could not allow for evaluation of $CP$ violation in the Higgs sector at the EWPT.

In this paper, we use the effective potential in the MSSM, which includes one-loop corrections from the top quark, top squarks, gauge bosons, charginos and neutralinos, to evaluate the masses of the neutral Higgs bosons and to examine the strength of the EWPT. The masses are approximated by the eigenvalues of the matrix whose elements are given by the second derivatives of the effective potential evaluated at the vacuum at zero temperature. Although the mass formulas for the neutral Higgs bosons have been found through fully-contained one-loop calculations and two-loop calculations, we adopt this method to make the calculations self-contained. Without $CP$ violation, the minimum of the effective potential is parameterized by the absolute value and the ratio of the VEVs of the two Higgs doublets. Extending the effective potential to include the $CP$-violating order parameter and employing a numerical method to search for a minimum of it, we find the magnitude of the induced $CP$ violation through radiative corrections from the superparticles when some of the parameters are complex-valued. To find the transition temperature $T_C$ and the magnitude of the VEVs of the Higgs fields at $T_C$, we numerically calculate the effective potential without use of the high-temperature expansion, and use the same numerical method as in the zero-temperature case to search for a minimum of the effective potential. $CP$ violation in the Higgs sector provides a boundary condition for the equations which dynamically determines the profile of the bubble wall.

This paper is organized as follows. In §2, we derive formulas for the neutral Higgs boson masses in the MSSM in the absence of $CP$ violation, and a numerical method is introduced which can be applied to the case with $CP$ violation. The method is extended to the effective potential at finite temperature in §3. Numerical results regarding the masses of the Higgs scalars and transition temperature and strength of the EWPT are summarized in §4. Section 5 is dedicated to concluding remarks. The formulas for the derivatives of the effective potential are summarized in the Appendix.
§2. Higgs boson masses

If we write the VEVs of the two Higgs doublets as $\varphi_d$ and $\varphi_u$, the tree-level potential is given by

$$V_0 = m_1^2 \varphi_d^\dagger \varphi_d + m_2^2 \varphi_u^\dagger \varphi_u + (\mu B \varphi_u \varphi_d + \text{h.c.}) + \frac{g_2^2 + g_1^2}{8} (\varphi_d^\dagger \varphi_d - \varphi_u^\dagger \varphi_u)^2 + \frac{g_2^2}{2} (\varphi_d^\dagger \varphi_d)(\varphi_u^\dagger \varphi_u),$$

where we take $m_3^2 \equiv \mu B$ to be real and positive by the phase convention of the fields. We parameterize the VEVs as

$$\varphi_d = \frac{1}{\sqrt{2}} \left( v_1 + iv_4 \right), \quad \varphi_u = \frac{1}{\sqrt{2}} \left( 0 + iv_3 \right).$$

Then the tree-level potential is expressed as

$$V_0 = \frac{1}{2} m_1^2 (v_1^2 + v_4^2) + \frac{1}{2} m_2^2 (v_2^2 + v_3^2) - m_3^2 (v_1 v_2 - v_3 v_4) + \frac{g_2^2 + g_1^2}{32} (v_1^2 + v_4^2 - v_2^2 - v_3^2)^2.$$  (2.3)

The minimum of this potential is given by

$$v_1 = v_0 \cos \beta, \quad v_2 = v_0 \sin \beta, \quad v_3 = v_4 = 0,$$  (2.4)

with

$$m_1^2 = m_3^2 \tan \beta - \frac{1}{2} m_2^2 \cos(2\beta), \quad m_2^2 = m_3^2 \cot \beta + \frac{1}{2} m_2^2 \cos(2\beta),$$  (2.5)

so that $CP$ is conserved. The effective potential at the one-loop level is given by

$$V_{\text{eff}}(v) = V_0(v) + \Delta V(v),$$  (2.6)

where

$$\Delta V(v) = \Delta_g V(v) + \Delta_t V(v) + \Delta_{\tilde{t}} V(v) + \Delta_{\chi^\pm} V(v) + \Delta_{\chi^0} V(v)$$  (2.7)

is the sum of the one-loop corrections. Each term is given as follows:

$$\Delta_g V(v) = 3 \cdot 2 F \left( m_W^2(v) \right) + 3 \left( m_Z^2(v) \right),$$  (2.8)

$$\Delta_t V(v) = -4 \cdot 3 \cdot F \left( m_t^2(v) \right),$$  (2.9)

$$\Delta_{\tilde{t}} V(v) = 2 \cdot 3 \cdot \sum_{a=1,2} F \left( m_{\tilde{t}_a}^2(v) \right),$$  (2.10)

$$\Delta_{\chi^\pm} V(v) = -4 \sum_{a=1,2} F \left( m_{\chi^\pm_a}^2(v) \right),$$  (2.11)

$$\Delta_{\chi^0} V(v) = -2 \sum_{a=1,2,3,4} F \left( m_{\chi^0_a}^2(v) \right),$$  (2.12)

---

* The order parameter $v_4$ can be eliminated by the gauge transformation. So we shall set $v_4 = 0$ except when we numerically calculate the Higgs boson masses in the presence of $CP$ violation.
where

$$F(m^2) = \frac{m^4}{64\pi^2} \left( \log \frac{m^2}{M_{\text{ren}}^2} - \frac{3}{2} \right), \quad (2.13)$$

which was renormalized by the DR-scheme. The renormalization scale $M_{\text{ren}}$ will be taken to be the weak scale. The masses-squared in (2.8) and (2.9) are given by

$$m_{W}^2(v) = \frac{g_2^2}{4} (v_1^2 + v_2^2 + v_3^2 + v_4^2), \quad m_Z^2(v) = \frac{g_2^2 + g_1^2}{4} (v_1^2 + v_2^2 + v_3^2 + v_4^2),$$

$$m_t^2(v) = \frac{g_2^2}{2} (v_2^2 + v_3^2). \quad (2.14)$$

The $m_{l_a}^2(v)$ in (2.10) are the eigenvalues of the matrix

$$M_{l_a}^2 = \begin{pmatrix}
    m_{l_a}^2(v_4) & \frac{3 g_2^2}{4} (v_3^2 - v_2^2 + v_1^2) & \frac{m_2^2}{\sqrt{2}} (\mu (v_1 + iv_4) + A_t (v_2 - iv_3)) \\
    \frac{m_2^2}{\sqrt{2}} (\mu (v_1 - iv_4) + A_t (v_2 + iv_3)) & m_{l_a}^2(v_1) & \frac{g_2^2}{2} (v_1^2 - v_2^2 + v_3^2) \\
    \frac{m_2^2}{\sqrt{2}} (\mu (v_1 - iv_4) + A_t (v_2 + iv_3)) & \frac{m_2^2}{\sqrt{2}} (\mu (v_1 + iv_4) + A_t (v_2 - iv_3)) & m_{l_a}^2(v_3) + \frac{g_2^2}{2} (v_1^2 - v_2^2 + v_3^2)
\end{pmatrix},$$

