Characteristics of Dressed Particles in the 1D Correlated Electron Systems

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(Received June 21, 1990)

Characteristic properties of the dressed particles, holon and spinon, in the one dimensional correlated electron systems are investigated by means of the exact solution of the Hubbard model. In particular, the nature of holon is clarified in detail in the system close to the Mott insulating phase. The dressed particles in magnetic fields are also formulated and discussed in terms of the bulk quantities. The exact results obtained for the correlation functions are mentioned in connection with the fractional charge of the dressed particles.

§ 1. Introduction

Since the discovery of the high $T_c$ superconductivity, there has been a renewed interest in the highly correlated electron system in low dimensions. In particular, the strong correlation problem near the Mott insulator has been expected to offer a key idea to solve this challenging problem. In this connection, the one dimensional (1D) correlated system has been studied intensively. The metallic properties of 1D correlated systems are not of usual Fermi liquid but of marginal liquid (Luttinger liquid, see Ref. 10) in detail. This fascinating liquid state has its origin, of course, in the low-dimensional quantum fluctuation. The elementary excitations in such metallic systems are described by the soliton like excitations in place of the quasiparticle excitations in the usual Fermi liquid theory. In order to describe the elementary excitations in 1D systems, it is necessary to introduce two kinds of the dressed particles related to the charge and spin fluctuations. In the metallic system close to the Mott insulator, the dressed particles are affected by the formation of the Hubbard gap, in particular for charge excitations. As a result, the corresponding bulk quantities are drastically modulated by the formation of the Hubbard gap.

In this paper, we discuss the characteristic properties of the dressed particles in the Hubbard chain by means of the exact solution. In § 2, we survey the results obtained for the charge excitations in the system close to the Mott insulating phase and clarify the properties of dressed particles in connection with the bulk quantities. The exact calculation of the correlation functions is also mentioned, because the basic formulation is directly related to the fractional charge of the dressed particles discussed here. In § 3, the dressed particles are investigated in the presence of the magnetic field. We show that the dressed particles introduced here clearly describe the elementary excitations in the Hubbard chain, especially in the
vicinity of the Mott insulating phase. We use terms holon and spinon, for simplicity, to express the dressed particles appearing in the exact solution.

§ 2. Elementary excitations in Mott insulating phase

We consider the one-dimensional Hubbard chain described by the Hamiltonian

\[ H = -t \sum_{\langle ij \rangle, \sigma} a_{i \sigma}^\dagger a_{j \sigma} + 4U \sum_i a_{i \uparrow}^\dagger a_{i \uparrow} a_{i \downarrow} a_{i \downarrow}, \]  

where the electron is assumed to transfer between the nearest-neighbor sites. The integrability of the above Hamiltonian is ensured by the Yang-Baxter relation which factorizes the many-body scattering matrix into the two-body one. The diagonalization of the scattering matrix is completed by introducing two kinds of the rapidities \( k_j \) and \( \Lambda_a \) which specify the elementary excitations concerning the charge and spin excitations generally. The resulting Bethe equation obtained by Lieb and Wu has the following form:

\[ \exp(ik_jN_a) = \prod_{j=1}^{N} e^{[\sin(k_j) - \Lambda_a]} \quad j=1 \sim N, \]  

\[ -\prod_{j=1}^{N} e^{[\sin(k_j) - \Lambda_a]} = \prod_{j=1}^{M} e^{[(\Lambda_a - \Lambda_a)/2]}, \quad a=1 \sim M, \]  

where \( e[x] = (x + iU/t)/(x - iU/t) \), \( N_a \) is the number of the lattice point and \( N(M) \) is the total (up-spin) electron number. For example, see references for the standard techniques to deal with the zero\textsuperscript{12,13}\textsuperscript{4} and finite temperature cases.\textsuperscript{15,4}  

