Statistical properties of spatial snowcover in mountainous catchments in Norway

Nordic Hydrology **Vol 35 No 2 pp 101–117** © IWA Publishing 2004

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Received 18 June 2002; accepted in revised form 1 September 2003

Abstract The spatial distribution of snowcover in a catchment is determined by complex interactions between meteorological and physiographical factors, integrated over time. The snowcover shows variability over scales ranging from centimeters up to hundreds of kilometers.

An important and necessary decision for modelers is to determine spatial resolution in a distributed model. Since the spatial variability in snowcover may be quite large, even within a few meters, it is difficult to use modeling units small enough so that the snow can be assumed evenly distributed within the unit. A possible method to compensate for this is to use larger units, and describe the snow distribution within each unit by a statistical model (e.g. normal, log-normal, gamma, etc). This technique requires information about spatial statistical properties of snowcover within a unit.

As many of the distributed hydrological models operate on a grid basis, it would be desirable to find a statistical distribution on a sub-grid scale. However, as an initial approach, the study presented here was done on a catchment scale. The catchment scale presented the possibility of incorporating data from several historical snow surveys. These surveys were taken at the time of maximum snow accumulation in various mountainous catchments in Norway. Comparing empirical distribution functions with different theoretical distribution functions, it was shown that a mixed distribution combining two separate log-normal distributions clearly gave the best fit in most of the catchments. This seems to indicate that a mixture of at least two different populations of SWE values exists.

Keywords Hydrological modeling; distribution function; mixed distribution function; snowcover; snow distribution; spatial distribution

Introduction

In order to model the hydrology of northern areas, it is necessary to acquire data on the snowcover. Such data are traditionally collected manually by measuring with stakes, graduated rods or fixed rulers. The principal uncertainty of the accumulation pattern constructed from point data is that the snow accumulation varies locally over the surface (Richardson *et al.* 1997). To gain enough information about the actual snowcover variability with traditional means, an extensive measurement design with a large number of snow courses is required. Such measurements are very time-demanding and costly. In many recent approaches, remote sensing data from airborne or satellite platforms are coupled with models of the spatial distribution of snowcover or snow water equivalent (SWE). These techniques are presently not developed enough to give proper results on snowcover properties like depth and density (Elder *et al.* 1998), but they can give important information on the snow covered area and albedo. In recent years, several authors have described the use of a ground based radar system to measure depth of snow (Andersen *et al.* 1987; Killingtveit and Sand 1988; Sand and Bruland 1998; Marchand *et al.* 2001). This technique makes it possible to collect snow depth samples over large areas, spending much less time than with traditional methods.

When performing distributed hydrological modeling, it is usually necessary to convert point values from field measurements to areal values. Due to various meteorological and physiographical factors, the snowcover in most natural catchments is highly unevenly. Thus, it cannot be assumed that the snow is distributed uniformly inside one model unit, irrespective of whether the unit is a cell in a grid-based model or an entire catchment. Therefore, even with accurate snow depth at a point, the transition to areal values will always be a source of error for the modeling results. Donald *et al.* (1995) described the consequence of the hypothesis of uniformly distributed snow as the "instant bare ground" effect in the modeling of snowmelt. This means that the uniformly distributed snowcover in the model is already depleted completely, while in reality only the shallow snow packs are depleted but the deeper ones are still leading melt-water runoff.

In order to build up a hydrological model, the modeler has to decide the appropriate scale of calculation. This decision is influenced not only by the availability of data but also by considerations of the computational costs. However, it is not possible to operate a hydrological model with a resolution that is high enough to ensure the snowcover to be distributed evenly within one pixel.

One possibility to avoid the "instant bare ground" effect is to characterize the distribution of the snowpack within grid cells or a catchment using a statistical model, i.e. a probability distribution function, such as a normal or log-normal distribution. For grid-based models it is hereby possible to do the calculations with a relatively large pixel size and, due to the distribution function, still take care of the natural snow variations inside the pixel. The advantage is that the calculation is more computationally effective since the total number of grid cells can be reduced. But also for lumped models, which are still widely used for operational purposes, the knowledge of a proper snow distribution function is vital for good simulation results. Since the natural distribution of snow is known to vary with geophysical and meteorological conditions, the use of different probability distributions according to varying geophysical and meteorological conditions is suggested. There is, therefore, a need to determine proper snowcover distribution functions for different types of landscape.

One possible solution to this problem is the method of stratified snow sampling. The idea is to divide the landscape into homogenous units, concerning snow accumulation, and take a sufficient number of samples in every unit. Snow samples collected within similar areal units usually exhibit similar frequency distributions (Stepphun and Dyck 1974). Several examples for terrain stratification were found in the literature from the last few decades. Stepphun and Dyck (1974) estimated the maximum winter snowcover over catchments in four climatic regions: prairie, parkland, forest and tundra. Applying a "tri-level design" they measured snow at regional, local and micro scale. They tested snow data for normality and found a normal frequency distribution in over 40% of the units in a prairie environment and in over 60% of those units within a forest. Class verification showed that the class means were significant different. Martinec (1991) and Martinec *et al.* (1991) used snow depth measurements, a snow settling model and satellite measurements for the evaluation of the snow reserves in mountain catchments and for the derivation of snowcover depletion curves (SDCs). The calculated snow water equivalents (SWEs) were within 6% of the values, measured at two reference stations. However, they did not suggest a certain statistical distribution function for the snowcover depletion curves. Donald *et al.* (1995) used the three-parameter log-normal distribution to derive SDCs for similar land cover-based units and compared the results to field measurements. A general model for land cover-based SDCs was developed for short grass, ploughed field and deciduous forest land cover types. Parameters for the SDCs were derived from empirical data. They concluded that the land cover-based SDCs provide a snow cover conceptualization that more closely approximates the melt of the snowcover in a catchment than does the catchment-wide SDC approach. Shook and Gray (1997) used the two-parameter log-normal probability density function to describe the frequency distribution of the SWE in different types of terrain in a prairie environment. According to these terrain classes they gave values for the coefficient of variation (CV). Based on extensive snowcover studies, Pomeroy *et al.* (1998) also suggested representative values for the CV of SWE on various landscapes in prairie, arctic and boreal forest environments in late winter.

In the present study, snow data from seven comprehensive studies in mainly mountainous catchments in Norway were analyzed statistically. In all the catchments, snow depth and density data were collected around the time of maximum snow accumulation. The analysis was done for the entire catchments, except for one catchment where it was possible to separate forested from open areas. The objective was to find statistical probability distribution functions that most closely fit the observed snow distributions. This "entire catchment approach" is valuable since the majority of operational hydrological models in the Nordic countries are still lumped models and require bulk snow distribution functions. The major part of the snow data was obtained from historical research reports (mainly in Norwegian), but the authors themselves collected the data for the Aursunden and Siso catchment.