(2.15)

the $m_{\chi_a^\pm}^2(v)$ in (2.11) are the eigenvalues of the matrix $M_{\chi_a^\pm}^\dagger M_{\chi_a^\pm}$ with

$$M_{\chi_a^\pm} = \begin{pmatrix}
    M_2 & -\frac{i}{\sqrt{2}} g_2 (v_2 - iv_3) \\
    \frac{i}{\sqrt{2}} g_2 (v_2 - iv_3) & -\mu
\end{pmatrix}, \quad (2.16)$$

and the $m_{\chi_a^0}^2(v)$ in (2.12) are the eigenvalues of the matrix $M_{\chi_a^0}^\dagger M_{\chi_a^0}$ with

$$M_{\chi_a^0} = \begin{pmatrix}
    M_2 & 0 & -\frac{i}{\sqrt{2}} g_2 (v_1 - iv_4) & \frac{i}{\sqrt{2}} g_2 (v_2 - iv_3) \\
    0 & M_1 & \frac{i}{\sqrt{2}} g_1 (v_1 - iv_4) & \frac{i}{\sqrt{2}} g_1 (v_2 - iv_3) \\
    -\frac{i}{\sqrt{2}} g_2 (v_1 - iv_4) & \frac{i}{\sqrt{2}} g_1 (v_1 - iv_4) & 0 & \mu \\
    \frac{i}{\sqrt{2}} g_2 (v_2 - iv_3) & \frac{i}{\sqrt{2}} g_1 (v_2 - iv_3) & -\mu & 0
\end{pmatrix}. \quad (2.17)$$

In general, $\mu$, $A_t$, $M_2$, and $M_1$ are complex-valued, but we assume them to be real until we discuss CP violation.

The minimization conditions of the effective potential relate the mass parameters in the Higgs potential to the VEVs of the Higgs fields:

$$m_1^2 = m_3^2 \tan \beta - \frac{1}{2} m_2^2 \cos 2\beta - \frac{1}{v_1} \frac{\partial \Delta V(v)}{\partial v_1},$$

$$m_2^2 = m_3^2 \cot \beta + \frac{1}{2} m_2^2 \cos 2\beta - \frac{1}{v_2} \frac{\partial \Delta V(v)}{\partial v_2}. \quad (2.18)$$

These relations are used to eliminate $m_1^2$ and $m_2^2$ in favor of $v_0 \equiv |v|$ and $\tan \beta = v_2/v_1$. These are equivalent to the tadpole conditions. The masses of the CP-even Higgs bosons are the eigenvalues of the matrix

$$M_h^2 = \begin{pmatrix}
    \frac{\partial^2 V_{\text{eff}}(v)}{\partial v_1^2} & \frac{\partial^2 V_{\text{eff}}(v)}{\partial v_1 \partial v_2} \\
    \frac{\partial^2 V_{\text{eff}}(v)}{\partial v_1 \partial v_2} & \frac{\partial^2 V_{\text{eff}}(v)}{\partial v_2^2}
\end{pmatrix}, \quad (2.19)$$
where the derivatives should be evaluated at the vacuum. The mass of the \( CP \)-odd scalar is given by
\[
m_A^2 = \frac{1}{\cos^2 \beta} \frac{\partial^2 V_{\text{eff}}(v)}{\partial v_3^2}.
\] (2.20)

By use of (2.18), the second derivatives evaluated at the vacuum are reduced to
\[
\frac{\partial^2 V_{\text{eff}}(v)}{\partial v_1^2} = m_3^2 \tan \beta + m_Z^2 \cos^2 \beta + v_1 \frac{\partial}{\partial v_1} \left( \frac{1}{v_1} \frac{\partial \Delta V(v)}{\partial v_1} \right),
\] (2.21)
\[
\frac{\partial^2 V_{\text{eff}}(v)}{\partial v_2^2} = m_3^2 \cot \beta + m_Z^2 \sin^2 \beta + v_2 \frac{\partial}{\partial v_2} \left( \frac{1}{v_2} \frac{\partial \Delta V(v)}{\partial v_2} \right),
\] (2.22)
\[
\frac{\partial^2 V_{\text{eff}}(v)}{\partial v_1 \partial v_2} = -m_3^2 - m_Z^2 \sin \beta \cos \beta + \frac{\partial^2 \Delta V(v)}{\partial v_1 \partial v_2},
\] (2.23)
\[
\frac{\partial^2 V_{\text{eff}}(v)}{\partial v_3^2} = m_3^2 \cot \beta - \frac{1}{v_2} \frac{\partial \Delta V(v)}{\partial v_2} + \frac{\partial^2 \Delta V(v)}{\partial v_2^2}.
\] (2.24)

The expressions of the derivatives of \( \Delta V(v) \) are summarized in the Appendix.

In addition to the evaluation of the masses by use of these formulas, we adopt a fully numerical method. In this method, the effective potential defined by (2.6) is calculated at every \( v = (v_1, v_2, v_3, v_4) \), where the mass eigenvalues are evaluated numerically. For a given set of \( (v_0, \tan \beta) \) and \( m_3^2 \), the mass parameters in the Higgs potential are determined by (2.18). The minimum of the effective potential is searched for by use of the downhill simplex algorithm, \(^{16}\) starting from a randomly generated simplex in the restricted space of \( (v_1, v_2, v_3) \) with \( v_1 = 0 \). Once the minimum is found, the second derivatives of the effective potential with respect to \( (v_1, v_2, v_3, v_4) \) are numerically evaluated. We have checked, in the absence of \( CP \) violation, that the minimum coincides with the prescribed \( (v_0, \tan \beta) \) and that the four-by-four matrix of the second derivatives is completely divided into the two sectors of \( CP \) eigenmodes and the eigenvalues coincides with those obtained by use of the formulas above. This numerical method can be applied to the case with \( CP \) violation. When \( CP \) is explicitly violated by the relative phases of the parameters in the MSSM, the \( CP \)-violating order parameter \( v_3 \) is induced, and the \( CP \) eigenstates of the Higgs sector mix to make the mass eigenstates.

\section{3. Finite-temperature effective potential}

At finite temperatures, the one-loop corrections in (2.8)–(2.12) are modified to include the finite-temperature effects. We have
\[
\Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) + 6 \frac{T^4}{2\pi^2} \sum_{a=1,2} I_B \left( \frac{m_a^2(v)}{T^2} \right),
\] (3.1)
\[
\Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) = \Delta_T V(v; T) + 6 \frac{T^4}{2\pi^2} \sum_{a=1,2} I_B \left( \frac{m_a^2(v)}{T^2} \right),
\] (3.2)
\[ \Delta \chi^\pm V(v;T) = \Delta \chi^\pm V(v) - 4 \cdot \frac{T^4}{2\pi^2} \sum_{a=1,2} I_F \left( \frac{m_{\chi^\pm}^2(v)}{T^2} \right), \quad (3.4) \]

\[ \Delta \chi^0 V(v;T) = \Delta \chi^0 V(v) - 2 \cdot \frac{T^4}{2\pi^2} \sum_{a=1,2,3,4} I_F \left( \frac{m_{\chi^0}^2(v)}{T^2} \right), \quad (3.5) \]

where the functions \( I_B(a^2) \) and \( I_F(a^2) \) are defined by

\[ I_{B,F}(a^2) = \int_0^\infty dx x^2 \log \left( 1 \pm e^{-\sqrt{x^2+a^2}} \right). \quad (3.6) \]

The effective potential at finite temperature is calculated at each \( v \) by numerically evaluating the mass-squared eigenvalues and inserting them into the expressions (3.1)–(3.5). The integrals defined in (3.6) are numerically calculated without use of the high-temperature expansions. \(^{14}\)

For a given set of parameters, the minimum of the effective potential is searched for at various temperatures by the method described in the previous section. Near the transition temperature, several numbers of starting simplexes are generated, and the minimum reached starting from each simplex is found. The temperature at which two degenerate minima are found is defined to be the transition temperature, \( T_C \), of the first-order EWPT. Then we examine whether the condition which must be satisfied for the sphaleron process to decouple just after the EWPT, \(^{17}\)

\[ \frac{v_C}{T_C} = \lim_{T \rightarrow T_C} \frac{|v(T)|}{T} > 1, \quad (3.7) \]

is in fact satisfied. If \( v_C = 0 \), the EWPT is of second order. We executed this minimum search for various sets of parameters to find the order of the EWPT and \( T_C \), and measured \( v_C \) and \( \tan \beta \) at \( T_C \) when the EWPT is of first order.

