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Lepton Number Violating 
\( e^- W^+ \rightarrow e^+ W^- \) and \( e^- e^- \rightarrow W^- W^- \) 
Processes in the Left-Right Gauge Model

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As new tests of the nature of neutrinos, lepton number violating 
\( e^- W^+ \rightarrow e^+ W^- \) and 
\( e^- e^- \rightarrow W^- W^- \) processes are studied within the \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) gauge model. They take place via exchange of a Majorana neutrino and a doubly charged Higgs particle. Differential cross sections are derived in the most general form. The angular distribution of the former process becomes resonant at 
\[ \cos \theta_j = -1 + 2 \left( M_a^2 M_b^2 - m_j^2 s \right) / (s - M_a^2)(s - M_b^2), \] 
from which the neutrino mass \( m_j \) can be deduced. Differential cross sections are estimated by using present bounds on the parameters. The cross section of the former process is about \( 10^8 \) times larger than the latter. Another process, \( e^- p \rightarrow e^+ W^- n \), which includes \( e^- W^+ \rightarrow e^+ W^- \) as a sub-process, is also discussed, and orders of magnitude of the cross section are estimated.

§1. Introduction

There has been particular interest in determining the properties of neutrinos to obtain clues for constructing a theory beyond the standard model. The intermediate weak interaction model based on the \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) gauge group was proposed from the speculation that there should be left-right symmetry above some higher energies.\(^1\),\(^2\) The observed predominance of the charged current \( V-A \) interaction is attributed to the fact that the vacuum is not invariant under spatial reflection, and there should exist \( V+A \) current as a relic of the left-right symmetry.\(^3\) Neutrinos naturally have non-vanishing masses in this model, since both left- and right-handed helicities of neutrinos are included. The observed small mass of left-handed neutrinos is explained reasonably well through the so-called seesaw mechanism if neutrinos are of the Majorana type and right-handed neutrinos are sufficiently heavy. Then, there arises the essential question of whether neutrinos are of the Dirac or Majorana type.

The neutrinoless double beta decay of the nucleus is considered to be useful in surveying the above-mentioned problems.\(^4\)-\(^6\) However, the double beta decay is so rare a phenomenon since half-lives of typical nuclei are longer than \( 10^{20} - 10^{23} \) [yr], and there are some other drawbacks: We are not able to control the decay at will, and we need extremely high accuracy to detect it experimentally. From a theoretical viewpoint, contamination from hadron physics comes in through the nuclear matrix elements, which makes analysis of experimental data ambiguous to some extent.

As a new test of properties of neutrinos, we propose here the lepton number

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violating process

\[ e^- W^+ \rightarrow e^+ W^- \]  \hspace{1cm} (1.1)

The initial \( e^- \) and \( W^+ \) are converted into a virtual neutrino at one vertex, and subsequently a virtual anti-neutrino decays into the final \( e^+ \) and \( W^- \) at another vertex. This can take place only if a neutrino and an anti-neutrino are identical and in addition if these two have the same helicity components. Thus, once this process is detected experimentally, one may conclude that neutrinos are of the Majorana type and that neutrinos have non-vanishing masses or form the \( V + A \) charged current. Experiments using weak bosons have been made possible thanks to the production of a pair of gauge bosons at CERN’s LEP collider, and the feasibility of such experiments should be examined.

The crossed channel process

\[ e^- e^- \rightarrow W^- W^- \]  \hspace{1cm} (1.2)

was first discussed by Rizzo \(^7\) as a future collider experiment. It has been studied by several authors, \(^8\)-\(^\text{11}\) although their results are incomplete and have no generality.

These lepton number violating processes are considered to be fundamental sub-processes of the neutrinoless double beta decay of the nucleus. However, they are advantageous compared with the double beta decay in some points: First, they are pure electroweak processes in which disturbance by hadronic phenomena does not enter, and theoretical methods for calculating cross sections are established. Second, they are second order processes with respect to gauge coupling constants, while the double beta decay is fourth order. Then one may expect to have larger reaction rates than the double beta decay. Third, they are controllable accelerator phenomena, although high accuracy will be required to some extent.

We note that \( e^- W^+ \rightarrow e^+ W^- \) scattering would be preferable to \( e^- e^- \rightarrow W^- W^- \) scattering. One reason for this is that the threshold energy of the former is lower than that of the latter, and another reason is that the electromagnetic effect favors the former. One may further expect to have a larger cross section for the former because Majorana neutrinos propagating virtually in the scattering channel give rise to a resonance structure in the scattering amplitude.

In this paper, detailed systematic study is made of lepton number violating processes in Eqs. (1.1) and (1.2). Our investigation relies on the left-right symmetric gauge model which is based on the electroweak \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) symmetry.

The general framework of our study is summarized in \( \S 2 \). Differential cross sections in the most general form are given in \( \S 3 \) for \( e^- W^+ \rightarrow e^+ W^- \) scattering, and in \( \S 4 \) for \( e^- e^- \rightarrow W^- W^- \) scattering. Their physical features are discussed in \( \S 5 \), and numerical estimations are also made by adopting the present experimental bounds on parameters. In \( \S 6 \), another lepton number violating process, \( e^- p \rightarrow e^+ W^- n \), which includes \( e^- W^+ \rightarrow e^+ W^- \) scattering as a sub-process, is investigated within the framework of the effective \( W \) approximation and the quark-parton picture of hadrons. In the Appendix, the derivation of the cross sections is sketched.
§2. General framework

The gauge model based on the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry group is applied throughout this work. We summarize here the general framework for our study of lepton number violating processes.

2.1. Mass of Majorana neutrinos

Let us first focus on the mass of neutrinos. In the left-right symmetric gauge models, charged leptons $\ell$ ($\ell = e, \mu, \tau$) and accompanying neutrinos $\nu_\ell$ are assigned to irreducible representations $\psi_\ell = (\nu_\ell, \ell)_L : (1/2, 0, -1)$ and $\psi_\ell = (\nu_\ell, \ell)_R : (0, 1/2, -1)$ for each of the chiral components $L$ and $R$, where the numbers in parentheses refer to transformation properties under the group in question. Then the general form of neutrino mass term in the Lagrangian is given by

$$L_m = -\frac{1}{2} (\nu_L^C, \nu_R) \left( \begin{array}{cc} M_L & m_D \\ m_D^T & M_R \end{array} \right) (\nu_L^C, \nu_R).$$

(2.1)

Here, charge conjugation of a spinor field $\psi$ is defined as $\psi^C = C \psi^T$ by using the charge conjugation matrix $C$. The Dirac mass $m_D$ in the mass matrix preserves lepton number, while Majorana masses $M_L$ and $M_R$ violate lepton number by two units.

We are assuming in our formulation that there are $n$-generations of lepton families for generality. The $n \times n$ Majorana masses, $M_L$ and $M_R$, and therefore the total $2n \times 2n$ neutrino mass matrix in Eq. (2.1) are symmetric, because identities $\bar{\psi}_1^C \psi_2^C = \bar{\psi}_2 \psi_1$ and $\bar{\psi}_1^C \psi_2^C = \bar{\psi}_2^C \psi_1$ hold for any two spinor fields $\psi_1$ and $\psi_2$. The neutrino mass matrix in Eq. (2.1) can then be diagonalized by a $2n \times 2n$ unitary transformation for neutrino fields:

$$\left( \begin{array}{c} \nu_L^C \\ (\nu_R)^C \end{array} \right) = \left( \begin{array}{cc} U & V^* \\ N_L & (N_R)^C \end{array} \right) \left( \begin{array}{c} \nu_L^C \\ (\nu_R)^C \end{array} \right).$$

(2.2)

Let us introduce Majorana neutrino fields $N_j$ ($j = 1, 2, \cdots, 2n$) which have definite masses $m_j$ by defining

$$N_j = N_j^L + (N_j^L)^C = (N_j)^C \quad \text{for} \quad j = 1, 2, \cdots, n,$$

$$N_j = (N_j^R)^C + N_j^R = (N_j)^C \quad \text{for} \quad j = n + 1, n + 2, \cdots, 2n.\quad \text{(2.3)}$$

Then weak-eigenstate neutrinos $\nu_\ell L$ and $\nu_\ell R$ are given by a superposition of these Majorana neutrino fields:

$$\nu_\ell L = \sum_{j=1}^{2n} U_{\ell j} (N_j)_L, \quad \nu_\ell R = \sum_{j=1}^{2n} V_{\ell j} (N_j)_R. \quad \text{(2.5)}$$