Study sites and snow measurement methods

The study sites were located in various landscapes and climatic conditions. However, all catchments were mountainous. An overview of the catchment locations is given in Figure 1 and a description of each catchment is given in Table 1.

All snow measurements were made approximately at the time of maximum snow accumulation. The historical snow measurements (Nea, Orkla, Olstappen, Sagelva and Sira-Kvina) were made using conventional techniques. This means that snow depth measurements were performed along numerous snow courses with equidistant placed sample points.

Figure 1 Location of the catchments

Table 1 Description of the study sites

	Latitude Longitude	Elevation range (m)	Elevation average	Area, km^2 (lake area, %)	Forested area (%)	Forest type	Climate (terrain character)
Siso	67°20′ N 15°54′ E	671–1530	945	235 (12.3) 22.5% glacier	~0	–	Maritime (highly mountainous)
Sagelva	63°19′ N 10°39′ E	266–514	341	9.16 (3.4)	85	Mixed coniferous boreal	Transition from coastal to continental (hilly)
Nea	63°04′ N 12°00′ E	678–1796	933	693 (13.4)	4	Birch	Transition from coastal to continental (hilly to highly mountainous)
Orkla	62°57′ N 9°50′ E	129–1641	880	2655 (2.1)	41	Coniferous /birch	Transition from coastal to continental (hilly to mountainous)
Olstappen	61°29′ N 9°21′ E	662–1825	1033	642 (4.5)	54	Birch	Continental climate (mountainous)
Sira-Kvina	58°45′ N 12°00′ E	524–1140	756	1909 (8.5)	34	Birch	Mild, maritime (gentle terrain)
Aursunden	62°41′ N 11°48′ E	690–1553	870	849 (11.8)	31.4	Birch	Continental (hilly to highly mountainous)

The location of single snow-courses attempted to represent all variations of the terrain in the catchment. To do this, Tveit (1980) developed a method that included the analysis of topographic and morphologic terrain parameters. The idea of this method is to compare the characteristics of the entire catchment with the characteristics of selected snow sample points, whereas the terrain parameters (see below) are the measure for the terrain character. The assumption is: if the complete set of snow sample points has the same biogeophysical characteristics as the entire catchment, then the snow data will be a representative sample of the entire population of snow depths. This means further that there is no involvement of extra or interpolated snow depth or density for unmeasured parts of the catchment. However, the method includes a control of the similarity between the catchment and snow sample point characteristics. If the similarity is not sufficient, the different snow sample points will get different weights to compensate for the divergence. The following biogeophysical parameters were used for terrain characterization:

- *X* coordinate (south–north direction)
- *Y* coordinate (west–east direction)
- elevation
- land-cover (lake, open country, forest of varying density)
- near-exposure (within a spot of approximately 15 m)
- near-slope (within a spot of approximately 15 m)
- near-curvature (concave positive, convex negative)
- large-scale north–south slope (south positive, north negative)[1]
- large-scale east-west slope (west positive, east negative)[2]
- large-scale curvature (concave positive, convex negative).

The values for these parameters were obtained partly from maps and partly from field surveys. This method has been used in the following catchments: Nea, Sira-Kvina, Olstappen, Orkla, Sagelva and Siso.

The most recent snow survey was the one in the Aursunden catchment. It was performed in spring 1999. Here, Geographic Information System (GIS) supported analyses were undertaken to determine catchment characteristics and parameters. The following terrain parameters were considered:

- elevation
- aspect
- curvature
- slope
- land-cover (forest or open field).

As in the approach of Tveit (1980), the snow depth values should be a representative sample of the entire population. Therefore the statistical moments (mainly min, max and mean) of the terrain parameter were used as a measure. Snow sample locations should represent as best as possible the entire catchment, e.g. if the entire catchment has a forest area of 30%, around 30% of the snow sample locations should be located in the forest. This was investigated prior to the snow survey using the GIS and snow sample locations were adjusted by trial and error until the fit between statistical moments of the terrain parameters at the snow sample locations and the statistical moments of the entire catchments was acceptable. Thus, the approach is similar to the historical surveys, except for the use of GIS and the hereby resulting

[1]This parameter is a combination of aspect and slope.

[2]This parameter is a combination of aspect and slope.

difference in resolution. The digital terrain model (DEM) had a resolution of 100 m. Thus, the terrain characteristics could not be investigated at the same small scale as some of the terrain parameters of the historical investigations were ("near-slope", "near-aspect", "near-curvature"). To cover macro scale variations, the snow sample locations were placed evenly over the whole catchment. This is in principal the same as adjusting for the "*X* coordinate" and "*Y* coordinate" parameters in the historical work. The parameters "Large-scale north–south slope" and "Large-scale east–west slope" are combinations of the parameters "slope" and "aspect". The terrain parameters used in the Aursunden catchment therefore were similar to that of the historical work, except for the missing micro scale parameters and a simplified land cover representation (characteristic distances of micro scale in snow hydrology: <50 m (Killingtveit and Sælthun 1995), 10–100 m (Pomeroy and Gray 1995)).

In the Aursunden, as well as in the Siso, catchment a georadar was used for the snow depth measurements. A snow scooter pulled the radar and the antenna was mounted in a sledge. The sampling interval was 0.5 m and 1 m in Siso and in Aursunden, respectively. However, due to the combination of radar and GPS in the Aursunden catchment the typical distance between samples after data processing was around 5 m. The quality of radar measurements was ensured and tested by manual control measurements using graduated rods. The radar-measured depth showed very good agreement with the control measurements. Comparison between radar measurements and manual measurements in previous studies gave typically R^2 values in the range of 0.90–0.97 (Bruland *et al.* 2001; Killingtveit *et al.* 1998; Sand and Bruland 1998). See also Marchand *et al.* (2001) for more details on measurements with georadar.

For the conversion from snow depth to SWE it is necessary to measure or estimate the snow density. Usually, the snow density does not vary as much as the snow depth (Stepphun and Dyck 1974). Thus, less density samples are required compared to the snow depth. Since the density varies with depth, it is useful to express snow density as a function of snow depth. This is especially important for deep snowpacks where the covariance between depth and density is stronger compared to shallow snowpacks (Pomeroy and Gray 1995). The required function can either be established with the help of measurements or a general formula can be used. For snow depths greater than 60 cm Pomeroy and Gray (1995) recommended a general formula by Tabler *et al.* (1990) (as found in Pomeroy and Gray (1995)) or a modified version of Tabler's formula which they had adapted to the Canadian Prairie.

