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Storage Coefficient Determination from Quasi-Steady State Flow

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A method has been proposed for determining a unique storage coefficient value for confined and unconfined aquifers tapped by a large diameter well. The prerequisites for the application of this method are estimation of the transmissivity value and the field measurements of well radius, pumping discharge and time-drawdown measurements at large times, or preferably at the steady or quasi-steady state flow conditions. The application of the method does not require any complicated mathematical procedure of the classical type curve matching procedures. It is recommended especially as a supplementary method to the existing techniques in determining the storage coefficient.

Introduction

Assessment of groundwater potential of any aquifer requires determination of its transmissivity and storage coefficients. Field methods of estimating these aquifer characteristics by means of either pumping or recovery tests have been used widely in practise. Generally, they include measurements of artificial discharge from an aquifer, or recharge into it, while at various intervals of time, the effects on water level as drawdown or recovery, is recorded in observation wells or in abstraction well itself. The methods of analysing the effects of lowering or raising the water levels, and the formula used to determine the aquifer characteristics have been thoroughly discussed by many researchers, such as Theis (1935), Jacob (1940), Chow (1952), Hantusch (1956), Ferris (1963), Boulton (1963), Şen (1986) and

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many others. However, the construction of an observation well is expensive and time consuming. It is therefore, desirable to have alternative methods for determining the aquifer characteristics using the pumping tests recorded in the abstraction well itself. The methods for this purpose are presented by Papadopulos and Cooper (1967) and Şen (1982). Although, the transmissivity can be very well estimated with a reasonable accuracy from the abstraction well data using these methods, it is not possible to calculate the storage coefficient from the pumped well data reliably.

Theim (1906) suggested a method for estimating the transmissivity of an aquifer using data from steady state groundwater flow towards wells. It is one of the most reliable field methods to determine the transmissivity. It does not require a complete pumping test. To the best of author's knowledge there is not yet any method to determine the storage coefficient from the steady state groundwater flow towards wells. The method presented in this paper makes use of the steady state flow measurements in the abstraction well only to estimate the storage coefficient. This method provides a quick and economical determination of the storage coefficient without any need for observation well.

Theoretical Background

Assuming the groundwater flow in an extensive, isotropic and homogeneous aquifer of uniform thickness to be Darcian, the unsteady flow equation of movement can be written as

$$\frac{\partial^2 s(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial s(r,t)}{\partial r} = \frac{S}{T}\frac{\partial s(r,t)}{\partial t} \qquad (1)$$

where

- $s(r, t)$ – drawdown
- r – distance from the well center
- t – time since pump start
- S – storage coefficient
- T – transmissivity.

However, for the steady state flow Eq. (1) becomes

$$\frac{d^2 s(r)}{dr^2} + \frac{1}{r}\frac{ds(r)}{dr} = 0 \qquad (2)$$

This equation does not include the aquifer characteristics explicitly.

On the other hand, according to the definition of the storage coefficient, the discharge must be equal to the product of the storage coefficient and the rate of decline in head integrated over the area affected by pumping. Hence,

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$$dQ(t) = Srd\theta dr \frac{\partial s(r,\theta,t)}{\partial t}$$

or in general,

$$Q(t) \equiv S\int_{r_w}^{\infty}\int_{0}^{2\pi} r\frac{\partial s(r,\theta,t)}{\partial t}\, d\theta dr \tag{3}$$

where

$s(r, \theta, t)$ = drawdown at time t and distance r from the well center
$Q(t)$ = discharge from the aquifer into the well