The unitarity of the mixing matrix imposes constraints on the transformation matrices:

$$\sum_j U_{\ell j}^* U_{\ell' j} = \sum_j V_{\ell j}^* V_{\ell' j} = \delta_{\ell \ell'}, \quad \sum_j U_{\ell j} V_{\ell' j} = 0. \quad \text{(2.6)}$$
It is straightforward to show that Majorana and Dirac masses in the mass matrix are related to definite neutrino masses \( m_j \) and mixing matrices in Eq. (2.5) through the equations

\[
(M_L)_{\ell\ell'} = \sum_j m_j U_{\ell j} U_{\ell' j}^*, \quad (M_R)_{\ell\ell'} = \sum_j m_j V_{\ell j} V_{\ell' j}, \quad (m_D)_{\ell\ell'} = \sum_j m_j U_{\ell j} V_{\ell' j}.
\] (2.7)

2.2. The charged current interaction Lagrangian

The charged current interaction Lagrangian inspired by the \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \) gauge models contains not only the left-handed \( (V-A) \) but also right-handed \( (V+A) \) charged currents as a relic of the left-right symmetry in the higher energy regime:

\[
\mathcal{L}_{cc} = \frac{1}{2\sqrt{2}} \left\{ g_L J_{L\mu} W_{L\mu}^- + g_R J_{R\mu} W_{R\mu}^- \right\} + \text{H.C.} \] (2.8)

Here, \( J_{L\mu} = \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_L \) and \( J_{R\mu} = \bar{\ell} \gamma_\mu (1 + \gamma_5) \nu_R \). The gauge bosons \( W_{L\mu}^- \) and \( W_{R\mu}^- \) are associated with \( SU(2)_L \) and \( SU(2)_R \) symmetries with coupling constants \( g_L \) and \( g_R \), respectively.

The two gauge bosons \( W_{L\mu}^- \) and \( W_{R\mu}^- \) are not in general in mass eigenstates, and their mass matrix is diagonalized through the orthogonal transformation

\[
\left( \begin{array}{c} W_{L\mu}^- \\
W_{R\mu}^- \end{array} \right) = \left( \begin{array}{cc} \cos \zeta & \sin \zeta \\
-\sin \zeta & \cos \zeta \end{array} \right) \left( \begin{array}{c} W_{1\mu}^- \\
W_{2\mu}^- \end{array} \right),
\] (2.9)

where the vector bosons \( W_{1\mu}^- \) and \( W_{2\mu}^- \) have the definite masses \( M_1 \) and \( M_2 \), respectively.

Then, by using Eqs. (2.5) and (2.9), the interaction Lagrangian in Eq. (2.8) can be rewritten as

\[
\mathcal{L}_{cc} = \frac{1}{2\sqrt{2}} \sum_j (J_{J\mu} J_{R\mu}) \rho_j \left( \begin{array}{c} W_{1\mu}^- \\
W_{2\mu}^- \end{array} \right) + \text{H.C.},
\] (2.10)

where the leptonic currents \( J_{J\mu} \) and \( J_{R\mu} \) are defined as

\[
J_{J\mu} = \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_j, \quad J_{R\mu} = \bar{\ell} \gamma_\mu (1 + \gamma_5) \nu_j,
\] (2.11)

and the \( 2 \times 2 \) matrix \( \rho_j \) is given by

\[
\rho_j = \left( \begin{array}{cc} (\rho_{j})_{L1} & (\rho_{j})_{L2} \\
(\rho_{j})_{R1} & (\rho_{j})_{R2} \end{array} \right) = \left( \begin{array}{cc} g_L U_{\ell j} \cos \zeta & g_L U_{\ell j} \sin \zeta \\
g_R V_{\ell j} \sin \zeta & g_R V_{\ell j} \cos \zeta \end{array} \right).
\] (2.12)

The matrix elements \( (\rho_{j})_{\alpha\alpha} (\alpha = L, R \text{ and } \alpha = 1, 2) \) represent coupling constants between a leptonic current \( J_{ja} \) and a charged vector boson \( W_{a\mu}^- \). It should be noted that only \( (\rho_{j})_{L1} \) is non-zero in models based on the \( SU(2)_L \) gauge group. The non-vanishing of others is nothing but evidence of the underlying \( SU(2)_R \) group.
Lepton Number Violating $e^- W^+ \rightarrow e^+ W^-$ and $e^- e^- \rightarrow W^- W^-$ Processes

Hereafter, throughout this paper the Greek suffices $\alpha$ and $\beta$ are used to refer to the chirality $L$ or $R$, while the Roman $a$ and $b$ refer to the mass-eigenstate 1 or 2 of the vector bosons $W_1$ and $W_2$.

The reaction $e^- W^+ \rightarrow e^+ W^-$ in Eq. (1.1) takes place through the second order perturbation of the Lagrangian in Eq. (2.10), and the Feynman diagrams are shown in Figs. 1(a) and (b). The initial and final gauge bosons are $W^+_a$ and $W^-_b$ with masses $M_a$ and $M_b$, respectively, and the Majorana neutrino $N_j$ with mass $m_j$ mediates the scattering.

2.3. The Higgs scalar interaction Lagrangian

If the Higgs scalar particle participates in a reaction process, we should take its effect into consideration in obtaining the reaction rate in closed form. Indeed, it is well known that the Higgs scalar particle plays an essential role in the neutrino reaction in order for the cross section to satisfy the unitary bound.

In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge model, the Higgs sector contains three multiplets $\phi : (\frac{1}{2}, \frac{1}{2}, 0)$, $\Delta_L : (1, 0, 2)$ and $\Delta_R : (0, 1, 2)$. The Higgs bi-doublet $\phi$ can give Dirac masses to quarks and leptons. The content of Higgs triplets $\Delta_\alpha (\alpha = L, R)$ is $(\Delta^{++}_\alpha, \Delta^+_\alpha, \Delta^0_\alpha)$ in a spherical base. This content gives Majorana mass to neutrinos. The doubly charged Higgs scalar $\Delta^{++}_\alpha$ may have some contribution to the lepton number violating process $e^- W^+ \rightarrow e^+ W^-$ through the Feynman diagram in Fig. 2. A relevant interaction Lagrangian will be derived.

According to the gauge principle, the Lagrangian for Higgs triplets is given by

$$L_\Delta = \sum_{\alpha=L,R} (D_{\alpha\mu} \Delta_\alpha^\dagger) \cdot (D^{\mu} \Delta_\alpha),$$

where the covariant derivative is defined as $D_{\alpha\mu} = \partial_\mu - ig_\alpha(T^i_\alpha A^i_{\alpha\mu}) - ig'_\beta B_\mu$. Here, $A^i_{\alpha\mu}$ and $B_\mu$ are gauge fields associated with the $SU(2)_\alpha$ and $U(1)_{B-L}$ groups with coupling constants $g_\alpha$ and $g'$, respectively. The product of the three dimensional
representation of generators $T^j_α$ and the gauge field $A^j_{αμ}$ is given by

$$ (T^j_α A^j_{αμ}) = \begin{pmatrix} A^3_{αμ} & W^+_{αμ} & 0 \\ W^−_{αμ} & 0 & W^−_{αμ} \\ 0 & W^−_{αμ} & −A^3_{αμ} \end{pmatrix}. \tag{2.14} $$

As a result of spontaneous breaking of the gauge symmetry, the neutral Higgs field $\Delta^0_α$ has a non-vanishing vacuum expectation value,

$$ ⟨\Delta^0_α⟩ = \frac{1}{\sqrt{2}} v_α. \tag{2.15} $$

The interaction between the doubly charged Higgs scalar $Δ^{++}_α$ and the singly charged gauge boson $W^−_α$ can be extracted by rearranging the Lagrangian in Eq. (2.13):

$$ ℒ_Δ = ∑_α \left( −\frac{v_α}{\sqrt{2}} g^α W^−_α W^−_μ Δ^{++}_α + \text{H.C.} \right). \tag{2.16} $$

The Yukawa interaction between lepton doublets $ψ_{ℓα} = (ν_{ℓα}, ℓ_α)^T$ and the Higgs triplet $Δ_α$ is given by

$$ ℒ_Y = ∑_α ∑_{ℓℓ′} h^{(α)}_{ℓℓ′} (ψ_{ℓα})^C (iτ^2 \vec{τ} \cdot \vec{Δ}_α) ψ_{ℓ′α} + \text{H.C.}, \tag{2.17} $$

where the inner product of $Δ_α$ and the Pauli spin matrix $\vec{τ}$ is

$$ \vec{τ} \cdot Δ_α = \begin{pmatrix} Δ^+_α & √2 Δ^{++}_α \\ √2 Δ^0_α & −Δ^+_α \end{pmatrix}. \tag{2.18} $$

The Yukawa coupling constant $h^{(α)}_{ℓℓ′}$ is symmetric about $ℓ$ and $ℓ′$ because of the identity $(ψ_{ℓα})^C ψ_{ℓ′α} = (ψ_{ℓ′α})^C ψ_{ℓα}$. The Majorana mass is generated by a non-zero vacuum expectation value of the neutral Higgs triplet in Eq. (2.15).