Figure 2 illustrates a comparison of both formulae and a function based on linear regression with a constant upper limit to the observed data. The linear function with constant upper limit was later used in the analysis and it is explained further in the next paragraph.

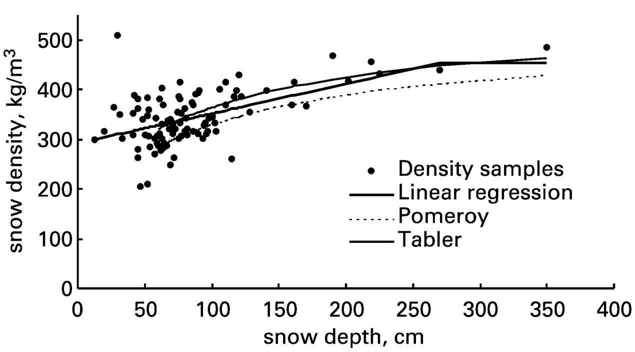

Figure 2 Comparison of measured snow density samples (Aursunden in 2001) with different theoretical functions for the conversion from snow depth to density. Two formulae were found in previous publications: Tabler (1990), as cited by Pomeroy (1995), and Pomeroy (1995). The third function, linear regression with constant maximum value, is similar to the technique used in this paper

Tabler's formula and the linear regression gave an almost equally good fit to the observed data. Pomeroy's formula did not give a good fit to the data observed in the Norwegian catchment Aursunden. Comparing the functions at snow depths greater than 60 cm by the Nash–Sutcliffe goodness-of-fit criterion, the R^2 values were 0.464, 0.468 and 0.194 for the linear regression, Tabler's formula and Pomeroy's formula, respectively.

Snow densities in the snow data used were measured with a "Songa tube". This tube is 1 m long and has a diameter of 0.113 m ($A = 100\,\text{cm}^2$). The samples were weighted and the bulk density, according to the snow depth, was calculated. Snow density samples were taken along the same courses as the snow depth samples, but less frequently. In the different catchments the sampling rate was varying. A typical density sampling rate was *ca*. 3 samples per snow course, with snow courses in the range of a few hundred meters to a few kilometers. For the conversion from snow depth to SWE, the following technique was used in the historical reports, as well as for the newer measurements. Based on the field measurements of density, a linear regression equation between depth and density was established for each catchment Since it is obvious that the snow density follows this regression line only up to a certain level (see, for example, Figures 2 and 3), a combination of this linear function and a constant maximum value was used for calculation of the density for the snow depth samples without density measurements. For the more recent measurements the breakpoint for the function (from linear to constant) was set to: mean density + 2 × standard deviation of the density measurements. The practice for setting the breakpoint was varied in the historical reports. As an example, the functions for the Aursunden catchment are given as follows:

Forest:

$$\text{density} = 0.0382 \times \text{depth} + 334.56 \qquad \text{for } 0.0382 \times \text{depth} + 334.56 < 390$$

$$\text{density} = 390 \qquad \text{for } 0.0382 \times \text{depth} + 334.56 \geq 390$$

Open field:

$$\text{density} = 0.025692 \times \text{depth} + 331.81 \qquad \text{for } 0.25692 \times \text{depth} + 331.81 < 420$$

$$\text{density} = 420 \qquad \text{for } 0.25692 \times \text{depth} + 331.81 \geq 420$$

A plot of the measured density samples against the snow depth, the regression functions and the R^2 values are shown in Figure 4. Based on the functions above, the density for all the measured snow depths was calculated and thus the SWE could be determined.

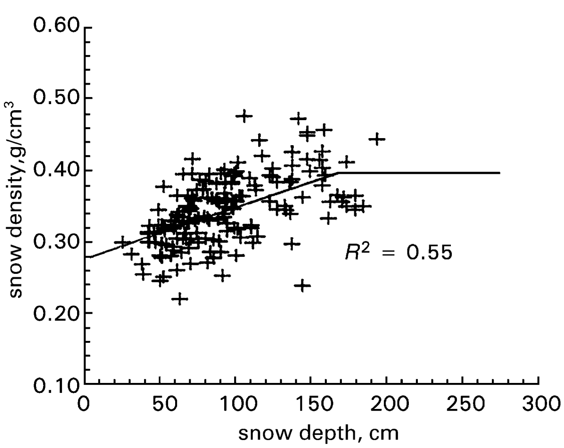

Figure 3 Example of the depth–density relationship measured in the Orkla catchment (1982) with regression line and R^2 value. Measurements are plotted as " + ". (After Sand and Killingtveit 1983)

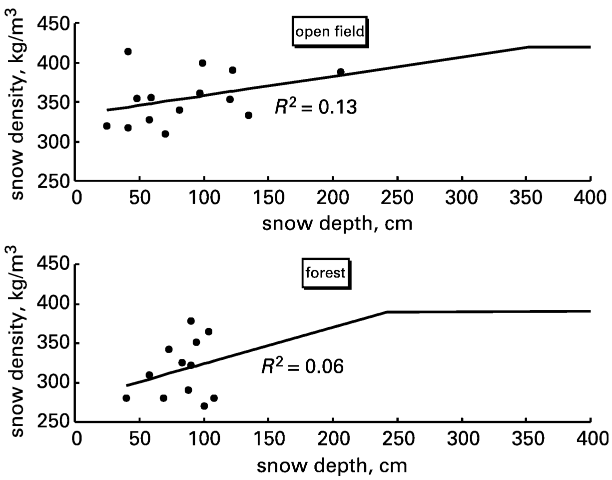

Figure 4 Relationship between snow density and depth of the samples measured in the Aursunden catchment (1999). Measurements are plotted as dots. The associated linear regression lines are given as well as the R^2 values

Snow survey results

The main results of the snow surveys are shown in Figure 5 and Table 2. Figure 5 gives an overview over the spatial distribution of the snow water equivalent in all catchments. On the *x* axis, the cumulative snow sample frequency is shown in percentages and on the *y* axis, the snow water equivalent is given in mm. The intersection of the snow distribution curve with the *x* axis can be interpreted as the percentage of the catchment that was snow-covered at the time of measurement. For example, for Sagelva this figure was 86%, while for Nea it was 78%.