2.1. Dressed particles

The classification of the spin and charge excitations is performed by examining the deviation of distribution for the rapidities from their ground state ones, \( \rho(k) \) and \( \sigma(\Lambda) \). This classification was first done by des Cloizeaux and Pearson\textsuperscript{16} for the Heisenberg model and by Ovchinnikov\textsuperscript{17} for the Hubbard model. Since the spin excitation has been discussed in detail elsewhere,\textsuperscript{16,17,18} we outline the formation of the dressed particle concerning the charge excitations in the Mott insulating phase. The simplest excitation including the change of the electron charge is obtained by inserting a hole into the half filled band at a given value of the charge rapidity \( k_0 ( -\pi \leq k_0 \leq \pi ) \).\textsuperscript{19,20,5} The creation of the hole state, at the same time, causes the change of the ground-state distribution both for the charge and spin rapidities. For instance, the change of the density function \( \rho_1(k) \) for the charge rapidity is given as,

\[ \rho_1(k) = -\delta(k - k_0) - \cos kR(\sin k - \sin k_0), \]  

which shows that the bare change (\( \delta \)-function part) is accompanied by the back flow effect, where \( R(x) \) is given by the Fourier transform of \( [1 + \exp(2U|\omega|t)]^{-1} \). This kind of excitation describes that the charged hole with no spin moment is doped into the Mott insulator.\textsuperscript{5} As a consequence of the back flow effect, the energy and momentum are strongly renormalized to form the dressed particle. The dressed particle specified by the above excitation is referred to as holon in the following
The dressed charge of holon $Z_c(k_0)$ is unity in units of $e$ and its energy is given by the sum of the bare energy change $(2t\cos k_0)$ and the back flow one,$^{5,19,20}$
\[
\varepsilon = 2t\cos k_0 + 2t \int_{-\infty}^{\infty} dy \frac{f(y)}{4U} \tanh \frac{\pi y}{2U},
\]
where
\[
f(y) = \int_{-\pi}^{\pi} d\cos k^2 \left( \frac{U/\pi t}{\sin k_0 - \sin k - y} \right)^{-1} + \left( \frac{U}{\pi t} \right)^{-1}.
\]
Since this formula is not suitable for the analytic calculation, we rewrite it into a more convenient form.$^5$ Integration over $k$ in $f(y)$ is performed along the unit circle in the complex plane and then the integral path of $y$ is shifted to the one along the branch cut including half of the contour integral around the pole $y = iU/t$. As a result, the following expression is obtained,
\[
\varepsilon = -2U + 4t\cos k_0 \theta(\pi/2 - |k_0|) + t^2 \int_{-\infty}^{\infty} \frac{dz}{U} (z^2 - 1)^{1/2} \sum_{r \neq 1} \tanh \frac{\pi t}{2U} (z + r\sin k_0),
\]
where $\theta(x)$ is a step function. It is noted that we can choose a successive integer as the quantum number which specifies the eliminated rapidity $k_0$. This fact enables us to calculate the density of states for holon.$^5$ The results calculated for the density of states $D_c(\omega)$ are shown in Fig. 1(a) for two typical values of the Coulomb interaction. Here we use the energy $\omega$ measured from the chemical potential ($\mu = -2U$) instead of $\varepsilon$. In addition, the density of states for the dressed particle of the upper Hubbard band is also shown. A noticeable feature caused by the Coulomb interaction is the divergent behavior of $D_c(\omega)$ in the vicinity of the Hubbard gap, which does not exist in the non-interacting case. This fact directly reflects the formation of the Hubbard gap accompanied with the Mott insulating phase. In order to understand this behavior more clearly, we derive the expression for $D_c(\omega)$ in a closed form in the energy region close to the gap-edge.$^5$
\[
D_c(\omega) \sim (2\pi \sqrt{t^*(0)})^{-1} [\omega - \Delta^2/2]^{-1/2},
\]
where the magnitude of the Hubbard gap $\Delta$ is
\[
\Delta = (4t^2/U) \int_{-1}^{1} dz (z^2 - 1)^{1/2}
\times \tanh (\pi tz/2U),
\]
and $t^*(0)$ is defined by
\[
t^*(0) = (\eta_0/\eta^2) t,
\]
where $\eta_0$ and $\eta$ are the Hubbard band parameters. For the case of $U/t = 0.5$ and $1.0$, respectively, the density of states $D_c(\omega)$ is shown in Fig. 1(b).
The renormalized transfer integral $t^*(0)$ becomes exponentially large with the decrease of the Coulomb interaction, while it leads to the bare value $t$ in the large $U$ region (spinless fermion). This behavior characterizes the crossover from the weak to strong coupling regime, as will be discussed below. It is worthwhile to note that the band width of the energy spectrum is not renormalized and takes the value of $4t$ irrespective of the strength of the Coulomb interaction.

