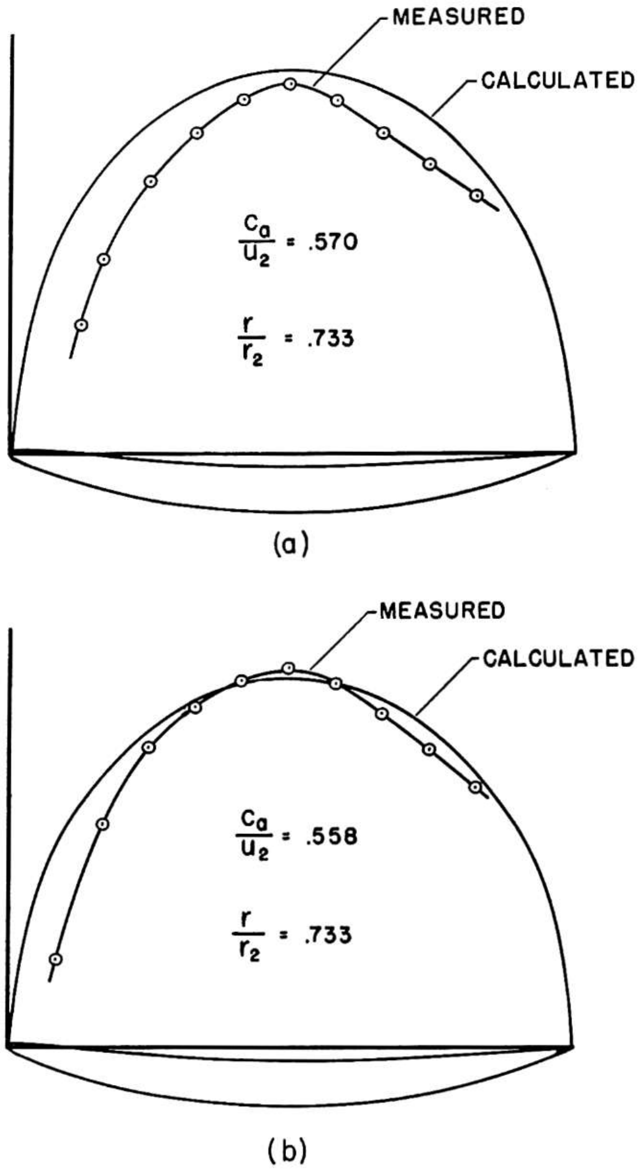

Fig. 8 Comparison of Calculated and Measured Differential Pressure
(*a*, At design flow rate; *b*, at 98 per cent of design flow rate.)

better. Even here, however, the loading is heavier toward the trailing edge, which is again indicative of a lower angle of attack than the optimum. For the next lower flow rate (0.543) the same comparison has shown that the pressure loading becomes greater toward the leading edge, indicating that at this flow the angle of attack is above the optimum.

Referring to Fig. 7, it will be noted that the distribution of pressure around the leading edge substantiates these conclusions, placing the best operating conditions slightly below a flow coefficient of 0.558. A flow coefficient of 0.550 is 3 per cent below the design point and is equivalent to a change in attitude of the blade on the hub of approximately 1 deg at the tip. No error of this magnitude occurred in the test vanes. By measurement it was found that only $^1/_3$ deg error in attitude existed. An increase in the axial velocity c_a, over the value used in the design is approximately equivalent to a change in attitude of the vane. Fig. 7(*b*) shows that, at the leading edge, the pressure falls generally below the duct static pressure. This can be attributed to the curvature imposed on the flow in the radial plane by the round end of the hub close upstream and to boundary layers. The resulting higher velocity at the leading edge, over the inner portion of the vane, would move the ideal inlet condition to a flow rate below the design point.

Conclusions

It appears that the airfoil theory, properly applied, permits a close prediction of the pressure distribution around the vane of pump impellers with few blades and, consequently, the head generated by the impeller. Such impellers are used today in pumps of high specific speed.

Where cavitation characteristics are important, it is possible to establish theoretically the lowest pressures occurring on the vane in the design stage. Over a very large range of flows the pressure distribution on the vane behaves regularly, and the static pressure near the leading edge is almost constant from root to tip. All radii are equally critical for cavitation.

The radial distribution of energy is uniform from about 60 to 100 per cent of design point. A very small (10 per cent) increase of flow beyond design point results in serious deterioration of the inlet conditions. Below design point there is a considerable range of flow rate wherein the entrance conditions are good. The experimental results establish the value of the techniques of design, manufacture, and test used in this work.

Acknowledgment

The authors acknowledge with pleasure the suggestions and assistance of the staff of the Hydrodynamics Laboratory of the California Institute of Technology.