In Table 2, the measurements from all catchments are described by SWE mean, median, standard deviation, the coefficient of variation and the percentage of snow-free area. The data show large variations concerning most of the statistical moments.

Statistical analysis and theory

The snow survey data for each catchment were converted from snow depths to SWE as described earlier. The SWE data were then used to construct histograms, i.e. the frequency was plotted against the snow water equivalent. The main objective of the statistical analyses was to find a statistical probability distribution function that fitted the observed snow distribution as well as possible.

In previous snow-distribution studies, mainly the log-normal probability distribution function was used as a theoretical distribution, though the normal distribution can also be of importance for snow cover analyses (Stepphun and Dyck 1974). The log-normal distribution is applicable to the study of the statistical distribution of many different types of geophysical and engineering data (Chow and Asce 1954) and it has been successfully used in previous research on snow distribution (Donald *et al.* 1995; Shook and Gray 1997).

Many of the data sets in this study had a significant number of zero snow depths, a very characteristic feature for snow cover in mountainous catchments. The occurrence of zero values is difficult to handle with the log-normal distribution in its usual form, since zero values cannot be log-transformed. The problem can be avoided by using a “zero-modified log-normal distribution”, sometimes also called a Δ distribution (Aitchison and Brown 1957). The zero-modified log-normal distribution is the combination of a log-normal distribution with a positive probability mass at 0. The zero-modified log-normal distribution

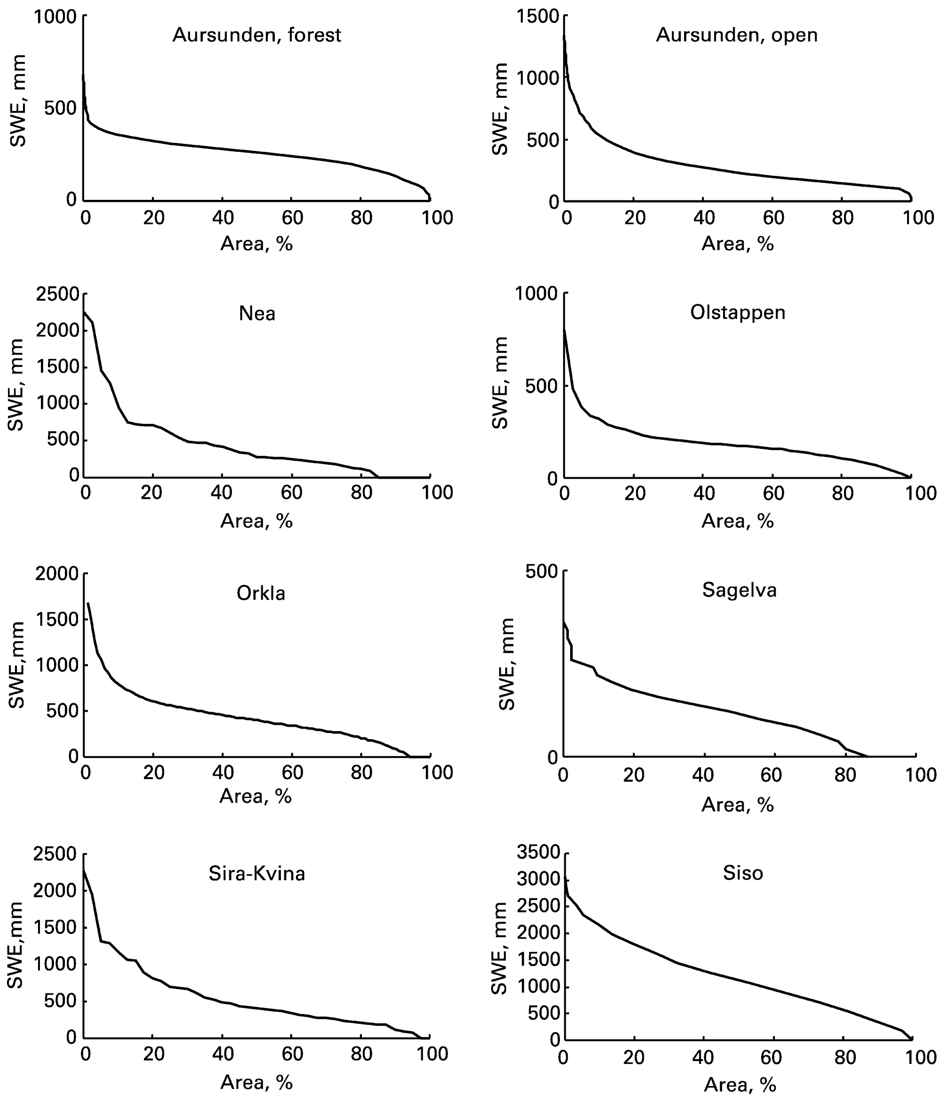

Figure 5 Areal snow distribution plotted as SWE versus percentage of catchment area

has been used to model data sets which have some values of zero, for example chemical concentration data where some observations are below the detection limits and daily rainfall data where some days have zero rainfall (Millard 2001). The zero-modified log-normal distribution $h(x)$ can be written as

$$h(x;\mu,\sigma,p)=\begin{cases} p & \text{for } x=0 \\ (1-p)f(x;\mu,\sigma) & \text{for } x>0 \end{cases}$$

where μ = mean of the non-zero values, σ = standard deviation of the non-zero values and p is the probability of zero values (or the percentage of values equalling zero). $f(x;\mu,\sigma)$ is the log-normal frequency distribution for the non-zero values in the dataset.

The percentage of snow-free area was obtained from the field data and used as the frequency p for zero snow. Table 2 gives the computed statistical parameters (μ, σ, p) for the

Table 2 Statistical summary for observed snow data. Snow water equivalent, in mm. (Average, median and st. dev. are computed for snow-covered area only.)