It is important to notice that for wells with storage capacity this discharge is not equal to the pumping discharge at all times during a pumping test. However, constant well discharge, Q, can be expressed as

$$Q \equiv Q(t) + \pi r_w^2 \frac{ds_w(t)}{dt}$$

where

r_w = well radius
$s_w(t)$ = drawdown in the abstraction well

The second term on the right hand side indicates the well storage effect. The aquifer discharge is

$$Q(t) = Q - \pi r_w^2 \frac{ds_w(t)}{dt} \tag{4}$$

For small diameter wells or wells with insignificant well storage effect the second term in Eq. (4) is negligible and therefore can be assumed as equal to zero for any time. In addition, it is also zero in the steady state flow case. Substitution of Eq. (4) into Eq. (3) yields

$$Q - \pi r_w^2 \frac{ds_w(t)}{dt} \equiv S\int_{r_w}^{\infty}\int_{0}^{2\pi} r\frac{\partial s(r,\theta,t)}{\partial t}\, d\theta dr \tag{5}$$

The integrations on the right hand side are with respect to θ and r only and therefore δt in the denominator can be thought as constant and hence Eq. (5) can be arranged as

$$Qdt - \pi r_w^2 ds(t) \equiv S\int_{r_w}^{\infty}\int_{0}^{2\pi} r\partial s(r,\theta,t)\, d\theta dr$$

or

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$$Qdt - \pi r_w^2 \; ds_w(t) = S\int_{r_w}^{\infty} 2\pi r \partial s(r,t)dr \qquad (6)$$

where the right hand side is equal to the increment in the depression cone volume $dV(t)$ within the aquifer, i.e.

$$dV(t) \equiv \int_{r_w}^{\infty} 2\pi r \partial s(r,t)dr$$

the substitution of which into Eq. (6) gives

$$Qdt - \pi r_w^2 ds_w(t) = SdV(t) \qquad (7)$$

The integration of Eq. (7) with relevant boundary conditions as $s_w(0)=0$ and $V(0)=0$, leads to

$$Qt - \pi r_w^2 s_w(t) = SV(t) \qquad (8)$$

where $V(t)$ is the depression cone volume at time t. Hence, the storage coefficient becomes

$$S \equiv \frac{Qt-\pi r_w^2 w_w(t)}{V(t)} \qquad (9)$$

Provided that the depression cone volume is known the storage coefficient can be evaluated from Eq. (9) with the drawdown measurement in the abstraction well. Eq. (9) proves to be useful especially for the steady or quasi-steady state cases similar to Jacob (1940) method requirement.

Confined aquifer – The aquifer is assumed to be pumped at a constant rate and the well penetrates the entire aquifer receiving water from the whole saturation thickness by horizontal flow. If the flow to the well is in steady state then the well discharge can be expressed as (Theim 1906)

$$Q \equiv \frac{2\pi T[s_w(t)-s(r,t)]}{\ln(r/r_w)}$$

Since we are interested in the depression cone volume, the expression for this cone can be found from this last equation by necessary arrangements as

$$r = r_w \exp\{= \frac{2\pi T[s_w(t)-s(r,t)]}{Q}\} \qquad (10)$$

The depression cone volume can be evaluated as a result of the following integration

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$$V(t) = \int_0^{s_w(t)} \pi r^2 ds(r,t) - \pi r_w^2 s_w(t) \tag{11}$$

leading to

$$V(t) = r_w Q\{\exp[\frac{4\pi T s_w(t)}{Q}]-1\}/4T - \pi r_w^2 s_w(t) \tag{12}$$

Hence, the substitution of Eq. (12) into Eq. (9) leads to

$$S = \frac{Qt - \pi r_w^2 s_w(t)}{r_w^2 Q\{\exp[\frac{4\pi T s_w(t)}{Q}]-1\}/4T - \pi r_w^2 s_w(t)} \tag{13}$$

Unconfined Aquifer – Analysis of groundwater flow in unconfined aquifers are relatively old and they are based on Dupuit (1863) assumptions. It is necessary to assume the velocity of flow to be proportional to the tangent of the hydraulic gradient instead of the sine as is actually the case in the nature. In addition, the flow is horizontal and uniform everywhere in a vertical section through the axis of the well. With these assumptions at hand the flow to the well in steady state case can be described as (Kruseman and DeRidder 1970)

$$Q \equiv \pi k \frac{[D-s(r,t)]^2 - [D-s_w(t)]^2}{\text{Ln}(r/r_w)}$$

where

D – aquifer thickness
k – hydraulic conductivity.