The neutrino mass term and the interaction relevant to the lepton number violating processes under consideration become as follows:

$$ ℒ_Y = ∑_α ∑_{ℓℓ′} \left\{ −\frac{1}{2} (ψ_{ℓα})^C (M_α)_{ℓℓ′} ν_α + \frac{√3}{2v_α} (M_α)_{ℓℓ′} ℓ_α Δ^{++}_α \right\} + \text{H.C.} \tag{2.19} $$

Here, neutrino masses $M_α$ are defined as

$$ (M_α)_{ℓℓ′} = 2v_α h^{(α)}_{ℓℓ′}. \tag{2.20} $$

These are nothing but the Majorana masses $M_L$ and $M_R$ in the neutrino mass matrix in Eq. (2.1). Thus, the mechanism to generate the Majorana mass leads inevitably to interactions that make the lepton number violating process occur through exchange of a Higgs particle.


§3. \( e^- W^+ \rightarrow e^+ W^- \) scattering

Within the framework described in the preceding section, we investigate the first lepton number violating process,

\[
e^-(p, \sigma, m_e) + W^+_a(k, \lambda, M_a) \rightarrow e^+(p', \sigma', m_e) + W^-_b(k', \lambda', M_b), \tag{3.1}
\]

where \( W_a \) and \( W_b \) are \( W_1 \) or \( W_2 \) defined in Eq. (2.9). The quantities in parenthesis represent the momentum, spin-polarization and mass of each particle. Initial and final vector bosons may be of the same kind or of different kinds, and all cases \((a = b = 1, 2 \text{ and } a \neq b)\) are treated in a unified way. The only approximation introduced is that the electron mass \( m_e \) is neglected in comparison with the gauge boson masses \( M_a \) and \( M_b \).

In a reaction between unpolarized \( e^- \) and \( W^+_a \) to form unpolarized \( e^+ \) and \( W^-_b \), the differential cross section takes the form

\[
\frac{d^2 \sigma^{(eW)}_{ab}}{dt} = \frac{1}{384\pi(M_a M_b)^2(s - M_a^2)^2} A^{(eW)}_{ab}(s, t, u). \tag{3.2}
\]

The Mandelstam variables \( t = (p - p')^2 \) and \( u = (p - k')^2 \) are related to the energy squared \( s = (p + k)^2 \) and the scattering angle \( \theta \) in the center of mass system:

\[
\left( \begin{array}{c} st \\ su - (M_a M_b)^2 \end{array} \right) = -\frac{1}{2}(s - M_a^2)(s - M_b^2)(1 \mp \cos \theta). \tag{3.3}
\]

The factor \( A^{(eW)}_{ab}(s, t, u) \) is calculated by using the Lagrangians given in the preceding section. It consists of three parts:

\[
A^{(eW)}_{ab}(s, t, u) = A^{(eW)}_{ab}(s, t, u)_{mu} + A^{(eW)}_{ab}(s, t, u)_{V+A} + A^{(eW)}_{ab}(s, t, u)_H. \tag{3.4}
\]

Here, the first two terms on the right-hand side come from the charged current interaction in Eq. (2.10), while the last one is related to the doubly charged Higgs scalar exchange diagram in Fig. 2. Their expressions are derived in the subsequent formulation.

3.1. The reaction through the charged current

The \( S \)-matrix element for the scattering process in Eq. (3.1) is given by the second order perturbation of the charged current weak interaction Lagrangian \( \mathcal{L}_{cc} \) in Eq. (2.10),

\[
S_{ab} = \frac{-i^2}{2!} \left( \frac{1}{\sqrt{2}} \right)^2 \sum_{\alpha, \beta = L, R} \sum_{j', j} \sum_{c, d = 1, 2} (\rho_j)_{a\alpha} (\rho_{j'})_{b\delta} \times \int d^4 x d^4 y \langle W^-_b | T \{ W^+_{c\mu}(x) W^{+\nu}_{d}(y) \} | W^+_{a} \rangle \\
\times \langle e^+ | T \{ e^{\gamma \mu}(x) P_{\alpha} \gamma_{\mu} (N^C_{j'}(x) N_{j'}(y)) \gamma_{\nu} P_{\beta} e(y) \} | e^- \rangle, \tag{3.5}
\]

where \( P_{L(R)} \) is a chiral projection operator defined as \( P_{L(R)} = (1 \mp \gamma_5)/2 \). In contrast to the case for Dirac-type neutrinos, the vacuum expectation does not vanish for a
product of Majorana neutrino fields, because they satisfy the condition \( N_j^F(x) = N_j(x) \). We have
\[
\langle 0 | T[N_j^F(x) N_j(y)] | 0 \rangle = i \delta_{jj} S_F(x - y; m_j),
\]
where \( S_F(x - y; m_j) \) is a Feynman propagation function for a spin 1/2 fermion with mass \( m_j \). This is the reason that the lepton number violating process can take place.

Thus the invariant amplitude becomes
\[
T_{ab} = \left( \frac{1}{\sqrt{2}} \right)^2 \varepsilon^\nu(\lambda') \varepsilon^\mu(\lambda) \sum_{\alpha, \beta} \sum_j (\rho_j)_{ab} (\rho_j)_{\alpha \alpha} \\
\times \frac{\sqrt{\alpha^\mu(p', \sigma')}}{p} \left[ P_\beta \gamma^\nu (\frac{p + k}{p + k}) + m_j \gamma_\mu P_\alpha + P_\alpha \gamma_\mu (\frac{p - k'}{p - k'}) + m_j \gamma_\mu P_\beta \right] u(p, \sigma),
\]
where \( \varepsilon^\nu(\lambda) \) and \( \varepsilon^\mu(\lambda') \) are the polarization vectors of the vector bosons \( W_a^+ \) and \( W_b^- \), respectively. The two terms on the right-hand side correspond to the Feynman diagrams shown in Figs. 1(a) and (b). It is seen from the numerators of the neutrino propagators that the non-vanishing mass of the neutrinos contributes to the scattering in the case that the chiralities at two vertices are the same, \((\alpha, \beta) = (L, L)\) and \((R, R)\). This will be referred to as the \( m_e \) part. The momentum part can contribute in the case \((\alpha, \beta) = (L, R)\) and \((R, L)\). This will be referred to as the \( V + A \) part. Thus, a lepton number violating process can occur only when neutrinos are Majorana particles and if they have non-vanishing masses and/or left- and right-handed charged currents coexist.

The factor \( A^{(eW)}(s, t, u) \) in Eq. (3.2) is obtained from the absolute square of \( T_{ab} \) in Eq. (3.7) by taking sum over polarization states of initial and final particles. The calculation is somewhat lengthy, and is outlined in the Appendix. It is shown there that the \( m_e \) and \( V + A \) parts contribute separately to the cross section through \( A^{(eW)}(s, t, u)_{m_e} \) and \( A^{(eW)}(s, t, u)_{V + A} \) in Eq. (3.4), respectively. The point is that the interference between the \( m_e \) and \( V + A \) parts vanishes in the approximation that the electron mass \( m_e \) can be ignored in comparison with the gauge boson masses \( M_a \) and \( M_b \).

The cross section due to the \( m_e \) part is through
\[
A^{(eW)}(s, t, u)_{m_e} = h_{ab}^L a^4 \left\{ -t \left| \sum_j m_j U_{ej}^2 \left( \frac{s}{s - m_j^2} + \frac{u}{u - m_j^2} \right) \right|^2 \\
- 2(M_a^2 + M_b^2)(s - M_a^2 M_b^2) \left| \sum_j \frac{m_j U_{ej}^2}{(s - m_j^2)(u - m_j^2)} \right|^2 \\
- 4(M_a M_b) t \left\{ \left| \sum_j \frac{m_j U_{ej}^2}{s - m_j^2} \right|^2 + \left| \sum_j \frac{m_j U_{ej}^2}{u - m_j^2} \right|^2 \right\} \right\}
+ (L \to R),
\]
where \((L \to R)\) represents the sum of the terms appearing above by replacing \((g_L, h_{ab}^L, U_{ej})\) with \((g_R, h_{ab}^R, V_{ej})\). Here, \( h_{ab}^L \) and \( h_{ab}^R \) are defined in terms of the mixing.
angle \( \zeta \) between \( W_L \) and \( W_R \):

\[
\begin{align*}
    h_{ab}^L &= \begin{cases} 
        \cos^2 \zeta & \text{for } a = b = 1, \\
        \sin^4 \zeta & \text{for } a = b = 2, \\
        (\sin \zeta \cos \zeta)^2 & \text{for } a \neq b,
    \end{cases} \\
    h_{ab}^R &= \begin{cases} 
        \sin^4 \zeta & \text{for } a = b = 1, \\
        \cos^4 \zeta & \text{for } a = b = 2, \\
        (\sin \zeta \cos \zeta)^2 & \text{for } a \neq b.
    \end{cases}
\end{align*}
\]

The \( m_\nu \) part has a larger contribution to the \( a = b \) case than the \( a \neq b \) case, as should be the case, since the latter is suppressed by a factor of \((\sin \zeta \cos \zeta)^2\). The largest cross section is expected for the \( a = b = 2 \) case because of the very heavy masses of right-handed neutrinos.