\section*{§4. Numerical results}

\subsection*{4.1. \( CP \)-conserving case}

Among the many parameters in the MSSM, the mass parameters \( m_1^2 \) and \( m_2^2 \) in the Higgs potential are determined by (2.18) in the absence of \( CP \) violation. Throughout this paper, we take \( v_0 = 246 \) GeV, \( m_W = 80.3 \) GeV, \( m_Z = 91.2 \) GeV and \( m_t = 175 \) GeV. The remaining parameters are \( m_3^2, \tan \beta, \mu, A_t, m_{\tilde{t}_L}, m_{\tilde{t}_R}, M_2 \) and \( M_1 \). For definiteness, we take \( M_2 = M_1, m_{\tilde{t}_L} = 400 \) GeV and \( A_t = 10 \) GeV.

Before presenting the numerical results on the Higgs masses and \( CP \) violation, we note that the contributions from the charginos and neutralinos are not negligible compared to those from the top quarks and squarks. For example, consider the contributions to the first derivatives appearing in (2.18), which have the form

\[ \frac{1}{v_i} \frac{\partial m_i^2}{\partial v_i} F^\prime(m^2). \]

The contributions from the gauge bosons are smaller than those from the top quark, since for \( \tan \beta = 5 \),

\[
\frac{1}{v_2} \frac{\partial m_2^2(v)}{\partial v_2} = \frac{2m_t^2}{v_0^2 \sin^2 \beta} = g_t^2 \simeq 1.0526, \quad \frac{1}{v_i} \frac{\partial m_i^2(v)}{\partial v_i} = \frac{g_i^2}{2} \simeq 0.2131, \quad (4.1)
\]
which multiply $-12F'(m^2_t)$ and $6F'(m^2_W)$, respectively. For the case of the stop and charginos, these factors are replaced with (A.13) and (A.22) or (A.23), which depend not only on the couplings but also on $\mu$, tan $\beta$ and the soft-SUSY-breaking masses. When $m^2_t$ is the same order as $m_{\chi^\pm}$, there is no reason for the stop contribution to become much larger than the chargino contributions. Now let us write

$$\Delta_l m^2_1 = \frac{1}{v_1} \frac{\partial \Delta l V(v)}{\partial v_1}, \quad \Delta_l m^2_2 = \frac{1}{v_2} \frac{\partial \Delta l V(v)}{\partial v_2},$$

$$\Delta_{\chi^\pm} m^2_1 = -\frac{1}{v_1} \frac{\partial \Delta_{\chi^\pm} V(v)}{\partial v_1}, \quad \Delta_{\chi^\pm} m^2_2 = -\frac{1}{v_2} \frac{\partial \Delta_{\chi^\pm} V(v)}{\partial v_2},$$

and calculate them for $M_2 = 300$ GeV. Several numerical values are presented in Table I. This suggests that when $M_2 \sim |\mu| \sim m_{\tilde{t}_R}$, $\Delta_l m^2_1$ and $\Delta_l m^2_2$ are of the same order as $\Delta_{\chi^\pm} m^2_1$ and $\Delta_{\chi^\pm} m^2_2$. As seen from (A.15) and (A.25), the factors multiplying $F'(m^2)$, which are the couplings squared in the case of the quarks and gauge bosons, are corrected by $n^{(1)}_l / R_l$, $n^{(1)}_R / R_R$ and $n^{(2)}_\chi / R_\chi$ for the stops and charginos, respectively. For the case tan $\beta = 5$, $\mu = -300$ GeV and $m_{\tilde{t}_R} = 0$ in Table I, $n^{(1)}_l / R_l \simeq 0.53$ while $n^{(1)}_R / R_R \simeq -13.4$ and $n^{(2)}_\chi / R_\chi \simeq 2.85$, which are large enough to compensate for the difference between the gauge and Yukawa coupling constants. As for the neutralino, although its contributions cannot be expressed in a compact form as (A.11), it is natural to expect the neutralino contributions to be of the same order as the chargino, as long as $M_2 \sim M_1$. For tan $\beta = 5$ and $\mu = -300$ GeV, we have

$$\frac{1}{v_1} \frac{\partial \Delta \chi^0 V(v)}{\partial v_1} = 2.374 \times 10^3 \text{ GeV}^2, \quad \frac{1}{v_2} \frac{\partial \Delta \chi^0 V(v)}{\partial v_2} = -1.013 \times 10^3 \text{ GeV}^2,$$

which are the same order as the chargino contributions. As for the second derivatives, we find that the contributions from the charginos and neutralinos are the same order as those from the stops for the parameters we adopted in the numerical analyses. We also observed that if we omit $\Delta_{\chi^\pm} V(v)$ and $\Delta_{\chi^0} V(v)$ to determine $m^2_1$ and $m^2_2$ in favor of $v_0$ and tan $\beta$ by (2-18), the numerical method used to search for the minimum results in a different point in the $(v_1, v_2)$-plane from $(v_0 \cos \beta, v_0 \sin \beta)$ with a deviation of about 5% for tan $\beta > 10$ and of about 70% for tan $\beta \lesssim 2$. Thus,
as long as the soft mass parameters $M_2$, $M_1$ and $m_{\tilde{t}_R}$ are of the same order, the contributions from the charginos and neutralinos are comparable to those from the top squarks.

Now we give results concerning the masses of the neutral Higgs bosons. We examine the dependence of the mass of the lighter scalar $m_h$ on the pseudoscalar mass $m_A$ and $M_2 = M_1$. In practice, we calculate $m_h$ and $m_A$ as functions of $(m_2^2, M_2)$ for a fixed set of $(\tan\beta, \mu, m_{\tilde{t}_R})$ and make a contour plot of $m_h$ in the $(M_2, m_A)$-plane. The mass of the lighter chargino $m_{\chi^\pm_1}$ is constrained to satisfy $m_{\chi^\pm_1} > 65.7 \text{ GeV}$, which restricts $\mu$ and $M_2$. According to the tree-level mass formula (A.21), the mass is plotted as a function of $M_2$ and $\mu$ in Fig. 1. This shows that the lower limit is satisfied for the entire range of $M_2$ we studied if we take $|\mu| \gtrsim 100 \text{ GeV}$. The limits on the masses of the lighter Higgs scalar and pseudoscalar are now $m_h > 62.5 \text{ GeV}$ and $m_A > 62.5 \text{ GeV}$, although more stringent bounds have been reported.