Catchment	Arithmetic average μ (mm)	Median (mm)	Standard deviation σ (mm)	Coeff. of variation $Cv = \mu/\sigma$	Snow-covered area (1-p) %	Snow-free area (p) %	Sample size (# of obs)	Method used for data collection
Aursunden, in forest	262	248	108	0.41	99.96	0.04	12250	Georadar survey
Aursunden, open field	403	293	323	0.80	99.96	0.04	16616	Georadar survey
Siso	1102	1021	650	0.59	100.0	0.0	27610	Georadar survey
Nea	450	321	392	0.87	78.0	22.0	350	Manual survey
Olstappen	197	176	135	0.69	97.6	2.4	41	Manual survey
Orkla	461	415	294	0.64	94.0	6.0	100	Manual survey
Sagelva	131	128	72	0.55	86.0	14.0	95	Manual survey
Sira	590	420	488	0.83	95.0	5	41	Manual survey

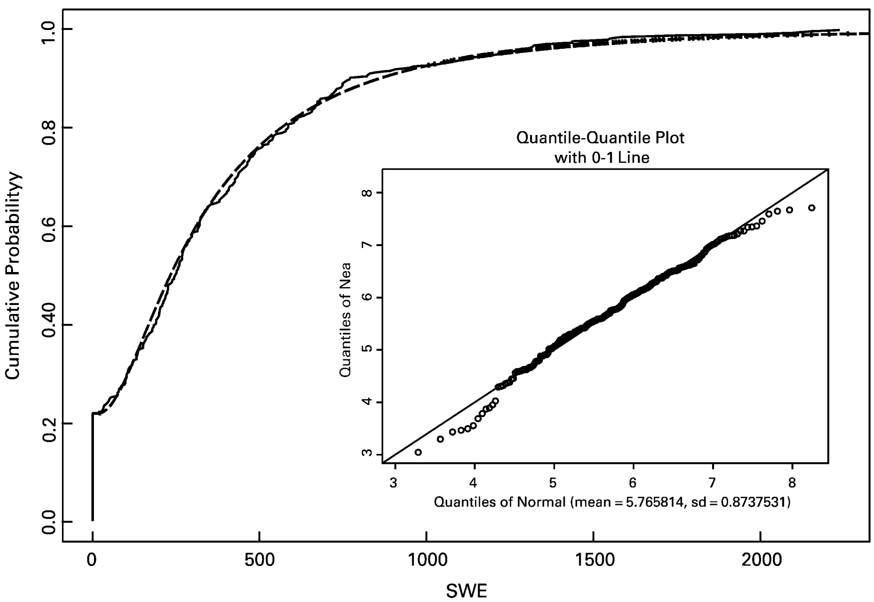

Figure 6 Observed (solid line) and fitted (dashed line) cumulative distribution functions for the Nea catchment. The inset shows a QQ plot for observed and modelled quantiles

different catchments. The plot in Figure 6 shows observed and computed cumulative distribution functions for the Nea catchment, one of the catchments with the highest percentage of snow-free area.

The zero-modified log-normal distribution was applied to all catchments, but still there was a problem fitting the distribution to the observed SWE values in most of the catchments. A detailed study of histograms and cumulative distribution functions, compared to the fitted log-normal distributions, revealed that in most catchments the empirical frequency distributions seem to consist of a mixture of two different distributions. Using only one distribution would not produce a sufficient close fit to the observed data. The problem could not be resolved by using other probability distributions like the Gamma distribution, log-normal 3 distribution or Generalized Extreme Value (GEV) distribution, which were tested. It was therefore decided to try the use of a mixed probability distribution, and a mix of two log-normal distributions (LN2 + LN2) was selected to model SWE distribution for the snow-covered areas (e.g. Figure 7).

The mixed probability distribution combining two log-normal distributions proved to give a better fit for SWE distribution in all the catchments except one, where the use of a mixed distribution did not improve the fit significantly.

The mixed log-normal frequency distribution ($f(x)$) can be written as a weighted sum of two ordinary log-normal distributions:

$$f(x) = \varepsilon \frac{\mathbf{exp}\left[-\frac{1}{2}\left\{\frac{\mathbf{ln}(x) - \mu_1}{\sigma_1}\right\}^2\right]}{\sqrt{2\pi}\sigma_2 x} + (1-\varepsilon)\frac{\mathbf{exp}\left[-\frac{1}{2}\left\{\frac{\mathbf{ln}(x) - \mu_2}{\sigma_2}\right\}^2\right]}{\sqrt{2\pi}\sigma_2 x}$$

The parameter ε represents the relative weight of distribution 1 and $(1-\varepsilon)$ the relative weight of distribution 2. The mixed-distribution $f(x)$ has 5 parameters (ε, σ_1, μ_1, σ_2, μ_2).

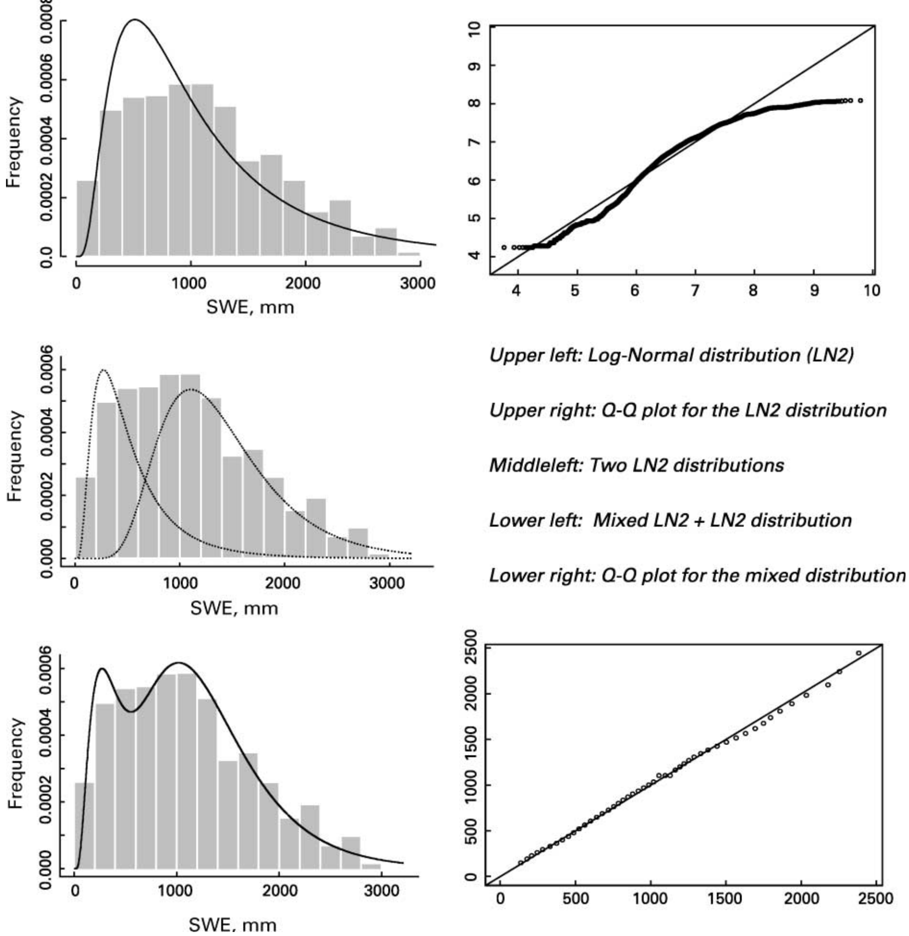

Figure 7 Analysis of data from the Siso catchment. In the frequency diagrams, the bar charts represent the empirical (observed) frequency and the lines represent the computed frequency