We again express the cross section due to the \( V + A \) part in a compact form. For the \( a = b \) case, we have

\[
A_{a=b}^{(eW)}(s, t, u)_{V+A} = 2(g_{LR})^2 (\sin \zeta \cos \zeta)^2 
\times \left\{ -(su + 4tM_a^2 - M_a^4)(s - u)^2 \left( \sum_j \frac{m_j^2 U_e j V_{ej}}{s - m_j^2} \right)^2 \\
- 4M_a^4 (s - m_a^2) \left( \left( \sum_j \frac{U_e j V_{ej}}{s - m_j^2} \right)^2 + \left| \sum_j \frac{U_e j V_{ej}}{u - m_j^2} \right|^2 \right) \\
- 16M_a^6 (t - 4M_a^2) \text{Re} \left[ \sum_j \frac{U_e j V_{ej} U_j^* V_{ej}^*}{s - m_j^2} \sum_{j' \neq j} \frac{U_j^* V_{ej}^*}{u - m_j'^2} \right] \right\}.
\]

The \( a \neq b \) case is given by

\[
A_{a \neq b}^{(eW)}(s, t, u)_{V+A} = (g_{LR})^2 
\times \left\{ -[su + 2t(M_a^2 + M_b^2) - (M_a M_b)^2] \\
\times \left( |\cos^2 \zeta \sum_j \frac{sU_e j V_{ej}}{s - m_j^2} + \sin^2 \zeta \sum_j \frac{uU_e j V_{ej}}{u - m_j^2}|^2 \\
+ |\sin^2 \zeta \sum_j \frac{sU_e j V_{ej}}{s - m_j^2} + \cos^2 \zeta \sum_j \frac{uU_e j V_{ej}}{u - m_j^2}|^2 \right) \\
- 4(\sin^4 \zeta + \cos^4 \zeta)(M_a M_b)^2 [su - (M_a M_b)^2] \left( \left| \sum_j \frac{U_e j V_{ej}}{s - m_j^2} \right|^2 + \left| \sum_j \frac{U_e j V_{ej}}{u - m_j^2} \right|^2 \right) \\
+ 16(\sin \zeta \cos \zeta)^2 (M_a M_b)^2 (M_a^2 + M_b^2) t \text{Re} \left[ \sum_j \frac{U_e j V_{ej} U_j^* V_{ej}^*}{s - m_j^2} \sum_{j' \neq j} \frac{U_j^* V_{ej}^*}{u - m_j'^2} \right] \right\}.
\]

In contrast to the case for the \( m_\nu \) part, the \( V + A \) part contributes dominantly in the \( a \neq b \) case.
3.2. The reaction through the Higgs scalar

The $S$-matrix element for a mechanism of the Higgs scalar exchange in Fig. 2 is given by the second order perturbation of the Lagrangians $\mathcal{L}_\Delta$ in Eq. (2.16) and $\mathcal{L}_Y$ in Eq. (2.19):

$$S_{ab} = \frac{i^2}{2!} \times 2 \sum_{\alpha, \beta = L, R} \left( \frac{-g^2}{2} \frac{v_\alpha}{v_\beta} \right) \int d^4x d^4y \langle W^-_0 | T[W^\pm_{\alpha \mu}(x) W^\mu_{\alpha}(x)] | W^+ \rangle \times \langle 0 | T[\Delta_{\alpha}^-(x) \Delta_{\beta}^{++}(y)] | 0 \rangle \langle e^+ | T[\gamma_\beta(y)(M_\beta)_{ee} e_\beta(y)] | e^- \rangle.$$  \hspace{1cm} (3.13)

After some manipulations, the invariant amplitude is derived as

$$T_{ab} = \frac{1}{4} \nu^\mu(\lambda) e^\mu(\lambda) \bar{\sigma}^C(p', \sigma') \Gamma^{(t)}_{\nu \mu}(p, \sigma),$$  \hspace{1cm} (3.14)

where

$$\Gamma^{(t)}_{\nu \mu}(p, \sigma) = 4g_{\nu \mu} \left\{ \frac{g_0^2 \omega_{L \bar{L}} \omega_{L \bar{a}} (M_L)_{ee}}{t - M_{\Delta_L}^2} (1 - \gamma_5) + \frac{g_2^2 \omega_{R \bar{R}} \omega_{R \bar{a}} (M_R)_{ee}}{t - M_{\Delta_R}^2} (1 + \gamma_5) \right\}.$$

Here $M_{\Delta_\alpha}$ ($\alpha = L, R$) is the mass of the doubly charged Higgs scalar $\Delta_{\alpha}^{++}$, and the notation $\omega_{\alpha a}$ represents the transformation matrix element of gauge bosons as introduced in Eq. (2.9):

$$\left( \begin{array}{cc} \omega_{L1} & \omega_{L2} \\ \omega_{R1} & \omega_{R2} \end{array} \right) = \left( \begin{array}{cc} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{array} \right).$$  \hspace{1cm} (3.16)

The total invariant amplitude is given by the sum of $T_{ab}$ in Eq. (A-1) in the Appendix and $T_{ab}'$ in Eq. (3.14). The calculation of the cross section is outlined in the Appendix. The factor $A_{ab}^{(\text{eW})}(s, t, u)_H$ in Eq. (3.4) is found to be

$$A_{ab}^{(\text{eW})}(s, t, u)_H = 4h_{ab}^L g_0^L (-i) \left\{ \left[ \frac{(s + u)^2 + 8(M_L M_b)^2}{(t - M_{\Delta_L}^2)^2} \right] [(M_L)_{ee}]^2 \right. \right.$$ 

$$+ \frac{(s + u)}{t - M_{\Delta_L}^2} \text{Re} \left[ (M_L)_{ee} \sum_j m_j U_{ej} \left( \frac{s}{s - m_j^2} + \frac{u}{u - m_j^2} \right) \right]$$ 

$$+ \frac{4(M_L M_b)^2}{t - M_{\Delta_L}^2} \text{Re} \left[ (M_L)_{ee} \sum_j m_j U_{ej}^2 \left( \frac{1}{s - m_j^2} + \frac{1}{u - m_j^2} \right) \right] \right\}$$

$$+ (L \to R).$$  \hspace{1cm} (3.17)

Here, $(L \to R)$ term represents the sum of the terms appearing above by replacing $(g_L, h_{ab}^L, U_{ej}, M_L, M_{\Delta_L})$ with $(g_R, h_{ab}^R, U_{ej}, M_R, M_{\Delta_R})$.

The first term on the right-hand side of Eq. (3.17) comes from the Higgs scalar exchange mechanism itself in Fig. 2, while the second and third terms represent interference between the Higgs scalar exchange mechanism and the $m_\nu$ part in the Majorana neutrino exchange mechanism in Figs. 1(a) and (b). Thus, the cross section due to the non-vanishing neutrino mass is through both $A_{ab}^{(\text{eW})}(s, t, u)_m$. 

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and $A_{ab}^{(eW)}(s,t,u)_{\mu}$ in Eq. (3-4). Here, we would like to mention that there is no need to consider interference between the Higgs scalar exchange mechanism and the $V + A$ part in Majorana neutrino exchange mechanism. Indeed, the interference is proportional to $m_{\nu}$ after summation over polarization states of an electron and a positron, because numbers of gamma matrices in the invariant amplitudes are even for the former mechanism and odd for the latter.

§4. $e^- e^- \rightarrow W^- W^-$ scattering

As the second lepton number violating process, we study the scattering

$$e^- (p, \sigma, m_\nu) + e^- (p', \sigma', m_\nu) \rightarrow W^-_{a'} (k, \lambda, M_a) + W^-_{b'} (k', \lambda', M_b),$$

(4-1)

where $W_a$ and also $W_b$ are $W_1$ or $W_2$. Quantities in parenthesis represent the momentum, spin-polarization and mass of each particle.