![Fig. 1. Contour plots of the mass of the lighter chargino as a function of the MSSM parameters $M_2$ and $\mu$ for $\tan\beta = 2$, 5 and 20, respectively. In all cases, the mass units are GeV.](https://academic.oup.com/ptp/article-abstract/101/2/415/1910461)

![Fig. 2. Contour plots of $m_h$ as a function of the MSSM parameters $M_2$ and $m_A$ for $\tan\beta = 2$, $\mu = -300 \text{ GeV}$ and $m_{\tilde{t}_R} = 0$, 100 GeV and 200 GeV, respectively. In all cases, the mass units are GeV.](https://academic.oup.com/ptp/article-abstract/101/2/415/1910461)
(\(m_h \gtrsim 75 \text{ GeV}\)).\(^{19}\) The results for \(m_h\) are plotted in Figs. 2 and 3 for \(\tan \beta = 2\), in Figs. 4 and 5 for \(\tan \beta = 5\) and in Figs. 6 and 7 for \(\tan \beta = 20\). We also calculated \(m_h\) for \(|\mu| = 100 \text{ GeV}\) and \(200 \text{ GeV}\) and found that \(m_h\) behaves in the same manner.
as in the case of $|\mu| = 300$ GeV, but here its value becomes slightly smaller for smaller $|\mu|$. For $\mu = 300$ GeV, in the blank region at small $m_A$ and large $M_2$ region, the point $v = (v_0 \cos \beta, v_0 \sin \beta)$ is not a local minimum but a saddle point. This region is broader for larger $\tan \beta$, which corresponds to a smaller top Yukawa coupling. This is because for larger $M_2$, the contributions from the charginos and neutralinos to the effective potential, which are negative, dominate the bosonic contributions and make the vacuum unstable. For $\tan \beta = 2$, the experimental bound on $m_A$ is satisfied for $m_A \gtrsim 200-300$ GeV, depending on $\mu$ and $M_2$. For $\tan \beta \geq 5$, it is satisfied for $m_A \gtrsim 100$ GeV.

At finite temperatures, the minimum of the effective potential differs from that at zero temperature. Here we are concerned with the order of the EWPT and the transition temperature $T_C$, at which two minima of the effective potential become degenerate, and the location of the minimum at $T_C$ when it is of first order. These are important ingredients for the electroweak baryogenesis. For definiteness, we take $\mu = -300$ GeV, $M_2 = M_1 = 350$ GeV, $m_{\tilde{t}_R} = 400$ GeV and $A_t = 10$ GeV. By use of the numerical method explained in the previous section, we search for the minimum of the effective potential at various temperatures to find the transition temperature.
This analysis is done for \( \tan \beta = 2, 5 \) and 20 with various \( m_{i_R} \geq 0 \) and two values of \( m_3^2 \) being tuned so that \( m_h \simeq 62.5 \text{ GeV} \) and \( m_h \simeq 80 \text{ GeV} \), respectively, for \( m_{i_R} = 0. \)

For \( \tan \beta = 2 \) and small \( m_{i_R} \), it is difficult to have \( m_h \) larger than 70 GeV, as seen from Fig. 2. We have adopted \( m_3^2 = 2.5 \times 10^4 \text{ GeV}^2 \), in which case \( m_h \simeq 62.5 \text{ GeV} \) when \( m_{i_R} = 0 \). The dependences of \( v_C/T_C \), \( \tan \beta(T_C) \) and the masses on \( m_{i_R} \) are plotted in Fig. 8. The condition for the sphaleron decoupling after the EWPT (3-7) is satisfied for \( m_{i_R} \leq 75 \text{ GeV} \), for which case \( m_h \leq 64 \text{ GeV} \) and \( m_A \simeq 239 \text{ GeV} \). The transition temperature varies from \( T_C = 77.3 \text{ GeV} \) (\( m_{i_R} = 0 \)) to 88.7 GeV (\( m_{i_R} = 120 \text{ GeV} \)) monotonically. \( \tan \beta \) at \( T_C \) is almost independent of \( m_{i_R} \) and remains the zero temperature value.

For \( \tan \beta = 5 \), we have taken \( m_3^2 = 3050 \text{ GeV}^2 \) and 4624 GeV\(^2\), which correspond to \( m_h \simeq 62.5 \text{ GeV} \) and 80 GeV, respectively. \( T_C \) is monotonically decreasing with respect to \( m_{i_R} \) and for the former case, \( 93.2 \text{ GeV} \leq T_C \leq 100.5 \text{ GeV} \), while \( 93.0 \text{ GeV} \leq T_C \leq 100.2 \text{ GeV} \) for the latter case. The dependence of \( T_C \) on \( m_3^2 \) (and thus on \( m_A \)) appears to be weak. \( v_C/T_C \), \( \tan \beta(T_C) \) and the masses are plotted in Figs. 9 and 10. The condition (3-7) is satisfied for \( m_{i_R} \leq 50 \text{ GeV} \). \( \tan \beta \) at \( T_C \) receives finite-temperature corrections to become about 20% larger than the zero-temperature value for the case of the larger \( m_A \).

For \( \tan \beta = 20 \), we adopted \( m_3^2 = 2308 \text{ GeV}^2 \) and 2440 GeV\(^2\). The dependence of \( T_C \) on \( m_{i_R} \) and \( m_3^2 \) is similar to that in the previous examples with \( \tan \beta = 5 \). Now \( 97.2 \text{ GeV} \leq T_C \leq 104.5 \text{ GeV} \) for both choices of \( m_3^2 \). \( v_C/T_C \), \( \tan \beta(T_C) \) and the masses are plotted in Figs. 11 and 12. In this case, \( \tan \beta(T_C) \) drastically deviates from the zero-temperature value.

In order to determine the profile of the bubble wall created at the first-order EWPT, we must know the global structure of the effective potential at \( T_C \). As an example, we display the contour plot of the effective potential at \( T = 0 \) and \( T = T_C \) for the case of \( \tan \beta = 5 \), \( m_3^2 = 4624 \text{ GeV} \) and \( m_{i_R} = 0 \) in Fig. 13. This indicates that two degenerate minima at \( T_C \) are connected by a valley with almost constant
Fig. 9. The same as Fig. 8 but for $\tan \beta = 5$ and $m_{3}^{2} = 3050 \text{ GeV}^{2}$.

Fig. 10. The same as Fig. 8 but for $\tan \beta = 5$ and $m_{3}^{2} = 4624 \text{ GeV}^{2}$.

Fig. 11. The same as Fig. 8 but for $\tan \beta = 20$ and $m_{3}^{2} = 2308 \text{ GeV}^{2}$.

Fig. 12. The same as Fig. 8 but for $\tan \beta = 20$ and $m_{3}^{2} = 2440 \text{ GeV}^{2}$.
Spontaneous CP Violation at Finite Temperature in the MSSM

4.2. CP-violating case

In the MSSM, there are many sources of CP violation other than the phase in the CKM matrix. Among these are the relative phases of the complex parameters $\mu$, $A_t$, $M_2$ and $M_1$. These also induce CP violation in the Higgs sector, which is the relative phase $\theta$ of the VEVs of the two Higgs doublets. This $\theta$ together with the other CP-violating phases affect such observables as the electric dipole moment of the neutron. Hence, knowledge of $\theta$ is necessary to find bounds on the phases of the complex parameters in the MSSM. The quantity $\theta = \text{Arg}(v_2 + iv_3)$ in the gauge with $v_4 = 0$ is determined by minimizing the effective potential at $T = 0$. Even when all the parameters are real, the effective potential could have a CP-violating vacuum. This is known as spontaneous CP violation, but in the MSSM it inevitably is accompanied by a scalar that is too light. 20 We found that this is the case. Indeed if we take $\tan \beta = 5$ and $m_t^R = 0$ and use $m_3^2 = 2113.3$ GeV$^2$, the effective potential has two degenerate minima which correspond to $\theta = \pm 0.318$ and $\tan \beta = 4.917$. Then the lightest scalar mass is 12.9 GeV, which differs from any mass from the mass formulas because of large mixing of the CP eigenstates.