Here, we briefly comment on the formation of spinon in the thermodynamic limit. As pointed out by Faddeev and Takhtajan, the dressed particle in this case is a kink-like doublet excitation. This is formulated in terms of the back-flow effect mentioned above. The insertion of the hole state into the spin rapidity causes the bare spin deviation $s_z=1$ in units of $g_{1B}$. As in holon case, the back flow effect is induced to partially cancel the spin deviation, resulting in the dressed spin particle with $s_z=1/2$, which is essentially same as discussed by Faddeev and Takhtajan. We show the density of states for the spin excitations in Fig. 1(b). It is seen that the width of the spectrum becomes narrower with the increase of the interaction strength. In the large $U$ limit the density of states is described by that for the Heisenberg model, as should be.

2.2. Relation to bulk quantities

The density of states for holon introduced above plays an essential role for the various physical quantities concerning the charge excitations near the insulating phase. For instance, we consider the compressibility at zero temperature. This quantity was calculated exactly in a closed form near half filling as,

$$\chi_c = \left[ 2\pi \bar{t}^*(0) (1-n) \right]^{-1} \quad \text{for} \quad n \approx 1, \quad (2.9)$$

where $n$ is the electron concentration with $n=1$ being for half filling. It should be noted that the quantity $\chi_c$ is considerably enhanced by the factor $(1-n)^{-1}$ near the Mott insulating phase according to the formation of the Hubbard gap. It is easily checked from (2.6) and (2.9) that the divergent behavior of $\chi_c$ for $n \rightarrow 1$ directly reflects the density of states for holon. An important thing is that it can be expressed in terms of $D_c(\omega)$ like a non-interacting form, which means holon being essential to express the charge excitations.

The specific heat coefficient has a similar divergent behavior near half filling ($n \approx 1$). We can derive the exact relation among the specific heat coefficient $\tilde{\gamma}$, spin susceptibility $\tilde{\chi}_s$ and compressibility $\tilde{\chi}_c$, \(^{(3,4)}\)

$$\tilde{\gamma} = \frac{1}{2} \left[ \tilde{\chi}_s + 2[Z_c(D)]^{-2} \tilde{\chi}_c \right] \quad (2.10)$$

for arbitrary filling $n$, where tilde means the quantities being normalized at $U=0$ and $n=1$. Here $Z_c(k)$ is the dressed charge of holon defined by
Characteristics of Dressed Particles in the 1D Correlated Electron Systems

\[ Z_c(k) = \int_{-D}^{D} \cos k'R(\sin k - \sin k') \times Z_c(k') \, dk' = 1, \quad (2\cdot11) \]

where \( D \) is the holon “Fermi level” which is determined by the condition that holon energy should vanish at ±\( D \). The dressed charge takes the value of unity at half filling, as discussed above, but generally takes the fractional values ranging \( 1 \leq Z_c \leq \sqrt{2} \) for \( n \neq 1 \). We can see from (2\cdot10) and (2\cdot11) that the specific heat coming from the charge excitation part is determined by the holon properties in a way slightly different from the compressibility. In the vicinity of half filling, the dressed charge takes the value of unity \( Z_c = 1 \), so that the specific heat coefficient is given by \( \tilde{\gamma} = 2Z_c \sim (1-n)^{-1} \) in this region\(^{3,4}\). Therefore the divergent behavior appearing in \( \tilde{\gamma} \) also reflects the density of states for holon. As a reference, we show the specific heat coefficient as a function of the electron concentration in Fig. 2. We note that the bulk quantities at finite temperatures were discussed in detail in Ref. 4).