The parameters were estimated using the maximum likelihood method and the parameters were found by maximizing the log-likelihood function for the mixed distribution:

$$l = \sum_{i=1}^{n} \mathbf{log}\left\{ \varepsilon \frac{\mathbf{exp}\left[-\frac{1}{2} \left\{ \frac{\boldsymbol{ln}(x_i) - \mu_1}{\sigma_1} \right\}^2 \right]}{\sqrt{2\pi}\sigma_2 x_i} + (1-\varepsilon) \frac{\mathbf{exp}\left[-\frac{1}{2} \left\{ \frac{\boldsymbol{ln}(x_i) - \mu_2}{\sigma_2} \right\}^2 \right]}{\sqrt{2\pi}\sigma_2 x_i} \right\}$$

The parameter estimation was done using the statistical program system S-Plus by writing the equations for the mixed log-normal distribution and the corresponding log-likelihood function in the S-Plus programming environment.

For each data set, various other probability distributions, like normal, log-normal 2 and log-normal 3, gamma distribution and the GEV distribution, were tested. The comparison with the mixed distribution is only shown for the log-normal 2 distribution since this is the most common type of distribution used for snow and since none of the other tested distributions gave a significantly better fit to the observed SWE values.

For each catchment it was tested whether the mixed distribution really improved the statistical description compared to a single log-normal distribution. This comparison was made using quantile–quantile plots (QQ plots). This type of plot shows the empirical quantiles along one axis compared to the modelled quantiles along the other axis. If the model is good, the values plotted in the QQ plot form a straight line. A deviation from the straight line indicates that the model fails to describe the observed (empirical) distribution of data.

Statistical results and discussion

Results from the statistical analysis are summarized in Table 3 and in Figures 7–13. The results generally show that it was possible to represent the empirical snow distribution with theoretical probability distribution functions, and that the use of a mixed distribution in general gave an improved fit compared to the log-normal distribution. The testing of different distributions is illustrated by Figure 7, where the main steps in the analysis are illustrated. The three bar charts to the left show the observed frequency of SWE plotted as a function of SWE, together with one or two theoretical frequency distributions. The upper left graph contains a log-normal 2 distribution. By studying the QQ plot (upper right graph) it can clearly be seen that the LN2 distribution does not fit the observed data very well. Then, the mixed model was fitted using the maximum likelihood method. The two LN2 frequency distributions are plotted in the middle left graph. The combined (mixed) distribution is plotted in the lower left graph and the corresponding QQ plot is shown in the lower right graph. By comparing the two QQ plots it is clear that the mixed distribution has a much better fit than the single LN2 distribution.

In one of the catchments (Aursunden), data could be separated into one data set for forested areas (Figure 8) and one data set for open areas (Figure 9). In each of the two figures a comparison is made between one ordinary log-normal distribution and the mixed distribution. By looking only at the histograms one could possibly conclude that the log-normal distribution seems to fit well, but the QQ plots show that the mixed distribution is superior to the single LN2 distribution both for open and forested areas.

For the other catchments, only one frequency distribution (the mixed distribution) and the corresponding QQ plots are shown (Figures 10, 12 and 13). For all the catchments except one (Sira, Figure 11) the mixed distribution clearly gave a better fit. In the Sira data set, the maximum likelihood estimation resulted in two equal LN2 distributions, each with weight 0.5, showing that there was no benefit in using a mixed distribution.

Table 3 Statistical parameters for two different theoretical snow distribution functions (Distribution functions are computed for snow-covered area only)

Catchment	Probability for no-snow	LN2 distribution			Mixed distribution (LN2 + LN2)			
		μ	σ	ε	μ_1	σ_1	μ_2	σ_2
Aursunden, in forest	0.0004	5.490	0.417	0.690	5.5515	0.2397	5.3500	0.6390
Aursunden, open field	0.0004	5.749	0.692	0.316	5.3156	0.3993	5.9498	0.7060
Siso	0.000	6.783	0.739	0.649	7.1689	0.4032	6.0678	0.6842
Nea	0.220	5.766	0.874	0.030	3.5392	0.2662	5.8348	0.7896
Olstappen	0.024	5.093	0.668	0.584	5.2158	0.3028	4.9194	0.9310
Orkla	0.060	5.898	0.823	0.120	4.6862	1.4128	6.0639	0.5107
Sagelva	0.140	4.566	1.083	0.085	1.4787	0.8957	4.8538	0.4698
Sira	0.050	6.088	0.785	No significant improvement from single LN2 distribution				

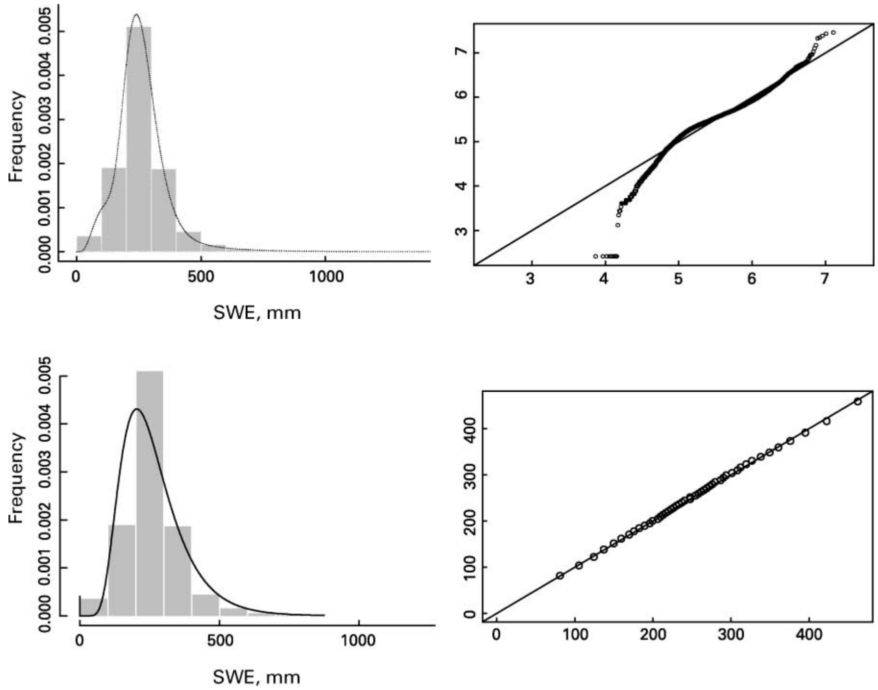

Figure 8 Aursunden – forested area. Upper left: LN2 distribution. Upper right: QQ plot for LN2 distribution. Lower left: mixed LN2 + LN2 distribution. Lower right: QQ plot for mixed distribution

