difference is that the former result is inaccurate. For instance, in the practical design, a panel size of 200 to 500 sq in. is not too big. However, the shear strain of the panel varies from point to point. Therefore the average value at the four corners of each panel cannot represent the shear strain in the whole panel.

Although both the discussers' theory and the author's were based on the assumption of small deformations, careful steps must be taken in order to obtain a reasonable result before the evidence of a large-deflection theory can be presented. Thus the author's method is more accurate, because he integrates the shear strain for the whole panel by mathematical integration and also takes care of the buckling state by using the effective shear modulus. Furthermore, if we consider the numerical calculations, the discussers' method is quite tedious, because they have to calculate the shear strain of every single panel. By using the author's method, it is only necessary to calculate one single value η for the whole structure.

Notice also there were many contradictions between their theory and statements, for instance:

1 The senior author's first paper (4) on the theory of semimonocoque structures under bending with one model test stated that both theory and experiment have a good agreement. His second revised paper (5) and third revised paper (3) offered the same statement. The writer admires his painstaking attitude to improve his theory from time to time. In the meantime it proves that a meager test is not reliable.

2 The discussers state in their comment, "In an excellent paper, published in 1935, J. L. Taylor considered both the torsion of the stringers and the shear in the cylinder subjected to compression, but they did not use it for checking their work. Besides, in 1946, they developed a rather inaccurate four-corner strain-energy method; they also used their bending theory to check the buckling load of the *Rainbow* in compression.

In fact, the present problem of the general instability of semimonocoque structures under compression was first suggested to the author by Dr. Th. von Kármán in 1938 and was taken up by Dr. S. Timoshenko in 1939, as the author's thesis for PhD (6). At least three of us considered it worth while to make a research on this subject, but the discussers said . . ., the author was mistaken in regard to the earlier work on the general instability of compressed semimonocoques. The author does not wish to voice his opinion on this. It would be better to let the readers pass judgment on this matter.

Finally, the author cannot give his approval of the following statement: "It is rather inconsistent, therefore, that he considers his own work on the general instability in bending as the final solution of that problem in spite of the fact that there he did not include the shear-strain energy in the calculating." The author did not make any statement like that. Furthermore, the author mentioned in a few places in that paper (7) that the shear effect of skin is taken care of by using empirical factors. Nevertheless, the author's solution well checked the experimental results of 74 model tests carried out at Caltech as previously mentioned, and thus his method is, at least, good for design purposes.

The author agrees entirely with the suggestion made by Prof. M. Z. Krzywoblocki. However, there are several reasons as follows for not presenting the discussions, as suggested in his paper: (*a*) Usually the space in the journal in which the papers appear is limited; (*b*) the author considered that it was unnecessary to do so because the function chosen has been used too many times by other authors; (*c*) after all, the theoretical analysis has to be checked by experiments anyway, regardless of the degree of accuracy of the chosen function achieved; (*d*) from the viewpoint of engineering, the presentation of the papers should not involve too much mathematics if it can be avoided.

BIBLIOGRAPHY

1 "Some Investigations of the General Instability of Stiffened Metal Cylinders. V-Stiffened Metal Cylinders Subjected to Pure Bending," Guggenheim Aeronautical Laboratory, California Institute of Technology, NACA TN No. 909, August, 1943.

2 "A New Theory for the Buckling of Thin Cylinders Under Axial Compression and Bending," by L. H. Donnell, Trans. ASME, vol. 56, 1934, pp. 795–806.

3 "The Inward Bulge Type Buckling of Monocoque Cylinders—III Revised Theory Which Considers the Shear Strain Energy," by N. J. Hoff and Bertram Klein, NACA TN No. 968, April, 1945.

4 "Instability of Monocoque Structures in Pure Bending," by N. J. Hoff, *Journal of the Royal Aeronautical Society*, vol. 42, April, 1938, p. 291.

5 "General Instability of Monocoque Cylinders (Revised Theory)," by N. J. Hoff, *Journal of the Aeronautical Sciences*, vol. 10, April, 1943, pp. 105–114.

6 "Stability of Semimonocoque Structures Under Pure Compression," by Tsun Kuei Wang, Dissertation under the direction of Prof. S. Timoshenko, Stanford University, 1940.