In scattering where all external particles are unpolarized, the differential cross section is expressed as

$$\frac{d\sigma_{ab}^{(ee)}}{dt} = \frac{\varepsilon_{ab}}{256\pi (M_a M_b)^2 s} A_{ab}^{(ee)} (s,t,u),$$

(4-2)

where $\varepsilon_{ab}$ is a statistical factor for the final vector bosons $W^-_a$ and $W^-_b$ and is defined such that $\varepsilon_{ab} = 1/2$ for $a = b$ and $\varepsilon_{ab} = 1$ for $a \neq b$. The Mandelstam variables $t = (p - k)^2$ and $u = (p - k')^2$ are related to $s = (p + p')^2$ and the scattering angle $\theta$ in the center of mass frame:

$$\begin{pmatrix} t \\ u \end{pmatrix} = \frac{s}{2} \left( 1 - \frac{M_a^2 + M_b^2}{s} \right) \left\{ 1 \mp \sqrt{1 - \left( \frac{2M_a M_b}{s - (M_a^2 + M_b^2)} \right)^2 \cos \theta} \right\}.$$ (4-3)

The Feynman diagrams here are the same as those in Figs. 1(a), (b) and 2 with appropriate change of direction of external lines. Thus, for all the $m_{\nu}$, $V + A$ and Higgs parts, the factor $A_{ab}^{(ee)} (s,t,u)$ in Eq. (4-2) is obtained from results for the preceding $e^- W^+_a \rightarrow e^+ W^-_b$ process by merely interchanging the Mandelstam variables $s \leftrightarrow t$:

$$A_{ab}^{(ee)} (s,t,u) = -A_{ab}^{(eW)} (t,s,u).$$ (4-4)

Here, $A_{ab}^{(eW)} (s,t,u)$ is given by Eqs. (3-8), (3-11), (3-12) and (3-17).

Our result $A_{ab}^{(ee)} (s,t,u)_{\mu\nu}$ for the $m_{\nu}$ part can be shown to be in accordance with that obtained by Rizzo $^7$ if we set $M_a = M_b = M_W$, $U_{ej} = \delta_{j1}$, $V_{ej} = \delta_{j1n+1}$ and $\zeta = 0$. However, our study here is far more general with regard to certain points. First, in our study mixings for both neutrinos and weak bosons are taken into account. Second, all cases ($W_a = W_b$ and $W_a \neq W_b$) are included in a unified way. Third, cross sections due to the $V + A$ part are also derived.

§5. Physical features

In this section, we discuss features of the cross sections derived in the preceding sections. The energy dependence will be examined first. The cross sections increase
initially, as the incident energy increases above the threshold. In the high energy limit in which masses of all particles can be neglected in comparison with $\sqrt{s}$, the cross section due to the non-vanishing neutrino mass comes to decrease like $1/s$. This is because the two terms $A_{ab}^{(W)}(s, t, u)_{m_v}$ in Eq. (3.8) and $A_{ab}^{(W)}(s, t, u)_H$ in Eq. (3.17) have the same asymptotic behavior but opposite signatures, so that their leading constant terms cancel:

$$A_{ab}^{(W)}(s, t, u)_{m_v} \simeq -A_{ab}^{(W)}(s, t, u)_H \simeq -4t \left\{ h_{ab}^L g^L_4 |(M_L)_{ee}|^2 + h_{ab}^R g^R_4 |(M_R)_{ee}|^2 \right\}.$$  

(5.1)

To obtain this expression, the relations in Eq. (2.7) have been utilized. Unless the Higgs particle exchange mechanism is taken into consideration, the cross section due to the $m_v$ part would be constant asymptotically. It should be noted that interference between the Higgs particle and the Majorana neutrino exchange mechanisms plays an essential role in this cancellation.

The cross section due to the $V + A$ part behaves asymptotically like $1/s$, too. This is evident for the $a = b$ case from $A_{a=b}^{(W)}(s, t, u)$ in Eq. (3.11). For the $a \neq b$ case, the leptonic GIM mechanism reduces the power behavior of $A_{a \neq b}^{(W)}(s, t, u)$ in Eq. (3.12) by two; that is, orthogonality between the mixing matrices $U_{jj}$ and $V_{jj}$ in Eq. (2.6) leads to the identity

$$\sum_j s U_{ej} V_{ej} = \sum_j m_j^2 U_{ej} V_{ej},$$  

(5.2)

and a similar relation obtained by replacing $s$ with $u$. The same holds for the $e^- e^- \rightarrow W^- W^-$ process. Thus the unitarity requirement that the cross section should decrease like $1/s$ is fulfilled.

Let us next study the angular dependence of the cross section. Differential cross sections for the $e^- W^+ \rightarrow e^+ W^-$ and $e^- e^- \rightarrow W^+ W^-$ scatterings exhibit analytically different behavior in the physical region of the energy and the scattering angle: The former process has characteristic features similar to Compton scattering, while the latter to the electron-positron annihilation into two photons, because the spin structure of particles participating in the reactions are the same.

In $e^- e^- \rightarrow W^- W^-$, both denominators $(t-m_j^2)$ and $(u-m_j^2)$ in $A^{(ee)}_{ab}(s, t, u)$ are negative definite, so that the cross section $d\sigma^{(ee)}_{ab}/d\cos \theta$ is finite. The maxima of the distribution curve are at $\cos \theta = \pm 1$. However, in $e^- W^+ \rightarrow e^+ W^-$, the differential cross section becomes singular at $s = m_j^2$ and $u = m_j^2$, because the denominators $(s-m_j^2)$ and $(u-m_j^2)$ in $A^{(W)}_{ab}(s, t, u)$ may be zero, as seen from Eq. (3.3). That is, the cross section $d\sigma^{(W)}_{ab}/d\cos \theta$ becomes resonant not only at the energy $\sqrt{s} = m_j$ but also at a scattering angle $\theta_j$ satisfying

$$\cos \theta_j = -1 + \frac{2(M_a^2 M_b^2 - m_j^2 s)}{(s - M_a^2)(s - M_b^2)}.$$  

(5.3)
These angular resonant peaks are to be seen for $s$ in the range

$$M_a^2 + M_b^2 - m_j^2 < s < \frac{M_a^2 M_b^2}{m_j^2} \quad (5-4)$$

if the neutrino mass satisfies the condition $m_j < \text{Min}(M_a, M_b)$. As the incident energy increases, these resonant peaks move backwards ($\cos \theta = -1$) just as in the case of Compton scattering, because gauge boson mass effects become less important. Thus the mass value of each neutrino can be deduced in principle by analyzing resonance structure of the angular distribution of $e^- W^+_a \rightarrow e^+ W^-_b$ in relatively low energy experiments above the threshold. This situation is different from that for other lepton number violating processes. For example, in neutrinoless double beta decay, one can only have a constraint on the weighted average of neutrino masses, $\sum_j m_j U^2_{ej}$ or $\sum_j m_j^{-1} U^2_{ej}$, where primed and double primed sums are only over lighter and heavier neutrinos, respectively.4) - 6)

In order to obtain a rough estimate of cross sections, a special case will be studied, where electron neutrinos are separated from other types of neutrinos in the flavor mixing. Masses of particles are taken such that $m_{\nu_e} \sim (M_L)_{ee} \sim 15 \text{ [eV]}$ for a light left-handed neutrino, $M_{\nu_e} \sim (M_R)_{ee} \sim 40 \text{ [GeV]}$ for a heavy right-handed neutrino, $M_\Delta \sim 46 \text{ [GeV]}$ for a doubly charged Higgs boson, $M_1 \sim 80 \text{ [GeV]}$ and $M_2 \sim 400 \text{ [GeV]}$ for gauge bosons. A further assumption is that $|\sin \zeta| \sim 0.04$ for the mixing between $W_L$ and $W_R$, and that $U_{e1} \sim V_{e1+1} \sim \cos \theta_{LR} \sim 1$ and $V_{e1+1} \sim - V_{e1} \sim \sin \theta_{LR} \sim 2.3 \times 10^{-5}$ for neutrino mixing. These values are taken from the present bounds given by the particle data group12) and are consistent in order of magnitude with those expected from the seesaw mechanism. As for the gauge coupling constants, the assumption $g_L \sim g_R \sim g$ with $g^2/(8M^2_F) \sim G_F/\sqrt{2}$ is made.

Angular distributions for $e^- W^+_a \rightarrow e^+ W^-_b$ scattering calculated by using Eq. (3-2) are shown in Fig. 3 for the non-vanishing neutrino mass part and in Fig. 4 for the $V + A$ part. Note that the non-vanishing neutrino mass part comes from the sum of $A_{ab}^{(eW)}(s, t, u)_{\nu_e}$ in Eq. (3-8) and $A_{ab}^{(eW)}(s, t, u)_{\nu_\mu}$ in Eq. (3-17). Distribution curves are drawn for an incident C.M. energy $\sqrt{s}$ in the range $M_a^2 + M_b^2 - m_j^2 < s < \frac{M_a^2 M_b^2}{m_j^2}$ with $m_j = m_{\nu_e}$ and $M_{\nu_e}$ in order to see the resonance structure. The case of $e^- e^+ W^-_a W^-_b$ scattering is depicted in Figs. 5 and 6 in the relatively low energy region above the threshold. Just above the threshold, relative magnitudes among cross sections of different channels are roughly given by $f_{ab}(\zeta) M^2_{\nu_e}/(M_a M_b)^2$ for the non-vanishing neutrino mass part and $g_{ab}(\zeta) \theta^2_{LR} M^4_{\nu_e}/(M_a M_b)^3$ for the $V + A$ part, where $f_{ab}(\zeta) = \zeta^4, \zeta^2$ and 1, and $g_{ab}(\zeta) = \zeta^2, 1$ and $\zeta^2$ for $(a, b) = (1, 1), (1, 2)$ and $(2, 2)$, respectively.