Now we study the effect of explicit CP violation in the complex parameters on the CP violation in the Higgs sector. As the first example, we take $\tan \beta = 5$, $m_t^R = 0$, $m_3^2 = 4624$ GeV$^2$, $\mu = -300 \cdot e^{i\delta_\mu}$, and the remaining parameters are set to the values adopted in the previous subsection. For nonzero $\delta_\mu$, $\theta$ has a nonzero value, and the scalar and pseudoscalar mixes to form the mass eigenstates. Using the numerical method, the minimum of the effective potential were searched for, and the second derivatives at the minimum were evaluated to calculate the masses of the Higgs bosons for $0 \leq \delta_\mu \leq 0.1$. The dependences of $\theta$, $\tan \beta$ and the masses

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig13.png}
\caption{Contour plots of the effective potential at $T = 0$ and $T = T_C$ for $\tan \beta = 5$, $m_3^2 = 4624$ GeV$^2$ and $m_t^R = 0$.}
\end{figure}
Fig. 14. Dependences of $\theta$ (solid curve), $\tan \beta$ (dashed curve) and the Higgs masses on $\delta_\mu$.

Fig. 15. Dependences of $\theta$ (solid curve), $\tan \beta$ (dashed curve) and the Higgs masses on $\delta_2$.

of the two light bosons on $\delta_\mu$ are depicted in Fig. 14. Within this range of $\delta_\mu$, the derivation of the masses from the values at $\delta_\mu$ is negligible. The induced $\theta$ is the same order and has the same sign as $\delta_\mu$. By linearly fitting, we find $\theta = 0.8265 \cdot \delta_\mu$. By redefining the fields, we find that the physical $CP$-violating phase in the mass matrices (2.15)–(2.17) is $\delta_\mu + \theta$. Hence $\delta_\mu$ enhances the magnitude of $CP$ violation.

As a second example, we put $M_2 = M_1 = 350 \cdot e^{i \delta_2}$ GeV, and the remaining parameters are taken to be real and set to the same values as in the previous example. The dependences of $\theta$, $\tan \beta$ and the masses of the two light bosons on $\delta_\mu$ are shown in Fig. 15. The induced $CP$ phase is fitted to $\theta = 0.8885 \delta_2$, which has the same sign as the original $\delta_2$. Since the physical $CP$ phase in the mass matrix (2.16) is $\delta_2 + \theta$, the $CP$-violating phase is enhanced by the radiative corrections.

In the scenario of electroweak baryogenesis, $CP$ violation around the

Fig. 16. Dependences of $\theta$ (solid curve) and $\tan \beta$ (dashed curve) at $T = 92$ GeV.
bubble wall is a key ingredient, and it is determined by solving the equations of motion with the effective potential at $T_C$. Then the VEVs in the two degenerate minima provide the boundary conditions for these equations. Although spontaneous $CP$ violation at $T \simeq T_C$ could occur, it would be accompanied by a scalar that is too light at zero temperature. In any case, explicit $CP$ violation is necessary to resolve the degeneracy in energy of the $CP$-conjugate pair of the bubble walls; otherwise no net baryon asymmetry would survive the EWPT. For the same parameters as the zero temperature case, we plot in Fig. 16 dependences of the induced $CP$-violating phase $\theta$ and $\tan \beta$ on the phase of the $\mu$-parameter. The behavior exhibited by each $\theta$ and $\tan \beta$ is almost the same as that at zero temperature, but $\theta = 0.8862 \cdot \delta_\mu$.

§5. Discussion

We have studied the masses of the neutral Higgs bosons and the electroweak phase transition of the MSSM by use of the one-loop effective potential. For the parameters we adopted, the contributions from the charginos and neutralinos are shown to be non-negligible. We have found that the EWPT is so strongly first order that the sphaleron process decouples after it, for $m_{\tilde{t}_R} < \sim 75$ GeV when $\tan \beta = 2$, $m_h = 62.8$ GeV and $m_A = 239$ GeV, for $m_{\tilde{t}_R} < \sim 50$ GeV when $\tan \beta = 5$, $m_h = 62.8$ GeV and $m_A = 70$ GeV, for $m_{\tilde{t}_R} \lesssim 53$ GeV when $\tan \beta = 5$, $m_h = 80$ GeV and $m_A = 114$ GeV, for $m_{\tilde{t}_R} \lesssim 46.7$ GeV when $\tan \beta = 20$, $m_h = 62.7$ GeV and $m_A = 62.6$ GeV, and for $m_{\tilde{t}_R} \lesssim 46.8$ GeV when $\tan \beta = 20$, $m_h = 80$ GeV and $m_A = 81$ GeV. These bounds on $m_{\tilde{t}_R}$ almost correspond to a bound on the lighter stop mass $m_{\tilde{t}_1} \lesssim m_t$. For parameter values which permit a strongly first-order EWPT, we have investigated $\tan \beta$ at the transition temperature. It receives larger temperature-corrections for larger $\tan \beta$ at zero temperature. This corresponds to the situation of a smaller Yukawa coupling of the top quark. This suggests the importance of finite-temperature contributions from particles other than the top quark and squarks. We also studied $CP$ violation in the Higgs sector, which is characterized by the relative phase $\theta$ of the expectation values of the two Higgs doublets. As is well known, the spontaneous mechanism to generate $\theta$ accompanies a scalar that is too light. Explicit $CP$ violation in the complex parameters induces a $\theta$ of the same order and sign as itself, through radiative and finite-temperature corrections. This implies that the physical phases in the mass matrices of the chargino, neutralino and stop are enhanced by the complex phases which are originally contained in these matrices. Hence one must take this effect into account to find bounds on the parameters in the MSSM obtained from such data as the neutron EDM.

Some of the mechanisms of electroweak baryogenesis in the MSSM require $\tan \beta$ to vary spatially. But at the transition temperature, it remains almost constant at $\tan \beta(T_C)$. Then, viable scenarios of electroweak baryogenesis rely on spatially varying $\theta$ and/or $|v|$ in the presence explicit $CP$ violation. The spatial dependence of $\theta$ and $|v|$ around the bubble wall created at the EWPT is examined in Ref. 15). The values of these variables in the broken phase at $T_C$ obtained here will serve as the boundary conditions for the dynamical equations of $(\theta(x), |v(x)|)$. These functions in the MSSM will be studied elsewhere.\(^{21}\)
In this paper, we extensively used the one-loop effective potential to make the calculation self-contained. Extension to the two-loop resummed potential would be straightforward. The two-loop resummed potential without the contributions from the charginos and neutralinos yields a strongly first-order EWPT for a wider range of parameters than the corresponding one-loop potential.\footnote{We expect that if higher-order effects are taken into account, the effective potential including the contributions from all the particles in the MSSM will provide a strongly first-order EWPT for a broader region in parameter space than that investigated here.}

**Acknowledgements**

The author thanks Y. Okada for informing him about recent papers on the Higgs mass in the MSSM. He also thanks A. Kakuto, S. Otsuki and F. Toyoda for discussions on electroweak baryogenesis. This work is supported in part by a Grant-in-Aid for Scientific Research on Priority Areas (Physics of CP violation, No. 10140220) and No. 09740207 from the Ministry of Education, Science, Sports and Culture of Japan.