This expression leads to the depression cone equation as

$$r = r_w \exp\{-\frac{\pi k[D-s_w(t)]^2}{Q}\}\exp\{\frac{\pi k[D-s(r,t)]^2}{Q}\} \tag{14}$$

Use of Eq. (11) after some algebraic calculations gives the depression cone volume in unconfined aquifers as

$$V(t) = \frac{1}{2} r_w^2 \sqrt{\frac{2\pi DQ}{T}} \exp\{-\frac{2\pi DT[1-s_w(t)D]^2}{Q}\}\int_a^b e^{x^2} dx \tag{15}$$

where

$$x = \sqrt{\frac{2\pi k}{Q}} [s(r,t)-D] \tag{16}$$

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$$a \equiv \sqrt{\frac{2\pi DT}{Q}} \tag{17}$$

and

$$b \equiv \sqrt{\frac{2\pi DT}{Q}}\left[1-\frac{s_w(t)}{D}\right] \tag{18}$$

Direct evaluation of the integration in the right hand side of Eq. (15) is not possible, but its evaluation is simple through numerical integration techniques such as Simpson's method. The substitution of Eq. (15) into Eq. (9) gives the storage coefficient expression for an unconfined aquifer as

$$S = \frac{Qt-\pi r_w^2 s_w(t)}{\frac{1}{2} r_w^2 \sqrt{\frac{2\pi DQ}{T}} \exp\{-\frac{2\pi DT[1-s_w(t)/D]^2}{Q}\}\int_a^b e^{x^2}dx-\pi r_w^2 s_w(t)} \tag{19}$$

Comparison of Eqs. (13) and (19) for the same values shows that the storage coefficient is always greater in the unconfined aquifer than confined aquifer.

Applications to Field Data

Unconfined Aquifer – Aquifer test data in wadi Qudaid, Saudi Arabia, for unconfined aquifer are already analyzed by Al-Hajeri (1977) using Papadopulos and Cooper type curves and Ferris (1963) analogous drain function method. The same data are selected for storage coefficient estimation by the methodology proposed herein. The existing wells that tap the aquifer are of large diameter with concrete or masonary casing. Their diameters vary between 1.5 m and 3.0 m. The aquifer is in the Quaternary deposits of the wadi which is underlain by weathered and fractured zone of Precambrian crystalline rocks. Four pumping test data from different portions of the wadi are subjected to the application of the methodology. Table 1 presents the necessary field measurements for the application. The pumping duration is rather short and the discharge is small which are the characteristics of large diameter wells. Satisfactory pumping tests on large diameter wells, lasting for several hours and indicating significant aquifer response, are possible only if the discharge is kept low and constant. Estimation of the storage coefficient can be achieved by applying Eq. (19) to the given data.

The aquifer parameter determinations resulting from various methods are presented in Table 2. It is obvious that although there is a good agreement between the analogous drain solution and the methodology developed herein for the storage coefficient values, they both differ significantly from the results of Papadopulos

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Tabel 1 – Field measurements (Unconfined aquifer).

Well No	Diameter (m)	Discharge (m^3/min)	Maximum drawdown (m)	Time (min)	Transmissivity (m^2/min)$\times 10^{-2}$
1	2.15	0.33	4.70	120	12.0
2	2.60	0.38	6.30	120	5.9
3	2.00	0.31	4.00	120	15.0
4	1.45	0.40	4.95	150	11.6

Table 2 – Storage coefficient estimations (Unconfined aquifer).