7 "General Instability of Semimonocoque Structures Under Bending," by Tsu Kuei Wang, *Journal of the Aeronautical Sciences*, vol. 13, Janary, 1946, pp. 29–37.

Experimental Studies of Biaxially Stressed Mild Steel in the Plastic Range[1]

D. C. DRUCKER.[2] The author's experimental data are of value in understanding the behavior of metals in the plastic range. Of special importance is the confirmation of the large deviation of the μ versus ν curve from the straight line, postulated by the simpler mathematical theories of plasticity. In this connection, a re-examination of Figs. 3, 5, 6, 7, of the paper, for stress ratios n of $^3/_4$, $^3/_8$, $^5/_8$, $1^3/_8$, respectively, is worth while. It shows quite conclusively that, in the range beyond an octahedral shearing strain of about 2 per cent, the difference $\mu - \nu$ is constant within experimental scatter.

This experimental result is a welcome confirmation of one of the bases of present theories. The usual assumption made is that the total strain or the strain increments can be broken up into elastic and plastic components, each of which can be treated separately. A further assumption adopted for work-hardening theories of the isotropic type is that the ratios of the plastic components of strain will remain constant if the stress ratios are fixed.[3] In terms of incremental theories, this means that the loading function, or criterion for further plastic deformation, is a homogeneous function of the components of stress (see P. Hodge and W. Prager[4]).

Therefore present theories predict that in the elastic range the μ versus ν curve will be a straight line, while, in the plastic range μ versus ν (plastic) will be a single curve, and as a consequence the μ versus ν (total strain) will lie between the two extremes, Fig. 1. However, for mild steel, the elastic strains are of the order of 0.001 and therefore have little influence on ν above a total strain of 2 per cent. Fig. 2 is a schematic representation of the variation of $\mu - \nu$ with increasing strain. The horizontal portion of the curve agrees with the results obtained by the author in the higher strain range. The low percentage

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[1] By S. J. Fraenkel, published in the September, 1948, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 70, pp. 193–200.

[2] Associate Professor of Engineering, Brown University, Providence, R. I. Jun. ASME.

[3] "Strain Hardening Under Combined Stresses," By W. Prager, published in the *Journal of Applied Physics*, vol. 16, 1945, pp. 837–840.

[4] "Variational Principal for Plastic Materials With Strain Hardening," By P. Hodge and W. Prager, published in the *Journal of Mathematics and Physics*, vol. 27, April, 1948, pp. 1–10.

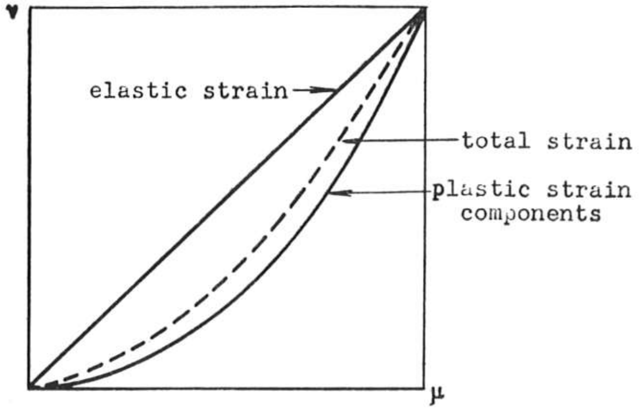

Fig. 1

accuracy in the small strain region, as mentioned in the paper, accounts for the high scatter there.

A minor point of criticism is that the use of the term "strain energy" to describe the work done is misleading, as the major portion is irrecoverable. Also, the expression given in the paper for the work required to produce a given maximum strain in the tube surface does not fit the data well. A very close fit is obtained by replacing $\int \bar{\sigma}_z d\,\bar{\epsilon}_z + \int \bar{\sigma}_t d\,\bar{\epsilon}_t$ by a constant times the sum of the products of the maximum values of stress and corresponding strain, assuming incompressibility, and using the μ versus ν curve. The results are independent of the shape of the stress-strain curve. Crudely taking $\mu = \nu$ gives