**Appendix A**

**Derivatives of the Effective Potential**

Here we present the formulas for the first and second derivatives of the effective potential evaluated at a CP-conserving vacuum. Since all the parameters are assumed to be real, CP is conserved so that $v_3 = v_4 = 0$ at the vacuum. However, we retain $v_3$ to derive the mass of the CP-odd scalar according to (2.20). To observe the contribution from each species, we give the derivatives of the correction to the effective potential from each particle.

**A.1. Gauge bosons**

The contributions to the effective potential is given by (2.8). Its first derivatives at the vacuum are

$$\frac{1}{v_1} \frac{\partial}{\partial v_1} (\frac{1}{v_1} \frac{\partial \Delta g V(v)}{\partial v_1}) = \frac{3 \cos^2 \beta}{8\pi^2 v_0^2} \left[ 2m_W^4 \log \frac{m_W^2(v)}{M_{\text{ren}}^2} + m_Z^4 \log \frac{m_Z^2(v)}{M_{\text{ren}}^2} \right],$$

(A.2)

$$\frac{1}{v_2} \frac{\partial}{\partial v_2} (\frac{1}{v_2} \frac{\partial \Delta g V(v)}{\partial v_2}) = \frac{3 \sin^2 \beta}{8\pi^2 v_0^2} \left[ 2m_W^4 \log \frac{m_W^2(v)}{M_{\text{ren}}^2} + m_Z^4 \log \frac{m_Z^2(v)}{M_{\text{ren}}^2} \right],$$

(A.3)

$$\frac{\partial^2 \Delta g V(v)}{\partial v_1 \partial v_2} = \frac{3 \sin \beta \cos \beta}{8\pi^2 v_0^2} \left[ 2m_W^4 \log \frac{m_W^2(v)}{M_{\text{ren}}^2} + m_Z^4 \log \frac{m_Z^2(v)}{M_{\text{ren}}^2} \right],$$

(A.4)
\[
\frac{\partial^2 \Delta_y V(v)}{\partial v_3^2} = \frac{1}{v_3} \frac{\partial \Delta_y V(v)}{\partial v_3} = \frac{1}{v_2} \frac{\partial \Delta_y V(v)}{\partial v_2}. \tag{A.5}
\]

### A.2. Top quark

Since the contribution from the top quark (2.9) is independent of \(v_1\), any derivative with respect to \(v_1\) vanishes:

\[
\frac{1}{v_1} \frac{\partial \Delta_y V(v)}{\partial v_1} = 0,
\]

\[
\frac{1}{v_2} \frac{\partial \Delta_y V(v)}{\partial v_2} = -12 \cdot \frac{m_t^2(v)}{32\pi^2} \left( \log \frac{m_t^2(v)}{M_{\text{ren}}^2} - 1 \right), \tag{A.6}
\]

and

\[
\frac{1}{v_2} \frac{\partial \Delta_y V(v)}{\partial v_2} = -12 \cdot \frac{4m_t^2(v)}{v_0^2 \sin^2 \beta} \frac{m_t^2(v)}{32\pi^2} \log \frac{m_t^2(v)}{M_{\text{ren}}^2}, \tag{A.7}
\]

\[
\frac{\partial^2 \Delta_y V(v)}{\partial v_3^2} = \frac{1}{v_2} \frac{\partial \Delta_y V(v)}{\partial v_2}. \tag{A.8}
\]

### A.3. Top squarks

The stop contribution is given by (2.10), in which the mass eigenvalues are

\[
m_{t_a}^2(v) = \frac{m_{t_a}^2 + m_{t_R}^2}{2} + \frac{g_2^2 + g_1^2}{8} (v_1^2 - v_2^2 - v_3^2) + m_t^2(v)
\]

\[
\pm \sqrt{\left( \frac{m_{t_L}^2 - m_{t_R}^2}{2} + \frac{3g_2^2 - 5g_1^2}{24} (v_1^2 - v_2^2 - v_3^2) \right)^2 + \frac{y_t^2}{2} [\mu v_1 + A_t v_2 + A_t^2 v_3^2]]. \tag{A.9}
\]

The first derivatives are given by

\[
\frac{1}{v_1} \frac{\partial \Delta_l V(v)}{\partial v_1} = 6 \sum_{a=1,2} \frac{1}{v_1} \frac{\partial m_{t_a}^2(v)}{\partial v_1} \cdot \frac{m_{t_a}^2(v)}{32\pi^2} \left( \log \frac{m_{t_a}^2(v)}{M_{\text{ren}}^2} - 1 \right), \tag{A.10}
\]

where the first derivatives of the stop mass-squared evaluated at the vacuum are

\[
\frac{1}{v_1} \frac{\partial m_{t_a}^2}{\partial v_1} = \frac{2m_{t_a}^2}{v_0^2 \sin^2 \beta} \left( \frac{m_Z^2 \sin^2 \beta}{2m_{t_a}^2} \pm \frac{n_i^{(1)}}{R_i} \right), \tag{A.11}
\]

\[
\frac{1}{v_2} \frac{\partial m_{t_a}^2}{\partial v_2} = \frac{2m_{t_a}^2}{v_0^2 \sin^2 \beta} \left( 1 - \frac{m_Z^2 \sin^2 \beta}{2m_{t_a}^2} \pm \frac{n_i^{(2)}}{R_i} \right), \tag{A.12}
\]

\[
\frac{1}{v_3} \frac{\partial m_{t_a}^2}{\partial v_3} = \frac{2m_{t_a}^2}{v_0^2 \sin^2 \beta} \left( 1 - \frac{m_Z^2 \sin^2 \beta}{2m_{t_a}^2} \pm \frac{8m_{t_a}^2 - 5m_{t_a}^2 d_i \sin^2 \beta + A_t^2}{6m_{t_a}^2} \right), \tag{A.13}
\]
with
\[ d_i = \frac{m_{1a}^2 - m_{2a}^2}{2} + \frac{8m_W^2 - 5m_Z^2}{6} \cos(2\beta), \]
\[ n_i^{(1)} = \frac{8m_W^2 - 5m_Z^2}{6m_i^2} d_i \sin^2 \beta + \frac{1}{2} \mu (\mu + A_t \tan \beta), \]
\[ n_i^{(2)} = -\frac{8m_W^2 - 5m_Z^2}{6m_i^2} d_i \sin^2 \beta + \frac{1}{2} A_t (\mu \cot \beta + A_t), \]
\[ R_i = \sqrt{d_i^2 + m_i^2 (\mu \cot \beta + A_t)^2}. \] (A.15)