2.3. Universal behavior, crossover and strong coupling

At half filling, Coulomb interaction opens the Hubbard gap for the charge excitation and the remaining degree of freedom is described by massless spinon excitation which has the same fixed point for the Heisenberg model \((U \to \infty)\). As for the charge excitation, holon has the fixed point same as the spinless fermion. Therefore the essential properties of holon are understood at least qualitatively in terms of the spinless fermion. This statement is consistent with the properties discussed above: holon behaves as a band edge hole irrespective of the Coulomb interaction as long as the Hubbard gap exists \((U > 0)\). There is a crossover from the weak to strong coupling regime with the increase of the Coulomb interaction\(^{5}\). Here we examine this crossover by investigating the properties of holon as well as the bulk quantities.

In weak coupling limit \((U \sim 0)\), we show that all the quantities concerning the charge excitations can be described by only one relevant parameter\(^{5}\). In this case, the density of states near the Hubbard gap is reduced to

\[ D_c(\omega) \sim \frac{1}{2\pi} \sqrt{\Delta_0/t^2} [\omega|\Delta_0/2)]^{-1/2}, \quad (2\cdot12a) \]

where the magnitude of the Hubbard gap takes the form in this limit,

\[ \Delta_0 \sim (16/\pi)tU \exp(-\pi t/2U). \quad (2\cdot12b) \]

On the other hand, the exact compressibility is given by\(^{5}\).
Therefore in the weak coupling regime, we have the universal behavior for the charge excitation near the Mott insulator with the relevant energy scale $\Delta_0$ which is exponentially small as compared with the bare interaction energy. This behavior is expected to be independent of models including the Hubbard gap and is essentially same as the one expected for the Tomonaga-Luttinger model including the Umklapp interaction. In this parameter regime, the insulating phase is formed by the electron-hole pair with opposite spins, which is considerably extended spatially. For instance, the correlation length of the electron-hole pair is roughly given by $\xi \sim t/\Delta_0$ in the lattice spacing unit. When the Coulomb interaction increases, the correlation length $\xi$ becomes small and the pair is localized at a specific site for the infinitely large interaction, which corresponds to the insulating phase in the strong coupling regime. The crossover is also characterized by the exponentially large transfer integral $t^\ast(0)$ in the weak coupling regime and the bare integral $t$ in the strong coupling regime.

2.4. Fractional charge and correlation functions

It was difficult to obtain the correlation functions within the framework of the Bethe Ansatz solution (see for example Ogata and Shiba for $U \to \infty$ case). This problem has been resolved recently by means of conformal field theory combined with the Bethe Ansatz method. Here we quote the results obtained for the Hubbard model (see also Ref. 26), because the basic idea is closely related to the charge of the dressed particles introduced above. The finite-size correction approach developed in conformal field theory enables one to exactly calculate the power-law exponent of the long-distance correlation functions in terms of the fractional charge of the corresponding dressed particles. In the metallic phase of the Hubbard model, there are two kinds of massless excitations, holon and spinon, both of which give rise to the power law behavior in the correlation functions. The long distance behavior of the spin correlation function has the $2k_F$ oscillating term as well as the non-oscillating term, while the density correlation function has another contribution of the $4k_F$ term (omitting logarithmic correction),

$$\begin{align*}
\langle s_\alpha(x)s_\alpha(0) \rangle &\sim B_0 x^{-2} + B_1 x^{-\delta_\alpha} \cos 2k_F x, \\
\langle \rho(x)\rho(0) \rangle &\sim \text{const} + A_0 x^{-2} + A_1 x^{-\delta_\alpha} \cos 2k_F x + A_2 x^{-\delta_\alpha} \cos 4k_F x
\end{align*}$$

with $k_F$ being the Fermi momentum. The exponents for these correlation functions are determined by dressed charges of spinon and holon. Note that the dressed charge of spinon corresponds to the effective spin $s_z$ mentioned before. The $4k_F$ oscillating part is caused purely by the charge degree of freedom, hence the corresponding exponent $\delta_c$ is determined by the fractional charge of holon as $\delta_c = 2Z_c(D)^2$, where $Z_c$ is defined in Eq. (2.11). On the other hand, the exponent of $2k_F$ oscillating part is given by the sum of the effective spin and the fractional charge of holon. Since the effective spin of spinon is given by $2s_z = 1$ irrespective of the interaction strength, $\delta_c = a_0 + \beta_c/4$. Using the Luttinger-liquid nature of the present system, we can also estimate the exponent $\theta$ of the power anomaly in the momentum distribu-
§ 3. Dressed particles in magnetic fields