Well No	Papadopulos and Cooper	Analogous drain	Steady state
1	7.3×10^{-6}	3.5×10^{-2}	8.2×10^{-2}
2	7.7×10^{-6}	2.0×10^{-2}	1.3×10^{-2}
3	5.9×10^{-6}	3.3×10^{-3}	1.4×10^{-2}
4	1.3×10^{-5}	8.5×10^{-2}	2.0×10^{-2}

and Cooper solution. In that Papadopulos and Cooper approach gives storage coefficients which are extremely low for an unconfined aquifer. Among some other factors this discrepancy is mainly due to the facts that their method is applicable with confidence to wells in confined aquifers and their type curves are very sensitive to S values. However, their method yields almost the same transmissivity values as for the analogous drain function. Analogous drain solution considers a large diameter well as equivalent to a horizontal drain (Ferris 1963). Such an analogy renders the two dimensional flow towards wells into a one dimensional flow towards horizontal drains. Such an approximation is acceptable close to the well or at the face of well itself. The basis of this analogy is that the flowing water towards the large diameter well depends on the seepage area of the well which is the same principle as the drain or gallery. As a result of such an analogy the aquifer parameters, T and S are calculated using the analogous drain function of Ferris (1963). In the application of the methodology proposed herein for the calculation of S value, the transmissivity values are calculated as the average of the two methods, namely, the analogous drain and Papadopulos and Cooper methods.

Confined Aquifer – Confined aquifer pumping test data have been obtained from the Druma sandstone formation in the central part of the Kingdom of Saudi Arabia. Al-Dakheel (1982) analyzed these data by using conventional Theis and Jacob methods. Two pumping test data are considered from this formation and the necessary field measurements for the application of the methodology, are given in Table 3. These wells have rather small diameters and therefore the well storage effect on the drawdown will not be as effective as it was for the wells of compera-

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Table 3 – Field measurements (Confined aquifer).

Well No	Diameter (m)	Discharge (m^3/min)	Maximum drawdown (m)	Time (min)	Transmissivity (m^2/min)$\times 10^{-2}$
6	0.30	1.20	5.45	180	14.3
7	0.30	0.55	2.30	360	17.7

Table 4 – Storage coefficient estimation (Confined aquifer).

Well No	Theis Method	Jacob Method	Steady state
6	1.2×10^{-1}	3.3×10^{-2}	2.2×10^{-1}
7	1.6×10^{-1}	2.0×10^{-1}	1.6×10^{-1}

tively large diameter in the Quaternary deposits. The average thickness of the Druma sandstone is 350 m and the wells tap water from the depth of 600 m. Geological composition of the Druma aquifer includes gray limestone, brown marly and calcarenitic limestone. It is mainly sandstone in the well locations with yellow and brown shale. Results of aquifer parameters determination from various techniques are presented in Table 4. Comparison of the storage coefficients indicates that herein developed method yields results which are in good agreement with conventional methods. A resent application of the methodology has been performed by Bazuhair (1986) during his research studies where he could not determine the storage coefficient with any other method available in the literature but only from the steady state case as suggested in this paper. He was unable to employ any readily available method because he did not have the complete set of pumping data but only the steady state measurements. As shown in Table 5 he applied this technique to four wells with satisfactory result.

Table 5 – Storage coefficient estimation (Confined and unconfined).

Well No	Diameter (m)	Discharge (m^3/min)	Maximum drawdown (m)	Time (min)	Transmissivity (m^2/min)	Storativity (–)
2	0.16	1.06	1.17	375	1.91	2.7×10^{-7}
4	0.16	3.78	9.19	580	0.45	3.2×10^{-2}
6	0.16	0.19	7.00	420	0.02	2.7×10^{-2}
10	0.16	3.49	3.05	268	1.99	1.5×10^{-5}

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Discussion and Conclusion

The two critical points underlying the derivation of Eqs. (13) and (19) are the negligence of well losses and the finite radius of influence. None of these equations are valid for infinitesimally small diameter wells, i.e., point sink. Through various applications in practise, it has been observed that the applicability of this method is valid when $4\pi T s_w(t)/Q \geqslant 4$. Therefore, it is necessary in the field either to keep the pumping discharge small or the drawdown as big as possible. In order to secure the latter condition it is always preferable to apply this technique for the case of steady or quasi-steady state flow situations where the Jacob straight line method is valid.