The general expression for the second derivatives is given by
\[ v_j \frac{\partial}{\partial v_i} \left( \frac{1}{v_j} \frac{\partial \Delta_i V(v)}{\partial v_j} \right) = 6 \sum_{a=1,2} \left[ v_j \frac{\partial}{\partial v_i} \left( \frac{1}{v_j} \frac{\partial m_{la}^2(v)}{\partial v_j} \right) \right] m_{la}^2(v) \left( \log \frac{m_{la}^2(v)}{M_{\text{ren}}^2} - 1 \right) + \frac{\partial m_{la}^2(v)}{\partial v_i} \frac{\partial m_{la}^2(v)}{\partial v_j} \frac{1}{32\pi^2} \log \frac{m_{la}^2(v)}{M_{\text{ren}}^2} \right]. \] (A.16)

The second derivatives relevant to this expression are
\[ v_1 \frac{\partial}{\partial v_1} \left( \frac{1}{v_1} \frac{\partial m_{1a}^2(v)}{\partial v_1} \right) = \frac{\cos^2 \beta}{v_0^2} \left[ \left( \frac{8m_W^2 - 5m_Z^2}{3} \right)^2 - \frac{\mu A_m m_{1a}^2}{\sin \beta \cos \beta} \right] R_i \frac{n_i^{(1)}}{R_i^3} \], (A.17)
\[ v_2 \frac{\partial}{\partial v_2} \left( \frac{1}{v_2} \frac{\partial m_{2a}^2(v)}{\partial v_2} \right) = \frac{\sin^2 \beta}{v_0^2} \left[ \left( \frac{8m_W^2 - 5m_Z^2}{3} \right)^2 - \frac{\mu A_m m_{2a}^2}{\sin \beta \cos \beta} \right] R_i \frac{n_i^{(2)}}{R_i^3} \], (A.18)
\[ \frac{\partial^2 m_{1a}^2}{\partial v_1 \partial v_2} = \frac{\sin \beta \cos \beta}{v_0^2} \left[ \left( \frac{8m_W^2 - 5m_Z^2}{3} \right)^2 + \frac{\mu A_m m_{1a}^2}{\sin \beta \cos \beta} \right] R_i \frac{n_i^{(1)}}{R_i^3} \], (A.19)
\[ v_3 \frac{\partial}{\partial v_3} \left( \frac{1}{v_3} \frac{\partial m_{3a}^2(v)}{\partial v_3} \right) = 0. \] (A.20)

A.4. Charginos

The derivatives of (2.11) are evaluated in the same manner as in the case of the stop. The mass-squared eigenvalues are
\[ m_{\tilde{\chi}}^2(v) = m_W^2(v) + \frac{M_t^2 + \mu^2}{2} \right] \pm \sqrt{ \left( \frac{M_t^2 - \mu^2}{2} + \frac{g_2^2}{4} (v_1^2 - v_2^2 - v_3^2) \right)^2 + \frac{g_2^2}{2} [(\mu v_1 + M_2 v_2)^2 + M_2^2 v_3^2]}. \] (A.21)
The first derivatives of the corrections to the effective potential are given by an expression similar to (A.11), in which the relevant derivatives of the masses-squared are

\[
\begin{align*}
\frac{1}{v_1} \frac{\partial m_{\chi^\pm}^2}{\partial v_1} &= \pm \frac{2m_W^2}{m_0^2} \left( 1 \pm \frac{n_{\chi}^{(1)}}{R_{\chi}} \right), \\
\frac{1}{v_2} \frac{\partial m_{\chi^\pm}^2}{\partial v_2} &= \pm \frac{2m_W^2}{m_0^2} \left( 1 \pm \frac{n_{\chi}^{(2)}}{R_{\chi}} \right), \\
\frac{1}{v_3} \frac{\partial m_{\chi^\pm}^2}{\partial v_3} &= \pm \frac{2m_W^2}{m_0^2} \left( 1 \pm \frac{M^2_{\chi^\pm} + \mu^2}{2} - \frac{m_W^2 \cos(2\beta)}{R_{\chi}} \right),
\end{align*}
\]

(A.22) \( \quad \) (A.23) \( \quad \) (A.24)

where

\[
\begin{align*}
n_{\chi}^{(1)} &= \frac{M^2_{\chi^\pm} + \mu^2}{2} + m_W^2 \cos(2\beta) + \mu M_{\chi^\pm} \tan\beta, \\
n_{\chi}^{(2)} &= \frac{M^2_{\chi^\pm} + \mu^2}{2} - m_W^2 \cos(2\beta) + \mu M_{\chi^\pm} \cot\beta, \\
R_{\chi} &= \sqrt{\left( \frac{M^2_{\chi^\pm} - \mu^2}{2} + m_W^2 \cos(2\beta) \right)^2 + 2m_W^2 (M_2 \sin\beta + \mu \cos\beta)^2}. \quad (A.25)
\end{align*}
\]

The second derivatives have the same form as (A.16), except for an overall coefficient. The relevant second derivatives are given by

\[
\begin{align*}
v_1 \frac{\partial}{\partial v_1} \left( \frac{1}{v_1} \frac{\partial m_{\chi^\pm}^2}{\partial v_1} \right) &= \pm \frac{2m_W^2}{m_0^2} \left[ \frac{2m_W^2 \cos^2\beta - \mu M_{\chi^\pm} \tan\beta}{R_{\chi}} - \frac{2m_W^2 \left( n_{\chi}^{(1)} \right)^2 \cos^2\beta}{R_{\chi}^2} \right], \\
v_2 \frac{\partial}{\partial v_2} \left( \frac{1}{v_2} \frac{\partial m_{\chi^\pm}^2}{\partial v_2} \right) &= \pm \frac{2m_W^2}{m_0^2} \left[ \frac{2m_W^2 \sin^2\beta - \mu M_{\chi^\pm} \cot\beta}{R_{\chi}} - \frac{2m_W^2 \left( n_{\chi}^{(1)} \right)^2 \sin^2\beta}{R_{\chi}^2} \right], \\
\frac{\partial^2 m_{\chi^\pm}^2}{\partial v_1 \partial v_2} &= \pm \frac{2m_W^2}{m_0^2} \left[ -2m_W^2 \sin\beta \cos\beta + \mu M_{\chi^\pm} - \frac{2m_W^2 \left( n_{\chi}^{(1)} \right) \left( n_{\chi}^{(2)} \right) \sin\beta \cos\beta}{R_{\chi}^2} \right], \\
v_3 \frac{\partial}{\partial v_3} \left( \frac{1}{v_3} \frac{\partial m_{\chi^\pm}^2}{\partial v_3} \right) &= 0.
\end{align*}
\]

(A.26) \( \quad \) (A.27) \( \quad \) (A.28) \( \quad \) (A.29)

A.5. Neutralinos

Although the neutralino contribution to the effective potential is given by (2.12) in terms of the mass eigenvalues, it is difficult to work out its derivatives, since the eigenvalues have complicated forms. To avoid this complexity, we return to the
original form of the neutralino contribution,
\[ \Delta_{\chi^0} V(v) = \frac{i}{2} \int_k \text{Tr} \log \left[ D^{-1}_{\chi^0}(k; v) \right], \]  
(A.30)

where \( f_k \) denotes an integral over the Minkowskian momentum, the trace is taken over the index of the 4-dimensional internal space and the spinor index, and \( D_{\chi^0} \) is the four-component Dirac operator defined by
\[ D_{\chi^0}^{-1}(k; v) = \frac{\not{k} - M_{\chi^0}}{2} - \frac{1 + \gamma_5}{2} \frac{1}{M_{\chi^0}}. \]
(A.31)