In this section, we consider the characteristics of the dressed particles in the presence of the magnetic field $H$. We introduce holon and spinon following the way in the previous section. In order to create holon, the $\delta$ function is inserted into the continuous distribution of the charge rapidity. The bare change due to the perturbation is accompanied by the back-flow effect. The deviation $\tilde{\rho}_1$ of the distribution function from the ground state is,

$$\tilde{\rho}_1(k) = \rho_1(k) - \cos k \int_{|\Lambda| > B} P(\Lambda - \sin k) \tilde{\sigma}_1(\Lambda) d\Lambda,$$

where $\rho_1(k)$ is the distribution function of holon in the absence of the magnetic field and

$$\tilde{\sigma}_1(\Lambda) = \int_{|\Lambda'| > B} R(\Lambda - \Lambda') \tilde{\sigma}_1(\Lambda') d\Lambda' = -P(\Lambda - \sin k_0)$$

These exponents were evaluated for arbitrary filling and interaction in Ref. 22. As a reference, we show the exponent $\theta$ as a function of the electron concentration in Fig. 3. It should be noted that in the vicinity of the half filling, all the exponents have the values expected for the infinite $U$ case, $\beta_c = 2$, $a_c = a_s = 3/2$ and $\theta = 1/8$, irrespective of the interaction strength. This result reflects the fact mentioned before that holon in the Mott insulator behaves as if it is a band edge hole as far as the Hubbard gap is retained.

Finally, we mention that the correlation functions for the one-dimensional $t$-$J$ model ($t=J$) were also calculated (Kawakami-Yang, Phys. Rev. Lett. 65 (1990), 2309). This model is formally derived in the strong correlation limit of the Hubbard model, but the resulting antiferromagnetic coupling constant $J$ is considered as a free parameter. It was proved that the same scaling relations listed above are valid for the $t$-$J$ model and that $\beta_c$ changes from 2 to 4 as the filling factor $n$ changes from 1 to the vanishing value. Near half filling $\beta_c$ is given by $\beta_c \sim 2 + 4(1 - n)$, while in the low density limit $\beta_c$ approaches 4 which is expected for the non-interacting model. We think that the latter peculiar behavior may not be understood in the strong correlation limit of the Hubbard model, but is inherent in the artificial $t$-$J$ model.
Fig. 4. Density of states for holon in magnetic fields. We show the behavior of the upper Hubbard band near the gap edge. The critical field $H_c$ is defined in text.

$$w = e + 2U.$$ with $P(\Lambda) = (t/4U) \text{sech}(\pi t \Lambda / 2U)$. Here the cutoff parameter $B$ corresponds to the “Fermi level” of spinon under the magnetic field. It is noted that the dressed charge of holon is not affected by the magnetic field and takes the value of unity, $Z_e = 1$, regardless of the magnetic field strength. This is because the sum rule

$$\int_{-\pi}^{\pi} \tilde{P}(k) dk = -1$$

holds even in the presence of the magnetic field.

The excitation energy of holon is given by

$$\epsilon = \epsilon(H=0) - \int_{|\Lambda| > B} P(\Lambda - \sin k_0) g_1(\Lambda) d\Lambda,$$

where

$$g_1(\Lambda) - \int_{|\Lambda| > B} R(\Lambda - \Lambda') g_1(\Lambda') d\Lambda' = 2t \int_{-\pi}^{\pi} P(\Lambda - \sin k) \cos^2 k dk - \mu_B H.$$