For the non-vanishing neutrino mass part, only the resonant peak due to the heavier mass $M_{\nu_e}$ is seen in Fig. 3. This is because the lighter mass $m_{\nu_e}$ is so small that its contribution is invisible behind the heavier mass effect. The cross section becomes larger for the heavier neutrino. This is because the cross section depends on the heavier neutrino mass $M_{\nu_e}$, like $M^2_{\nu_e} [s/(s - M^2_{\nu_e}) + u/(u - M^2_{\nu_e})]$ in the lower energy region and like $M^4_{\nu_e}/(s + 1)$ in the sufficiently high energy limit. This is
Fig. 3. Angular distribution of $e^- W^+_a \to e^- W^-_b$ (mass part).
Numbers in parentheses represent $(a, b, \sqrt{s} \text{ [GeV]})$.

Fig. 4. Angular distribution of $e^- W^+_a \to e^- W^-_b$ (V+A part).
Numbers in parentheses represent $(a, b, \sqrt{s} \text{ [GeV]})$.

crucially different from the case for neutrinoless double beta decay, where the contribution from the heavier neutrino is less important, because its Compton wavelength $1/M_{\nu_e}$ is very short in comparison with the average distance $1/40 [\text{MeV}^{-1}]$ between two decaying nucleons in the nucleus. The cross section is insensitive to the value of $M_{\Delta}$. For example, with $M_{\Delta} = 100 [\text{GeV}]$, numerical results change by at most 10%.

For the $V + A$ part, two resonant peaks coming from light and heavy neutrino masses, $m_j = m_{\nu_e}$ and $M_{\nu_e}$, are clearly seen in Fig. 4. That is, in contrast to the non-vanishing neutrino mass part, even lighter neutrinos can play a substantial role in the resonance region, as seen from Eqs. (3-11) and (3-12). The cross section is suppressed relative to the non-vanishing neutrino mass part by about $(g_{ab}(\zeta)/f_{ab}(\zeta)) \theta_{LR}^2 M_{\nu_e}^2/(M_a M_b)$, and therefore it becomes very small due to the small mixing angle $\theta_{LR}$. However, there may be the possibility that these resonant peaks coming from lighter neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) add up coherently to enhance the cross section considerably, because the resonant angles are close for these lighter neutrinos, as seen from Eq. (5-3). Thus one may expect to have a measurable value for the cross section, even for the $V + A$ part.

The $e^- e^- \to W^- a W^- b$ process has smooth distribution curves as shown in Figs. 5 and 6, and its rate is less than $10^{-2}$ times as small as that of the $e^- W^+_a \to e^+ W^-_b$ process for each $(a, b)$. However, the Higgs particle $\Delta^{++}$ is exchanged in the scattering channel, and its contribution becomes resonant at an energy $\sqrt{s} = M_{\Delta}$, provided the condition $M_{\Delta} > M_a + M_b$ is satisfied. If this happens, the cross section may be somewhat enhanced.
§6. Discussion

Let us discuss the experimental feasibility of the $e^- W^+ \rightarrow e^+ W^-$ process as a method to study properties of neutrinos. A direct method would be provided by allowing real gauge bosons produced at the $e^+ e^-$ collider to rescatter with electrons in the beam. The cross section for this is given by Eqs. (3.2) and (3.4). But it may be rather difficult to use such real gauge bosons, because the number of produced gauge bosons is too small at present.

Another way is to utilize such reactions that include $e^- W^+ \rightarrow e^+ W^-$ scattering as a sub-process. Among several candidates, the reaction

$$e^- p \rightarrow e^- W^+ n \rightarrow e^+ W^- n,$$

seems to be most promising. Here, an incident $p$ converts into a virtual $W^+$ and a final $n$, and then $e^- W^+ \rightarrow e^+ W^-$ scattering follows, as depicted in Fig. 7. The candidate $e^- \gamma \rightarrow e^- W^+ W^- \rightarrow e^+ W^- W^-$ requires a higher threshold energy, while $\gamma \gamma \rightarrow e^+ e^- W^+ W^- \rightarrow e^+ e^- W^- W^-$ is still a higher order process with respect to the gauge coupling constant.

A detailed investigation of the reaction in Eq. (6-1) is of interest, and we should rely on QCD. However, in order to obtain a rough estimation of cross sections, a quark-parton picture for hadrons will be adopted here. We will use an effective $W$ approximation, which is essentially the equivalent photon method of Weizäcker and Williams, and is viewed to be in accordance with the parton picture: This approximation is based on the assumption that the hadron moves nearly straight forward during the reaction, emitting a virtual gauge boson, and that the emitted
Fig. 7. Feynman diagram for the lepton number violating $e^- p^+ \rightarrow e^+ W^+ n$ scattering which includes the $e^- W^+ \rightarrow e^+ W^-$ reaction as a sub-process. The momentum of each particle is denoted in parenthesis.

$W^+$ rescatters with another incident particle $e^-$ as a real particle on the mass shell. This is plausible when the incident $p$ is of sufficiently high energy. Furthermore, it can be generally argued that the longitudinal component of a virtual $W$ contributes mainly to the cross section in the case of the vector and axial vector types of interactions.\textsuperscript{13}

Within these approximations, the cross section for Eq. (6.1) is given by

$$d\sigma^{(ep)}(s,t) \simeq \int_{x_{\text{min}}}^1 dx f_{p/W}(x)d\sigma^{(eW)}(\hat{s},\hat{t}), \quad (6.2)$$

where $d\sigma^{(eW)}(\hat{s},\hat{t})$ is the cross section for the sub-process $e^- W^+ \rightarrow e^+ W^-$ for on-shell particles, and is given by Eqs. (3.2) and (3.4). The Mandelstam variables $\hat{s}$, $\hat{t}$ and $\hat{u}$ are defined in this sub-process and restricted by the constraint $\hat{s} + \hat{t} + \hat{u} = M_a^2 + M_b^2$. The quantity $x$ represents the momentum fraction of the proton carried by a virtual $W^+$, that is, $k = xK$ in Fig. 7 and $x_{\text{min}} = M_W/E$ with $E$ being the energy of a proton. The variables $s = (p + K)^2$ and $t = (p' - p)^2$ are for the reaction in Eq. (6.1). There are the relations $\hat{s} \simeq x s$ and $\hat{t} = t$.

The distribution function $f_{p/W}(x)$ represents the number of a virtual $W^+$ in the proton. For a point-like quark, the distribution function is given asymptotically by

$$f_{q/W}(x) \simeq \frac{g^2}{16\pi^2} \left( \frac{1 - x}{x} \right), \quad (6.3)$$

for the longitudinal gauge boson. The distribution function $f_{p/W}(x)$ is obtained by integrating it with the quark distribution function $f_u(x)$ in the proton:

$$f_{p/W}(x) = \int_x^1 \frac{dy}{y} f_u(y)f_{q/W} \left( \frac{x}{y} \right). \quad (6.4)$$

For the function $f_u(x)$, we adopt the form\textsuperscript{14} $xf_u(x) = 1.78x^{0.5}(1 - x^{1.51})^{3.5}$ with $\Lambda_{\text{QCD}} = 0.5$ [GeV]. The $Q^2$ dependence of the quark structure functions can be safely neglected, since the treatment of the vector bosons as partons fixes the relevant $Q^2$ to be $Q^2 \sim M_W^2$. 

\textsuperscript{13} The approximation $\hat{s} = x$ and $\hat{t} = t$.

\textsuperscript{14} The parameter values are fixed to $\alpha_s(M_Z^2) = 0.12$, $\Lambda_{\text{QCD}} = 0.5$ [GeV].
Lepton Number Violating $e^-W^+ \to e^+W^n$ and $e^-e^- \to W^-W^-$ Processes 655

Table I. The total cross section for the $e^-p \to e^+W^n$ reaction. The $m_{\nu}$ part represents contributions from both $\Lambda^{(\nu)}(s,t,u)_{m_{\nu}}$ and $\Lambda^{(\nu)}(s,t,u)_{V+A}$ while the $V + A$ part is from $\Lambda^{(\nu)}(s,t,u)_{V+A}$. The type of the final gauge boson is designated as $W_1$ and $W_2$. The units for the cross sections are [pb].