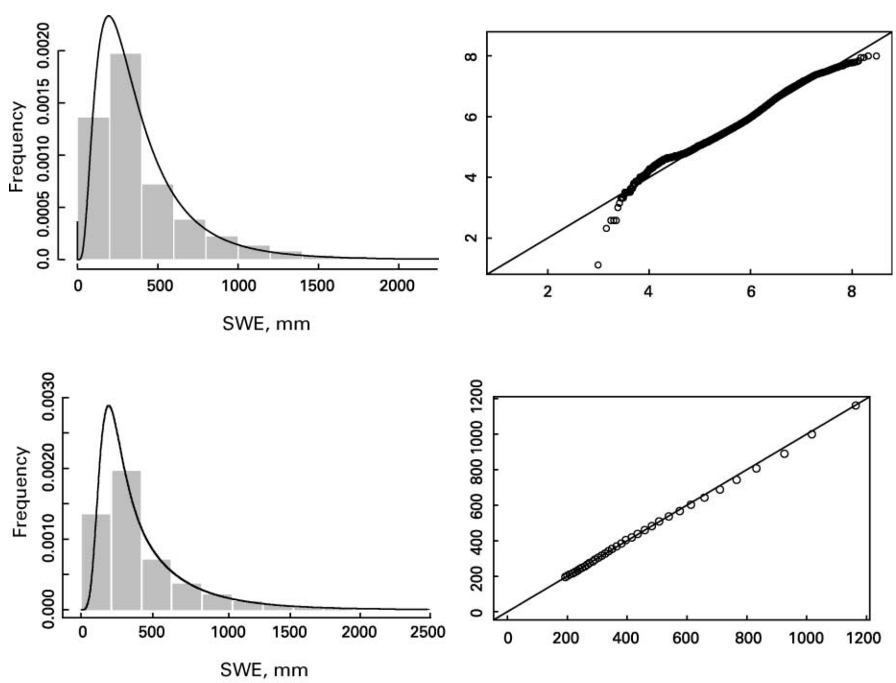

Figure 9 Aursunden-open field. Upper left: LN2 distribution. Upper right: QQ plot for LN2 distribution. Lower left: mixed LN2 + LN2 distribution. Lower right: QQ plot for mixed distribution

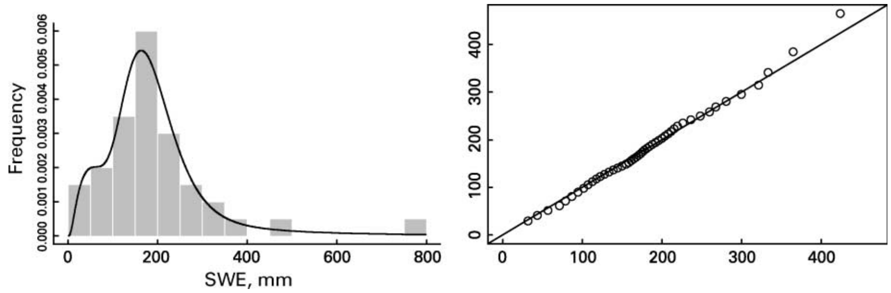

Figure 10 The Olstappen catchment. Left: mixed LN2 + LN2 distribution. Right: QQ plot for mixed distribution

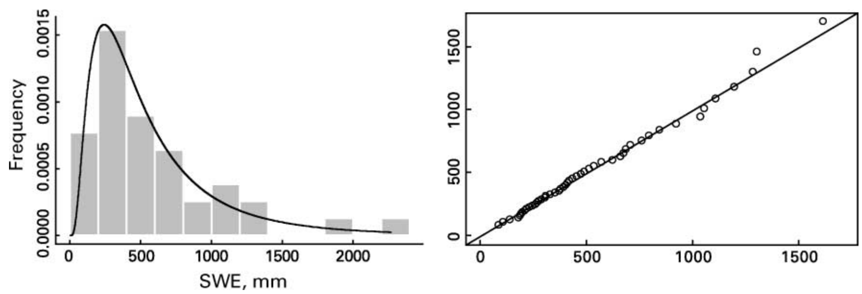

Figure 11 The Sira catchment. Left: LN2 distribution. Right: QQ plot for the LN2 distribution

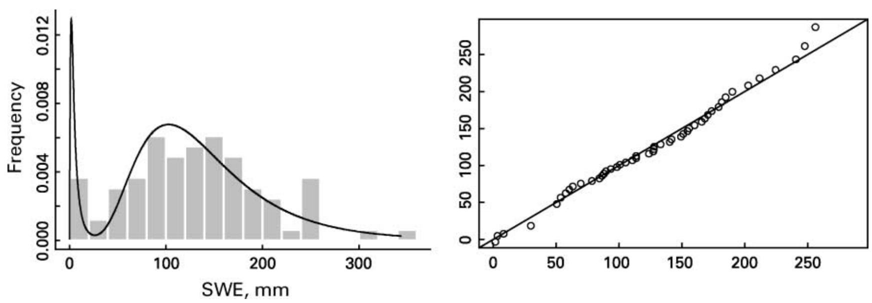

Figure 12 The Sagelva catchment. Left: mixed LN2 + LN2 distribution. Right: QQ plot for mixed distribution

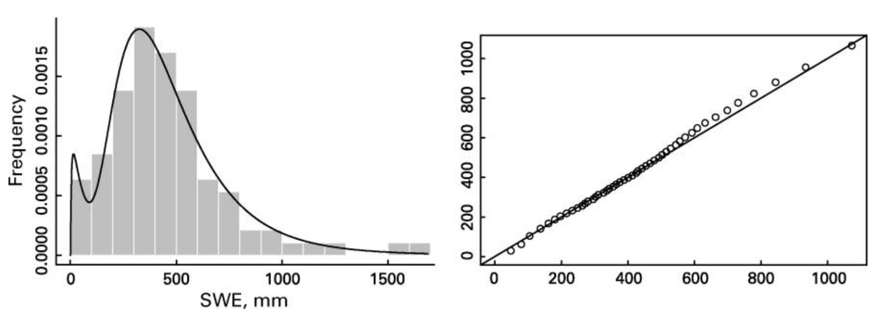

Figure 13 The Nea catchment. Left: mixed LN2 + LN2 distribution. Right: QQ plot for mixed distribution

Summary and conclusion

In this study, data from seven snow survey campaigns in mountainous areas in Norway were analyzed statistically. In five of the surveys, snow depths were measured conventionally, i.e. with graduated rods, whereas in two catchments the snow depth measurements were done with a georadar system.