The density of states for holon in the magnetic field is calculated following the procedure in the preceding section. The results are shown in Fig. 4 for several magnetic fields. We note that the band width of holon takes the value of $4t$ regardless of the magnetic field. In the vicinity of the gap edge, the density of states for holon is expressed analytically as

$$D_e(\omega) = \frac{1}{2\pi \sqrt{t^*(H)}} (|\omega| - \Delta_\Phi(H)/2)^{-1/2},$$

where $\omega = \epsilon + 2U$. Here the renormalized transfer integral $t^*(H)$ is

$$t^*(H) = \eta_0 t/[2\pi \rho(\pi)^2],$$

where

$$\eta_0 = \eta_0 - \frac{1}{2} \int_{|\Lambda| > B} \frac{d^2 P(\Lambda)}{d^2 \Lambda} g_1(\Lambda) d\Lambda$$

and $\rho(k)$ is the ground state distribution of the charge rapidity at $H \neq 0$. It is seen that the effects of the magnetic field as well as the correlation effects are incorporated into the renormalized transfer integral $t^*(H)$. It becomes bare value when the applied field is larger than the critical value $H_c$.

$$\mu_B H_c = 2t [\sqrt{(U/t)^2 + 1 - U/t}],$$

where the system is completely polarized magnetically. On the other hand, in weak
fields, $t^*(H)$ is expanded in power series of $H$ as,

$$t^*(H) \sim t^*(0)[1 + a_0(\mu_B H/t)^2], \quad (3\cdot10)$$

where

$$a_0 = \frac{\pi t^2}{16 U^2 I_0(\pi t^2/2U)^2} \frac{t I_0(\pi t|2U|)}{8\pi U I_0(\pi t|2U|^2) \rho(H=0)} \quad (3\cdot11)$$

and $I_0(z)$ and $I_1(z)$ are the modified Bessel functions. In a similar way, the compressibility in the magnetic field is obtained near half filling as,

$$\chi_c = \frac{1}{2\pi^2 t^*(H)} (1-n)^{-1}. \quad (3\cdot12)$$

Therefore, even in the magnetic fields, holon inserted into the Mott insulator behaves as a band edge hole with the renormalized transfer $t^*(H)$ which depends on the strength of the magnetic field as well as the Coulomb interaction. The bulk quantities are also expressed simply in terms of $t^*(H)$.

An essential difference from the zero field case is that holon cannot be treated as a spinless hole anymore under the magnetic field. The magnetic field induces the effective spin $\bar{s}_h$ of holon as,

$$\bar{s}_h = \frac{1}{2} \int_{|A| \geq B} \sigma_1(A) d\Lambda. \quad (3\cdot13)$$

We show the field dependence of the effective spin $\bar{s}_h$ in Fig. 5. In weak fields, one gets the expression for the effective spin of holon,

$$\bar{s}_h = \frac{1}{I_1(\pi t|2U|)} (\mu_B H/t), \quad (3\cdot14)$$

and it is monotonically increased up to the maximum value $\bar{s}_h = 1/2$ at $H = H_c$. It is a crucial problem whether the effective spin of holon in magnetic fields is physically meaningful or not. Below, we show that the field dependence of the bulk quantities near half filling can be indeed expressed in terms of the effective spin of holon and hence holon is well-defined particle even in the magnetic field as far as we are concerned with the system close to the insulating phase.

First, we calculate the field dependence of the Hubbard gap by the physical consideration, which is more or less helpful to understand the meaning of the effective spin of holon. We estimate the energy increase by creating holon following the zero-field case. We then take into account that holon has the effective spin given by Eq. (3\cdot13) and add
0.4

\[ \Delta_{\text{Hubbard}}(H) = \Delta_{\text{Hubbard}}(0) + \int_{|\Lambda| \gg \Lambda_0} d\Lambda \int_{-\pi}^{\pi} dk \left[ 4tP(\Lambda - \sin k)\cos^2 k - 2\mu_B \right] \sigma_1(\Lambda). \] 

(3.15)

It is shown that above formula derived by the physical consideration is the same as that calculated directly from the lowest energy of the particle-hole pair excitations. This fact means that we may consider holon with the effective spin (3.13) as the dressed particle concerning the charge excitations in magnetic fields. We plot the field dependence of the Hubbard gap in Fig. 6. In weak fields, the Wiener-Hopf method enables us to write down the explicit form of the Hubbard gap,

\[ \Delta_{\text{Hubbard}}(H) \sim \Delta_{\text{Hubbard}}(0) + \frac{t}{\pi I_1(\pi t/2U)}(\mu_B H/t)^2, \] 

(3.16)

which has the form explicitly including the effective spin of holon (see (3.14)).