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [TeV]</th>
<th>$m_{\nu}$ part</th>
<th>$V + A$ part</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W_1$</td>
<td>$W_2$</td>
</tr>
<tr>
<td>0.2</td>
<td>$9.7 \cdot 10^{-5}$</td>
<td>$-$</td>
</tr>
<tr>
<td>0.5</td>
<td>$4.8 \cdot 10^{-9}$</td>
<td>$4.4 \cdot 10^{-7}$</td>
</tr>
<tr>
<td>0.7</td>
<td>$1.5 \cdot 10^{-6}$</td>
<td>$6.4 \cdot 10^{-6}$</td>
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<td>$1.9 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>2.0</td>
<td>$4.6 \cdot 10^{-4}$</td>
<td>$1.0 \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

The distribution function $f_{p/W}(x)$ in Eq. (6.2) is interpreted as a so-called luminosity function. It is a decreasing function of $x$, and its magnitude is about $10^{-1}\text{--}10^{-2}$ at $x \approx 0.1$, while $10^{-4}$ at $x \approx 0.5$. The distribution function $f_{p/W}(x)$ becomes larger than $f_{q/W}(x)$ in the smaller $x$ region. This implies that the reaction in Eq. (6.1) becomes important at higher energy, because the contribution from a smaller $x$ dominates as the incident energy increases ($x_{\text{min}} \sim M_W/\sqrt{s}$).

The energy dependence of cross sections for the $e^-p^+ \to e^+W^-n$ reaction is weakened in comparison with the $e^-W^+ \to e^+W^-$ reaction. This is because the distribution function $f_{p/W}(x)$ makes the cross section $d\sigma^{(ep)}(s,t)$ increase as the incident energy grows, although the cross section $d\sigma^{(eW)}(\hat{s},\hat{t})$ itself decreases, as discussed in the preceding section.

The total cross sections are estimated by adopting the same values for parameters as in the preceding section. In performing the $t$-integral numerically, the resonant singularity in the $\hat{u}$-channel is cut off with the electron mass squared. The resultant orders of magnitude are tabulated in Table I for some incident energies. The cross section due to the $V + A$ part is suppressed by about $10^{-12}$ in comparison with the non-vanishing neutrino mass part because of a very small mixing angle $\theta_{LR}$ between left- and right-handed neutrinos.

The estimation here is somewhat crude, and numerical results depend largely on values of parameters. But it may be said that cross sections of the $e^-p \to e^+W^-n$ reaction are of measurable order of magnitude in lower energy experiments. It is an interesting problem to refine the treatment given here, and this will be done in a forthcoming paper.

In summary, the $e^-W^+_a \to e^+W^-_b$ process is a useful test of the Majorana nature of neutrinos. Each neutrino mass $m_j$ can be determined by analyzing the angular distribution, which is resonant at $\cos \theta_j = -1 + 2(M^2_{\nu_j} - m_{\nu_j}^2 s)/(s - M^2_{\nu_j})(s - M^2_{\nu_b})$. The magnitude of the cross section was estimated by using present bounds on the parameters. The cross sections are about $10^{-1}\text{--}10^{-3}$ [pb] for a non-vanishing neutrino mass, but they are highly suppressed for the $V + A$ part due to the small mixing between left- and right-handed neutrinos. The $e^-e^- \to W^-W^-$ process has about a $10^{-2}$ times smaller cross section than the $e^-W^+_a \to e^+W^-_b$ process. Other processes $e^-p \to e^+W^-n$ which include $e^-W^+ \to e^+W^-$ as a sub-process were also discussed,
and the magnitude of their cross sections was estimated.

**Appendix A**

**Derivation of the Cross Section**

Derivation of the differential cross section is sketched for the \( e^{-} W_{a}^{+} \rightarrow e^{+} W_{b}^{-} \) process. The other \( e^{-} e^{-} \rightarrow W_{a}^{-} W_{b}^{-} \) case is obtained merely by interchanging Mandelstam variables appropriately.

**A.1. The case of neutrino exchange**

By taking the sum over chiralities \( \alpha \) and \( \beta \) for the charged current, the invariant amplitude \( T_{ab} \) in Eq. (3.7) can be rewritten as

\[
T_{ab} = \frac{1}{4} e^{\nu}(\lambda') e^{\mu}(\lambda) \sigma^{C}(p', \sigma') \left( \Gamma_{\nu \mu}^{(s)} + \Gamma_{\nu \mu}^{(u)} \right) u(p, \sigma),
\]

where

\[
\Gamma_{\nu \mu}^{(s)} = \frac{\gamma_{\nu}[m_{j}^{(+)} + m_{j}^{(-)} \gamma_{5} + 2 \zeta_{j}^{(+)}(\not{p} + \not{k}) + 2 \zeta_{j}^{(-)} \gamma_{5}(\not{p} + \not{k})] \gamma_{\mu}}{s - m_{j}^{2}},
\]

\[
\Gamma_{\nu \mu}^{(u)} = \frac{\gamma_{\nu}[m_{j}^{(+)} + m_{j}^{(-)} \gamma_{5} + 2 \zeta_{j}^{(+)}(\not{p} - \not{k'}) - 2 \zeta_{j}^{(-)} \gamma_{5}(\not{p} - \not{k'})] \gamma_{\mu}}{u - m_{j}^{2}}.
\]

The \( m_{\nu} \) and \( V + A \) parts enter through parameters \( m_{j}^{(\pm)} \) and \( \zeta_{j}^{(\pm)} \), respectively, which are defined as

\[
m_{j}^{(\pm)} = m_{j} \left[ (\rho_{j})_{Lb}(\rho_{j})_{La} \pm (\rho_{j})_{Rb}(\rho_{j})_{Ra} \right],
\]

\[
\zeta_{j}^{(\pm)} = \frac{1}{2} \left[ (\rho_{j})_{Lb}(\rho_{j})_{Ra} \pm (\rho_{j})_{Rb}(\rho_{j})_{La} \right],
\]

where the coupling parameter \( \rho_{j} \) is defined in Eq. (2.12). Note that the number of gamma matrices in \( \Gamma_{\nu \mu}^{(s)} \) and \( \Gamma_{\nu \mu}^{(u)} \) is even for the \( m_{\nu} \) part and odd for the \( V + A \) part.

The function \( A_{ab}^{(W)}(s, t, u) \) in Eq. (3.2) is an invariant amplitude squared \( |T_{ab}|^{2} \) for which summation over initial and final spin polarization states are taken. It takes the form

\[
A_{ab}^{(W)} = \sum_{i=1}^{6} \sum_{j,j'} \xi_{jj'}^{(i)} \left[ \frac{F_{ab}^{(i)}(s, t, u)}{(s - m_{j}^{2})(s - m_{j'}^{2})} + \frac{C_{ab}^{(i)}(s, t, u)}{(s - m_{j}^{2})(u - m_{j'}^{2})} \right] + \left( \begin{array}{c} s \leftrightarrow u \\ M_{a} \leftrightarrow M_{b} \end{array} \right).
\]

The six internal parameters \( \xi_{jj'}^{(i)} \) are quadratic in \( m_{j}^{(\pm)} \) and \( \zeta_{j}^{(\pm)} \):

\[
\xi_{jj'}^{(1)} = \frac{1}{2m^{2}} \text{Re}[m_{j}^{(+)} m_{j'}^{(+)*} + m_{j}^{(-)} m_{j'}^{(-)*}],
\]

\[
\xi_{jj'}^{(2)} = \frac{1}{2m^{2}} \text{Re}[m_{j}^{(+)} m_{j'}^{(+)*)} \bar{m}_{j}^{(-)} m_{j'}^{(-)*}],
\]
The mass scale \(m\) has been introduced in the definition of \(\xi_{ij}^{(i)} (i = 1, 2, 3, 4, 5, 6)\) merely to make them dimensionless. Among these six internal parameters, the two with \(i = 1, 2\) stem from the \(m_\nu\) part, the two with \(i = 3, 4\) are from the \(V + A\) part, and the final two with \(i = 5, 6\) are the interference between the \(m_\nu\) and \(V + A\) parts.

On the other hand, the external factors \(F_{ab}^{(i)}(s, t, u)\) and \(G_{ab}^{(i)}(s, t, u)\) are given in terms of external momenta and gamma matrices. They become dependent solely on the Mandelstam variables and masses of scattering particles. The \(F_{ab}^{(i)}(s, t, u)\) factors originate from an invariant amplitude squared corresponding to Fig. 1(a), while the \(G_{ab}^{(i)}(s, t, u)\) factors are from the interference between Figs. 1(a) and (b); that is, the former are from the absolute square of \(\Gamma^{(s)}_{\nu\mu}\), while the latter are from the product of \(\Gamma^{(s)}_{\nu\mu}\) and \(\Gamma^{(u)}_{\nu\mu}\). In the approximation that the electron mass is ignored in comparison with gauge bosons masses, the spin sum projection for the electron (positron) may be replaced by \(\sum_{\sigma} u(p, \sigma)\overline{u}(p, \sigma) = \not{p} + m_\nu \simeq \not{p}\), because \(|p| \sim O(M_a, M_b)\) even just above the threshold. Then, out of twelve external factors, only \(F_{ab}^{(i)}(s, t, u)\) with \(i = 1, 3\) and \(G_{ab}^{(i)}(s, t, u)\) with \(i = 1, 4\) should be retained. Indeed, it can be seen by counting numbers of gamma matrices that \(F_{ab}^{(i)}(s, t, u)\) and \(G_{ab}^{(i)}(s, t, u)\) with \(i = 5, 6\) are proportional to \(m_\nu\) and that \(F_{ab}^{(i)}(s, t, u)\) with \(i = 2, 4\) and \(G_{ab}^{(i)}(s, t, u)\) with \(i = 2, 3\) are proportional to \(m^2_\nu\), if the spin sum is taken without any approximations.