Here, the mass matrix \( M_{\chi^0} \) is defined by (2.17). The first and second derivatives of (A.30) have the forms
\[ \frac{\partial \Delta_{\chi^0} V(v)}{\partial v_i} = \frac{i}{2} \int_k \text{Tr} \left[ D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_i} \right], \]
\[ \frac{\partial^2 \Delta_{\chi^0} V(v)}{\partial v_i \partial v_j} = -\frac{i}{2} \int_k \text{Tr} \left[ D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_i} D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_j} \right]. \]  
(A.32)

The integrand of the first derivative evaluated at the vacuum has the following compact form
\[ \frac{1}{v_i} \text{Tr} \left[ D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_i} \right] = -\frac{2m_Z^2}{v_0^2} \text{Tr} \left[ \frac{\not{k} + \mu_i}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right], \]  
(A.33)

where
\[ D_1(k) = \frac{\cos^2 \theta_W}{k - M_2} + \frac{\sin^2 \theta_W}{k - M_1}, \quad D_2(k) = \frac{\not{k} + \mu \sin(2\beta)}{k^2 - \mu^2}, \]
(A.34)

and
\[ \mu_1 \equiv \mu \tan \beta, \quad \mu_2 \equiv \mu \cot \beta. \]
(A.35)

The integrands of the second derivatives are reduced to
\[ \text{Tr} \left[ D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_1} \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_1} \right] = \frac{2m_Z^2}{v_0^2} \text{Tr} \left[ \frac{\not{k}}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right] 
\quad \quad + \frac{4m_Z^2 \cos^2 \beta}{v_0^2} \text{Tr} \left[ \left( \frac{\not{k} + \mu \tan \beta}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right)^2 \right], \]  
(A.36)

\[ \text{Tr} \left[ D_{\chi^0}(k; v) \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_2} \frac{\partial D_{\chi^0}^{-1}(k; v)}{\partial v_2} \right] \]
\[
= \frac{2m_Z^2}{v_0^2} \text{Tr} \left[ \frac{\kappa}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right] \\
+ \frac{4m_Z^2 \sin^2 \beta}{v_0^2} \text{Tr} \left[ \left( \frac{\kappa + \mu \cot \beta}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right)^2 \right], \quad (A.37)
\]

\[
\text{Tr} \left[ D_{\chi^0}(k; \nu) \frac{\partial D_{\chi^0}^{-1}(k; \nu)}{\partial \nu_1} D_{\chi^0}(k; \nu) \frac{\partial D_{\chi^0}^{-1}(k; \nu)}{\partial \nu_2} \right] \\
= \frac{2m_Z^2}{v_0^2} \text{Tr} \left[ \frac{\kappa}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right] \\
+ \frac{4m_Z^2 \sin \beta \cos \beta}{v_0^2} \text{Tr} \left[ \left( \frac{\kappa + \mu \tan \beta}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right)^2 \right], \quad (A.38)
\]

\[
\text{Tr} \left[ D_{\chi^0}(k; \nu) \frac{\partial D_{\chi^0}^{-1}(k; \nu)}{\partial \nu_3} D_{\chi^0}(k; \nu) \frac{\partial D_{\chi^0}^{-1}(k; \nu)}{\partial \nu_3} \right] \\
= \frac{2m_Z^2}{v_0^2} \text{Tr} \left[ \frac{\kappa}{k^2 - \mu^2} \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right] \\
- \frac{4m_Z^2 \cos^2 \beta}{v_0^2} \left( \frac{\mu^2}{(k^2 - \mu^2)^2} \text{Tr} \left[ \frac{D_1(k)}{1 - m_Z^2 D_1(k) D_2(k)} \right] \right)^2 \right] \quad (A.39)
\]

For the special case \( M_2 = M_1 \), which is extensively investigated in this paper, we have the following formulas for the derivatives relevant to the masses of the neutral Higgs bosons:

\[
\frac{1}{v_1} \frac{\partial \Delta_{\chi^0} V(\nu)}{\partial \nu_1} \\
= \frac{m_Z^2}{4\pi^2 v_0^2} \left\{ -\left( \mu^2 + m_Z^2 \right) \left( \log \frac{\mu^2 + m_Z^2}{M_2^2} - 1 \right) + M_2(M_2 + \mu) L(M_2^2, \mu^2 + m_Z^2) \\
+ \left( 1 + \frac{\mu \sin(2\beta)}{M_2} \right) m_Z^2 \left[ -F_1 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \\
+ 2 \left( 1 + \frac{\mu_1}{M_2} \right) F_2 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \right] \\
+ \left( 1 + \frac{\mu \sin(2\beta)}{M_2} \right)^2 m_Z^4 \left[ F_3 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \\
- \left( 1 + \frac{\mu_1}{M_2} \right) F_4 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \right] \right\}, \quad (A.40)
\]

\[
- \frac{1}{v_1} \frac{\partial \Delta_{\chi^0} V(\nu)}{\partial \nu_1} + \frac{\partial^2 \Delta_{\chi^0} V(\nu)}{\partial \nu_1^2}
\]
\[
\begin{align*}
-4m_Z^2 \cos^2 \beta & G_{11} \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right), \\
& -4m_Z^2 \sin^2 \beta G_{22} \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \\
& \frac{\partial^2 \Delta_\psi V(v)}{\partial v_2 \partial v_1} \\
& \frac{\partial^2 \Delta_\psi V(v)}{\partial v_2^2} \\
& \frac{\partial \Delta_\psi V(v)}{\partial v_2} \\
& \frac{\partial}{\partial v_2} \Delta_\psi V(v)
\end{align*}
\]
Here various functions arising from the momentum integrals are defined by

\[-2m_Z^2 \sin(2\beta) G_{12} \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \] \quad (A.43)

\[-\frac{1}{v_2} \frac{\partial \Delta \phi V(v)}{\partial v_2} + \frac{\partial^2 \Delta \phi V(v)}{\partial v_2^2} \]

\[= \frac{m_Z^2}{4\pi^2 v_0^2} \left\{ -\mu_2 M_2 L(M_2, \mu^2 + m_Z^2) + \frac{\mu_2}{M_2} m_Z^2 F_1 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \right. \]

\[-2 \frac{\mu_2}{M_2} \left( 1 + \mu \sin(2\beta) \right) m_Z^2 F_2 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \]

\[+ \frac{\mu_2}{M_2} \left( 1 + \mu \sin(2\beta) \right)^2 m_Z^2 F_4 \left( \frac{\mu}{M_2}, \frac{m_Z}{M_2}, \tan \beta \right) \} \quad (A.44)\]

where \(a_i = a \cdot \mu_i / \mu\).
References

1) For a review see, A. Cohen, D. Kaplan and A. Nelson, Ann. Rev. Nucl. Part. Sci. 43
(1993), 27.
3) For nonperturbative studies on the lattice, see F. Csikor, Z. Fodor and J. Heitger,
hep-lat/9807021 and references therein.
183.
B262 (1991), 54.
387.
1105.
(1996), 771.
98 (1997), 427.
16) W. H. Press, B. P. Flannery, S. A. Teukovsky and W. T. Vetterling, Numerical Recipes in
C (Cambrigde University Press, 1988).