Discussion

G. F. Wislicenus.[6] This paper presents the first results of a most promising investigation which, it is hoped, will be continued to include systems of greater solidity than the one investigated. The well-known excellence of the experimental work of the Hydrodynamics Laboratory gives the paper a more than ordinary importance. It is for this reason that one cannot help but wish for a somewhat greater clarity in the comparison between the experimental and the theoretical results presented.

It is feared that the significance of the basic Equation [1] of the the paper cannot be understood fully from the information presented. Particular reference is made here to the role played by the ratio z_1/r. It would seem that the controlling parameter for the various lines from *A* to *B* in Fig. 3 of the paper rather should be an expression of the solidity of the vane system at the particular radius considered than the ratio of the axial depth of the system to the local radius. The parameter suggested in the paper implies a three-dimensional effect of the arrangement of the system within a cylindrical space of revolution. If the authors really wish to describe such a three-dimensional effect it would seem necessary to give a complete explanation for it at the point of its introduction.

The foregoing criticism is made particularly because the method of design suggested by the authors impresses this writer as in other respects most constructive and useful. In spite of the fact that the lines shown in Fig. 3 are, in general, of course not straight lines, the straight-line approximation suggested is probably sufficient for most practical cases of design provided the problem of the inclination of the lines mentioned before can be resolved in a satisfactory manner.

Returning for a moment to the afore-mentioned comparison of measured and calculated test data, this writer would like to point out that the "detailed knowledge" regarding the theoretical pres-

---

[6] Chairman, Department of Mechanical Engineering, The Johns Hopkins University, Baltimore, Md. Mem. ASME.

sure distribution appears to be well within the reach of the existing two-dimensional theory of cascade flow. Instead of selecting one of the standard NACA blade profiles it might have been better to select for this investigation a profile that lends itself fairly readily to a theoretical solution of its flow problem. What is meant is a procedure in which the convenience of the conformal mapping of the system onto a circle is permitted to dictate to some extent the vane shapes selected. If it should have been found that curved vanes are too inconvenient for this purpose, it might have been better to select for the first experiment straight-line profiles for which the exact solution has long been given by Weining and others. With such theoretical data on hand the comparison between the actual and calculated pressure distribution could have been brought into a much more conclusive form.

As matters stand, this writer is not convinced that the data presented constitute either a confirmation or a contradiction to the design methods suggested. This statement is based largely on the fact mentioned by the authors immediately below Equation [5], where it is pointed out that for the single-vane pump investigated the correction of the streamline curvature at the root of the blades amounted only to 3 per cent of the airfoil camber required to develop a lift coefficient of unity. Since such a small change in the curvature of the airfoil would have an effect of the order of magnitude of only $^1/_2$ per cent on the lift coefficient of the blade it seems doubtful whether the data presented permit any conclusions with respect to such an effect. It is hoped that the authors will be able to clarify this apparent dilemma, but more, so that they will be able to extend their interesting investigations to configurations where it is known that the cascade effects approximated by Fig. 3 of the paper have a quite decisive influence on hydrodynamic characteristics of individual blades.

AUTHORS' CLOSURE

Mr. Wislicenus brings attention to the inclusion of the ratio z_1/r as an important parameter in the evaluation of curvature of the streamlines due to the trailing vorticity downstream of the blade. Since the vane system is designed for free vortex flow downstream and has only one vane the centerline trailing vortex is the only one contributing to the curvature of the incident flow. If the axial extent of the blade is very small compared to the radius, it is sufficient to evaluate the tangential induced velocity at midchord and correct the direction of the relative velocity in the manner of reference (3) of the text of the paper. When the axial extent of the blade becomes large compared with the radius under consideration a considerable streamline curvature exists as indicated by Fig. 3 of the text. This streamline curvature reduces the effective camber of the airfoil in direct ratio of the per cent camber of the streamline to the per cent camber of the airfoil. How Mr. Wislicenus proceeds from a 3 per cent streamline curvature to only $^1/_2$ per cent correction to the lift coefficient is not clear to the authors. Three per cent correction on camber is 3 per cent correction on lift coefficient.

It has been the authors' intention to avoid the usual procedure which reduces radial arrays of vanes to a two-dimensional cascade because their interests lie with rotors wherein few blades of large solidity are arranged on a relatively small hub. It is not evident that the two-dimensional approximation is adequate for these cases. Work is continuing along the same lines as presented in the text while theoretical work with manageable camber-line shapes in two dimensional cascades is being carried on by others. The simple straight camber line gives a very unsatisfactory pressure distribution as has been demonstrated by the rotor shown in Fig. 5 for which complete experimental data has been obtained.