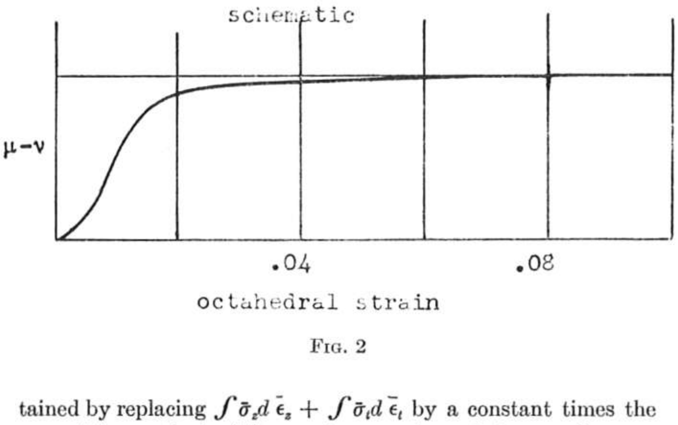

Fig. 2

$$W = W_t \frac{\sqrt{n^2 - n + 1}}{1 - n/2}$$

which lies much closer to the experimental results than the very high $W_t(1 + n^2)$ given in the paper.

The investigation by the author of the effect of the path of loading is another forward step. His conclusion, "the octahedral shearing stress associated with a certain set of orthogonal strains is independent of the order in which these strains were applied, provided the metal is not unloaded," is extremely interesting. However, great caution should be exercised in its application. Unloading is likely to occur in practice. Furthermore, a general superposition is certainly not permissible; even though the octahedral stress may be nearly the same, the same stress components will not always be present. Also, the path of loading chosen is limited, compared with the possible variations. The principal directions were kept constant and the stress path was in the tension-tension quadrant and was not so very different from the path for a stress ratio of unity (author's Fig. 11).

It is to be hoped that the author will continue his investigation along these and similar lines to settle the many questions that are still unanswered.

William Prager.[5] In the opinion of the writer, many experimental investigations of the mechanical behavior of mild steel suffer from the fact that they follow a pattern which, by now, has been firmly established. Thus numerous investigators have established the fact that the octahedral shearing stress correlates not too badly with the octahedral shearing strain, independently of the manner of stressing, provided that the principal axes of strain and the ratios between the principal strains are kept constant during the test. Similarly, several investigators have established the fact that the μ versus ν diagram is not a straight line. These two isolated facts, however, are far from constituting a complete description of the mechanical behavior of mild steel in the plastic range. Therefore the author is to be congratulated for his courageous departure from this set pattern. In particular, the study of the influence of the strain path merits the attention of future investigators.

Without wishing to detract from the value of the paper, the writer would like to caution against undue generalization of the author's results. (1) When he states, "the path of loading was found to be immaterial," he means that, for a certain strain, the same octahedral shearing stress was obtained regardless of the strain path. The octahedral shearing stress is only a scalar measure of the stress intensity, however, and, by no means describes completely the state of stress. It would seem desirable to study the influence of the strain path on the complete state of stress and not only on the octahedral shearing stress. (2) The principal directions of strain were kept constant during the author's tests. It seems legitimate to raise the question as to what degree the author's results would have been affected by a rotation of the principal axes of strain.

K. Hohenemser and W. Prager[6] studied the influence of the strain path on the state of stress in thin-walled tubes under combined tension and torsion. Obviously, these tests involve a rotation of the principal axes of strain. The results of Hohenemser and Prager show that under certain conditions the strain path is indeed immaterial. These conditions are rather restrictive, however, and a full description of the mechanical behavior of thin-walled tubes of mild steel under combined tension and torsion would exceed the space available for this discussion. Therefore the reader is referred to the original paper.

Author's Closure

The interest shown in the paper and the comments by Drs. Prager and Drucker are appreciated.

The author agrees with the thought that a good deal of additional work along this line is required, especially with respect to a possible generalization of the effect of the path of loading for other loading patterns and for the case of rotating axes of stress and strain.

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[5] Professor of Applied Mechanics, Brown University, Providence, R. I. Mem. ASME.

[6] "Beitrag zur Mechanik des bildsamen Verhaltens von Flusstahl," by K. Hohenemser and W. Prager, *Zeitschrift für Angewandte Mathematik und Mechanik*, vol. 12, 1933, pp. 1–14. An English translation of this paper is available as R.T.P. Translation No. 2468 (Durand Reprinting Committee, in care of California Institute of Technology, Pasadena 4, Calif.).