In particular Papadopulos and Cooper methods yield reliable transmissivity values, but the storativity values obtained thereoff are questionable because the field data can be fit to type curves which have slight differences in their appearance. However, the method presented herein provides a unique answer for the storativity value. It is, therefore, recommended to apply the Papadopulos and Cooper method for finding transmissivities and subsequently with these values the method presented in this paper should be applied in order to arrive at more reliable storativity values.

The application of this method is especially useful in the case of only one time drawdown reading after a long time. Although Jacob straight line method is applicable also for large times it requires many time drawdown records. The application of the method presented herein becomes very effective when it is coupled with the Theim formula.

His formula yields the estimation of the transmissivity value which is then employed in the method presented above to arrive at the storage coefficient value estimation.

References

Al-Dakheel, A. M. (1982) Groundwater potential and quality of upper Druma aquifer in Al-Zilfi area, unpublished M. Sc. Thesis, King Abdulaziz University, 146pp.

Al-Hajeri, F. (1977) Groundwater studies of Wadi Qudaid, Institute of Applied Geology, Research Series, No. 2, Jeddah, Saudi Arabia, 222pp.

Bazuhair, A. S. A. (1986) Comperative hydrogeological study of Al-Kharj and Al-Yamaniyah areas of Saudi Arabia, unpublished Ph. D. Thesis, University of London, University College, 558pp.

Boulton, N. S. (1963) Analysis of data from non-equilibrium pumping test allowing for delayed yield from storage, *Proc. Inst. Civ. Eng., Vol. 28,* pp. 603-610.

Chow, V. T. (1952) On the determination of transmissivity and storage coefficients from pumping test data, *Am. Geophys. Union Trans, Vol. 33,* pp. 397-404.

Davis, S. N., and DeWiest, R. J. M. (1966) *Hydrogeology,* John Wiley and Sons, New York, 520pp.

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Dupuit, J. (1863) *Etude theorique et pratiques sur la movement des eaux dans les canaux decouverts et a travers les terrains permeables,* 2eme edition, Dunot, Paris, 304pp.

Ferris, J. G. (1963) Correction of drawdowns caused by a pumped well tapping less than the full thickness of an aquifer. In »Methods of Determining Permeability Transmissivity and Drawdown« Compiled by R. Pentall, USGS Water Supply Paper, 1536, I.

Hantush, M. S. (1956) Analysis of data from pumping test in leaky aquifers, *Am. Geophys. Union Trans., Vol. 37,* pp. 702-714.

Jacob, C. E. (1940) On the flow of water in an elastic aquifer, *Am. Geophys. Union Trans., Vol. 72,* Part II, pp. 574-586.

Kruseman, G. P., and DeRidder, N. A. (1970) Analysis and evaluation of pumping test data, Bulletin 11, Institute for Land Reclamation and Improvement, Wageningen, the Netherlands, 200pp.

Papadopulos, I. S., and Cooper, H. H. (1967) Drawdown in a well of large diameter, *Water Resources Research, Vol. 3,* pp. 241-244.

Şen, Z. (1982) Type curves for large diameter wells near barriers, *Ground Water, Vol. 20, No. 3,* pp. 274-277.

Şen, Z. (1986) Determination of aquifer parameters by the slope matching method, *Ground Water, Vol. 24, No. 2,* pp. 217-223.

Theim, G. (1906) *Hydrologishe Methoden,* Gebhart, Leipzig, 56pp.

Theis, C. V. (1935) The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Am. Geophys. Union Trans., Vol. 16,* pp. 519-524.

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