The spin susceptibility in the vicinity of the half filling is expanded in terms of the hole concentration \((1-n)\),

\[ \chi_s \sim \frac{I_0(\pi t/2U)}{I_1(\pi t/2U)} - \frac{1}{I_1(\pi t/2U)}(1-n). \] 

(3.17)

We should stress again that the correction term due to the deviation from half filling directly reflects the effective spin of holon.

Next we consider the spinon excitations. To create spinon, we insert the \(\delta(\Lambda - \Lambda_0)\) function into the continuous distribution of the spin rapidity \(\Lambda\). It can be proved that the dressed energy of spinon is given by the function \(|\sigma_1(\Lambda_0)|\) defined in Eq. (3.5). In Fig. 7 the excitation spectrum for spinon is shown as a function of the momentum \(Q\). It is seen that the spectrum is affected by the magnetic field. There is a zero energy excitation at a characteristic momentum \(Q = Q_s\) as well as at \(Q = 0\). Since \(Q_s\) is decreased monotonically with the increase of the magnetic field, the Fermi radius of spinon may be considered to be reduced by the magnetic field. As will be mentioned in the discussions, however, it does not naively mean the real Fermi
radius being reduced. We should take into account both of the holon and spinon contributions.

Finally we make a brief comment on the effective spin of spinon in magnetic fields, which is given by the equation,\(^{22}\)

\[
s_z(A_0) - \int_{|\Lambda'\rangle > B} R(A_0, \Lambda') s_z(A') dA' = 1/2.
\]

It is noted that \(s_z(A_0)\) changes its value according to the strength of the magnetic field. As discussed for the dressed charge of holon, the effective spin in fields can solely determine the critical exponent of the \(2k_F\) oscillating part of the spin correlation function just at half filling: \( \alpha_s = 2|s_z(B)|^2 \). Therefore it is easily seen that the exponent \( \alpha_s \) at half filling is increased from unity to two with the increase of the magnetic field.\(^{22}\)

§ 4. Summary and discussion

We investigated the characteristic properties of the dressed particles in the one dimensional Hubbard model. In particular, the nature of holon was clarified in detail in connection with the bulk quantities in the metallic system near the Mott insulating phase. The dressed particles introduced here can explain the essential properties of bulk quantities and also the correlation functions in terms of the dressed energy, the dressed charge and so on. It is, however, necessary to treat physical quantities carefully, because the specification of the physical quantities is not so trivial; whether they are inherent in the charge or spin excitations. For instance, we see that the \(4k_F\) oscillating part of the density correlation function is controlled only by the holon excitation but the \(2k_F\) part has both of the spinon and holon contributions. Furthermore, because the Fermi momentum \(k_F\) is determined by eliminating one electron from the Fermi sea, it should be determined by the sum of the “Fermi momentum” of holon and spinon.\(^7\)

We were concerned chiefly with the characteristics of the metallic system very close to the insulating phase. For an arbitrary filling \((n = 1)\), both of the holon and spinon excitations are massless. In such a general case also, the dressed particles can explain the essential properties of the charge and spin excitations. For instance, the exact relation (2·10) can be derived by using the density of states and the fractional charge of the holon and spinon. It is worthwhile to point out that the present specification of the dressed particles is essentially the same as that appearing in the bosonization method\(^9,10\) for the Tomonaga-Luttinger model, as far as the low energy excitations are concerned. In both classifications, the elementary excitation spectra are formulated in a non-interacting particle form. We note that the correlation functions mentioned here have been also derived by the generalized bosonization method.\(^{26}\)

The present treatment of the dressed particles can be extended to dynamical quantities. For instance, the low frequency dynamical correlation functions are calculated with the aid of the conformal field theory. The corresponding critical exponents are expressed by the fractional charge mentioned here, which will be
discussed elsewhere.

**Acknowledgements**

We express our sincere thanks to T. Usuki for useful discussions. The present work is partly supported by the Grant-in-Aid for Scientific Research on Priority Areas from the Ministry of Education, Science and Culture.

**References**

8) See for example,