After lengthy manipulations to take the trace of gamma matrices, the following results are obtained:

\[
F_{ab}^{(1)}(s, t, u) = \frac{-m^2}{(M_a M_b)^2} \left\{ s|st + 2(M_a^2 + M_b^2)u \\
+ 2(M_a M_b)^2[2t - (M_a^2 + M_b^2)] \right\},
\]

(A.13)

\[
F_{ab}^{(3)}(s, t, u) = \frac{-2}{(M_a M_b)^2} \left\{ s^2[su + 2(M_a^2 + M_b^2)t - (M_a M_b)^2] \\
+ 4(M_a M_b)^2[su - (M_a M_b)^2] \right\},
\]

(A.14)

\[
G_{ab}^{(1)}(s, t, u) = \frac{-m^2}{(M_a M_b)^2} \left\{ su[t - 2(M_a^2 + M_b^2)] \\
+ 2(M_a M_b)^2(M_a^2 + M_b^2) \right\},
\]

(A.15)

\[
G_{ab}^{(4)}(s, t, u) = \frac{2}{(M_a M_b)^2} \left\{ su[su + 2(M_a^2 + M_b^2)t - (M_a M_b)^2] \\
- 4(M_a M_b)^2(M_a^2 + M_b^2)t \right\},
\]

(A.16)
Here, repeated use has been made of the identity $s + t + u = M^2_a + M^2_b$ to express these factors compactly. We would like to emphasize again that the $m_\nu$ ($i = 1$) and $V + A$ ($i = 3, 4$) parts are completely separated in $A^{(eW)}_{ab}(s, t, u)$, and that the interference ($i = 5, 6$) between them does not play any substantial role in the scattering process.

The internal parameters $\xi_{jj'}^{(i)}$ can be explicitly written down by substituting the definition of $\rho_j$ in Eq. (2.12) into Eqs. (A.7)–(A.12):

$$
\xi_{jj'}^{(1)} = \frac{m_jm_{j'}}{m^2}\left\{h^L_{ab}g^4\Re[(U_{ej}U_{ej}')^2] + h^R_{ab}g^4\Re[(V_{ej}V_{ej}')^2]\right\}, 
$$

$$
\xi_{jj'}^{(3)} = f_{ab}(gLGR)^2\Re[(U_{ej}V_{ej})(U_{ej'}V_{ej'})^*], 
$$

$$
\xi_{jj'}^{(4)} = g_{ab}(gLGR)^2\Re[(U_{ej}V_{ej})(U_{ej'}V_{ej'})^*].
$$

Here, $h^L_{ab}$ and $h^R_{ab}$ are given in Eqs. (3.9) and (3.10), and $f_{ab}$ and $g_{ab}$ are defined as

$$
f_{ab} = \begin{cases} 
(\sin \zeta \cos \zeta)^2 & \text{for } a = b, 1, 2, \\
\frac{1}{2}(\sin^4 \zeta + \cos^4 \zeta) & \text{for } a \neq b.
\end{cases}
$$

$$
g_{ab} = \begin{cases} 
(\sin \zeta \cos \zeta)^2 & \text{for } a = b, 1, 2, \\
-(\sin \zeta \cos \zeta)^2 & \text{for } a \neq b.
\end{cases}
$$

The cross section for the $m_\nu$ part is obtained from Eq. (A-6) by using $F_{ab}^{(1)}(s, t, u)$ of Eq. (A-13), $G_{ab}^{(1)}(s, t, u)$ of Eq. (A-15) and $\xi_{jj'}^{(1)}$ of Eq. (A-17). Some manipulations would be necessary to have a final simple form for $A^{(eW)}_{ab}(s, t, u)_{m_\nu}$ in Eq. (3.8).

Similarly, the $V + A$ part $A^{(eW)}_{ab}(s, t, u)_{V + A}$ in Eqs. (3.11) and (3.12) is obtained from external factors $F_{ab}^{(3)}(s, t, u)$ and $G_{ab}^{(3)}(s, t, u)$ in Eqs. (A-14) and (A-16) and the internal factors $\xi_{jj'}^{(3)}$ and $\xi_{jj'}^{(4)}$ in Eqs. (A-18) and (A-19).

### A.2. The case of Higgs particle exchange

Let us study the case in which contribution from the Higgs particle exchange mechanism is taken into account. The total invariant amplitude is given by the sum $T_{ab} + T'_{ab}$, where $T_{ab}$ due to the neutrino exchange is in Eq. (A-1), and the other $T'_{ab}$ due to the Higgs particle exchange is in Eq. (3.14). In order to obtain the full cross section, the term

$$
A'^{(eW)}_{ba} = \sum_{a=L,R} \left\{ \xi^{(1)}(\alpha) H^{(1)}_{ab}(s, t, u) \frac{H_{ab}^{(2)}(s, t, u)}{(t - M^2_{\Delta_a})(t - M^2_{\Delta_R})} + \sum_{i=3}^{6} \sum_j \xi^{(i)}_{jj'}(\alpha) \frac{H^{(i)}_{ab}(s, t, u)}{(t - M^2_{\Delta_a})(s - m^2_j)} + \left( \begin{array}{c} s \\ u \\ M_a \\ M_b 
\end{array} \right) \right\},
$$

should be added to $A_{ab}^{(eW)}$ in Eq. (A-6). Here, new internal quantities are defined as follows:

$$
\xi^{(1)}(L) = \frac{1}{m^2}g^4_L(\omega_{LL}\omega_{La})^2|\langle M_L, c e |^2,
$$

(A-23)
amplitudes that the external factors only

H

that the electron mass can be neglected in comparison with gauge boson masses,

is the interference between the

Eqs. (2.12) and (3.16) into Eqs. (A.23) and (A.25):

The relevant internal quantities can be explicitly written down by substituting

Here,

h

with the

neutrino exchange (Figs. 1(a) and (b)); that is, those with

i

stand for the interference of the Higgs scalar exchange mechanism (Fig. 2) with the

m

part, and those with

i

= 3, 4 are the interference with the

m

v

tax and those with

i

= 5, 6 are the interference with the

V

+ A part.

It can be seen by counting the number of gamma matrices in the invariant

amplitudes that the external factors

H

ab

(s, t, u) with

i

= 5, 6 are proportional to

m

e

and that

H

ab

(s, t, u) with

i

= 2, 4 are proportional to

m

2

e

. In the approximation

that the electron mass can be neglected in comparison with gauge boson masses,

only

H

ab

(s, t, u) and

H

ab

(s, t, u) contribute to the scattering. They are calculated as

\[
H^{(1)}_{ab}(s, t, u) = \frac{-4m^2}{(M_aM_b)^2} t [(s + u)^2 + 8(M_aM_b)^2],
\]

(A-29)

\[
H^{(3)}_{ab}(s, t, u) = \frac{-4m^2}{(M_aM_b)^2} t [s(s + u) + 4(M_aM_b)^2].
\]

(A-30)

The relevant internal quantities can be explicitly written down by substituting

Eqs. (2.12) and (3.16) into Eqs. (A.23) and (A.25):

\[
\xi^{(1)}(L) = \frac{1}{m^2} h^{L}_{ab} g^L_{1} |(M_L)_{ee}|^2,
\]

(A-31)

\[
\xi^{(3)}_j(L) = \frac{m_j}{m^2} h^{L}_{ab} g^L_{1} \text{Re} [U_{ej}^2 (M_L)_{ee}^*],
\]

(A-32)

\[
\xi^{(1)}(R) = \frac{1}{m^2} h^{R}_{ab} g^R_{1} |(M_R)_{ee}|^2,
\]

(A-33)

\[
\xi^{(3)}_j(R) = \frac{m_j}{m^2} h^{R}_{ab} g^R_{1} \text{Re} [V_{ej}^2 (M_R)_{ee}^*].
\]

(A-34)

Here, \(h^{L}_{ab}\) and \(h^{R}_{ab}\) are defined in Eqs. (3.9) and (3.10). By substituting Eqs. (A-29)–

(A-32) into Eq. (A-22), the final form of \(A^{(\text{elW})}_{ab}(s, t, u)_H\) in Eq. (3.17) is derived.
References