In all cases it was possible to fit a theoretical probability distribution function to the empirical SWE distributions. The presence of snow-free areas in some catchments could be handled by using the zero-modified log-normal distribution, combining a discrete probability function at $x = 0$ with a continuous frequency distribution for the area with snow cover ($x > 0$). For the snow-covered area the log-normal distribution function was applied but it did not generally produce a good enough fit.

A mixed distribution combining two separate log-normal distributions clearly gave a better fit in all except one of the catchments. This seems to indicate that a mixture of at least two different populations of SWE values exists. For the data sets that were analyzed, two distributions (and two populations) seemed to be enough: there were no indications that an even more complex mixed model was needed.

Acknowledgements

The research presented here is part of the project “New Generation Hydrological Models”. The Norwegian Research Council is acknowledged for financing the project (grant no 100457). Further thanks are expressed to John Tveit for helping with data acquisition from old reports. Finally, the authors gratefully acknowledge the assistance of Asgeir Petersen-Øverleir at the Norwegian Water Resources and Energy Directorate, who wrote a major part of the S-plus code for parameter estimation in the mixed LN2 + LN2 distributions.

References

Aitchison, J. and Brown, J.A.C. (1957). *The Lognormal Distribution (with special references to its uses in economics)*, Cambridge University Press, Cambridge.

Andersen, T., Lundteigen Fossdal, M., Killingtveit, Å. and Sand, K. (1987). The snow radar: a new device for areal snow depth measurements. *Hydropower '87. Int. Conf. on Hydropower in Norway*, *Trondheim*, *Norway*, Tapir, Trondheim.

Bruland, O., Sand, K. and Killingtveit, A. (2001). Snow distribution at a high Arctic site at Svalbard. *Nord. Hydrol.*, **32**(1), 1–12.

Chow, V.T. and Asce, A.M. (1954). *The Log-Probability Law and Its Engineering Applications*, American Society of Civil Engineers, New York.

Donald, J.R., Soulis, E.D., Kouwen, N. and Pietroniro, A. (1995). A land cover-based snow cover representation for distributed hydrologic models. *Wat. Res. Res.*, **31**(4), 995–1009.

Elder, K., Rosenthal, W. and Davis, R.E. (1998). Estimating the spatial distribution of snow water equivalence in a montane watershed. *Hydrol. Process.*, **12**(10–11), 1793–1808.

Killingtveit, Å., Marchand, W. and Kiplesund, G.H. (1998). Erfaringer fra snømåling med radar i Siso-feltet, ENFO-Seminar i Kraftverkshydrologi og produksjonsplanlegging, Røros, Norway (in Norwegian).

Killingtveit, Å., Sælthun, N.R. (1995). *Hydrol*. **7**. Norwegian Institute of Technology, Division of Hydraulic Engineering.

Killingtveit, Å. and Sand, K. (1988). Snow-radar: an efficient tool for areal snow pack assessments. *7th Northern Research Basins Symp. and Workshop, Ilulisat, Greenland*, Danish Society for Arctic Technology, Copenhagen.

Marchand, W.D., Bruland, O. and Killingtveit, A. (2001). Improved measurements and analysis of spatial snow cover by combining a ground based radar system with a differential global positioning system receiver. *Nord. Hydrol.*, **32**(3), 181–194.

Martinec, J. (1991). Indirect evaluation of snow reserves in mountain basins. *Snow, Hydrology and Forest in High Alpine Areas, 20th General Assembly of the International Union of Geodesy and Geophysics, Vienna, Austria*, IAHS (International Association of Hydrological Sciences), Wallingford, UK.

Martinec, J., Burkart, U. and Baumann, R. (1991). Areal modelling of snow water equivalent based on remote sensing techniques. *Snow, Hydrology and Forest in High Alpine Areas. 20th General Assembly of the International Union of Geodesy and Geophysics, Vienna, Austria*, IAHS (International Association of Hydrological Sciences), Wallingford, UK.

Millard, S.P. (2001). *EnvironmentalStats for S-PLUS Help*, Version 2.0, Probability, Statistics & Information, Seattle, WA.

Pomeroy, J.W. and Gray, D.M. (1995). *Snowcover-Accumulation, Relocation and Management*, National Hydrology Research Institute, Environment Canada, Saskatoon, Sask.

Pomeroy, J.W., Gray, D.M., Shook, K.R., Toth, B., Essery, R.L.H., Pietroniro, A. and Hedstrom, N. (1998). An evaluation of snow accumulation and ablation processes for land surface modelling. *Hydrol. Process.*, **12**(15), 2339–2367.

Richardson, C., Aarholt, E., Hamran, S.E., Holmlund, P. and Isaksson, E. (1997). Spatial distribution of snow in western Dronning Maud Land, East Antarctica, mapped by a ground-based snow radar. *J. Geophys. Res.-Solid Earth*, **102**(B9), 20343–20353.

Sand, K. and Killingtveit, Å. (1983). *Snøforhold i Orkla-feltet, Studie av snøfordeling og foreslag til snømåleopplegg*, 2-83016. Norges hydrodynamiske laboratorier, SINTEF, Trondheim, Norway (in Norwegian).

Sand, K. and Bruland, O. (1998). Application of georadar for snow cover surveying. *Nord. Hydrol.*, **29**(4–5), 361–370.

Shook, K. and Gray, D.M. (1997). Synthesizing shallow seasonal snow covers. *Wat. Res. Res.*, **33**(3), 419–426.

Stepphun, H. and Dyck, G.E. (1974). Estimating the true basin snowcover. *Advances Concepts and Techniques in the Study of Snow and Ice Resources, Asilomar Conference Grounds, Monterey, CA, USA*, National Academy of Sciences, Washington, D.C..

Tveit, J. (1980). *Representativitet av snømålesystem frå topografiske og morfometriske parameter*. B-2-1980-1, Dept. of Hydraulic and Environmental Engineering, Norwegian Universiti of Science and Technology, Trondheim, Norway (in Norwegian with English summary).