Recovery of aspherical earth structure from observations of normal mode amplitudes

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SUMMARY
On an aspherical earth, the normal mode amplitude pattern is perturbed by along-branch coupling. In the geometrical optics approximation, the effect of aspherical earth structure on normal mode amplitudes through along-branch coupling is equivalent to a perturbation $\Phi$ in the spatial phase of the amplitude pattern. This observable quantity is related to earth structure through a WKBJ phase integral along the source-receiver minor arc. The amplitude pattern is strongly sensitive to odd-degree earth structure and complements the information contained in the frequency shift pattern, which is sensitive to only even-degree structure. We have developed a method to estimate the amplitudes of unresolvably split multiplets from long-period spectra and have applied it to a large number of IDA and GEOSCOPE recordings to obtain a global data set of normal mode amplitudes for spheroidal modes $S_{26}^o$-$S_{43}^o$. Normal mode amplitude estimation requires that the first surface wave arrival $R_1$ be available in the seismogram, and coherent amplitude patterns have been obtained for 172 data records.

The analysis of normal mode amplitudes affords several advantages not shared by surface wave phase velocity analysis or time-domain waveform fitting. Our method allows the estimation of errors in the model and the ability to cull away modes in the frequency domain, and it makes use of the information contained in the entire seismogram. In addition, our method makes no assumptions concerning the spectral shape of isolated multiplets in the frequency domain. The small data set employed here is only a fraction of the size of previous global data sets, but has the advantage that fewer assumptions are built in to the theory on which the structure inversion is based. Several factors, including uncertainty in the moment tensor scaling and focusing and defocusing effects, prohibit recovery of $\Phi$ for individual multiplets and limit us to the recovery of the average $\Phi$ over the mode band $l = 26-43$, for each source-receiver pair. Through a preferential weighting of subsets of this mode band, we may obtain estimates of $\Phi$ within narrower mode bands centred, respectively, on $l = 31$, $l = 34$, and $l = 37$. Least squares inversion of $\Phi$ within the various mode bands, utilizing approximately 100 source-receiver pairs, yields the geographic distribution of local eigenfrequency perturbation $\delta\omega_{\text{local}}(\theta, \phi)$ in a truncated spherical harmonic expansion. Inversion is first performed for the best fitting degree-6 earth model using all available modes within the band $l = 26-43$ with equal weight. The degree-6 earth model is obtained by inverting for degrees 1, 3, and 5 with degrees 2, 4 and 6 fixed according to published results from the frequency studies. Narrower band inversions are performed within $l = 26-36$, $l = 29-39$, and $l = 31-43$. Comparison with model M84A shows that the odd degree-1 and 3 components of aspherical earth structure may be reproduced from the amplitude data set. The degree-5 structure differs significantly from that of model M84A.

Key words: coupled modes, free oscillations, lateral heterogeneity.

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1 INTRODUCTION

Investigations in long-period seismology have led to a number of global models of deep earth structure in recent years. A common feature of these models is their success in accounting for a moderate part of the observed seismic signal. The most successful of such studies are those which relate the fundamental spheroidal mode frequency shift, or multiplet location parameter (Jordan 1978), to earth structure (Silver & Jordan 1981; Masters et al. 1982; Davis 1987; Romanowicz, Roult & Kohl 1987; Smith & Masters 1989). The principal limitation of these frequency shift studies is that they allow the recovery of only the even-degree earth structure, since the multiplet location parameter is sensitive to only the even-degree components of earth structure (Backus 1964; Jordan 1978). Global studies aimed at recovering the odd-degree structure have relied upon phase anomalies of surface wave arrivals (Nakanishi & Anderson 1984; Nataf, Nakanishi, & Anderson 1984; Tanimoto 1985; Tanimoto & Anderson 1985) or time-domain waveform fitting of the seismic signal (Woodhouse & Dziewonski 1984; Tanimoto 1988). In the surface wave studies, the observable quantity that is related to earth structure is the phase delay of surface wave packets, which is expressed asymptotically as an integral of local phase velocity perturbation along the source–receiver major and minor arcs. The geometrical optics approximations made by Woodhouse & Dziewonski (1984) make their approach essentially equivalent to the surface wave approach. These studies generally do not make use of the first surface wave arrival \( R_1 \) or \( L_1 \), and this greatly reduces the potential resolution of odd-degree earth structure. Tanimoto & Anderson (1985) have pointed out the lack of odd-degree resolution in the surface wave studies, based on the sensitivity of the structure kernels to antipodal earth structure. The resulting earth models (which are models of the uppermost 670 km) explain a great deal of the waveform perturbations of local earth structure. Also Park (1988) has shown that the variance reduction achieved by matching waveforms from even-degree perturbations is much greater than that achieved from matching equivalent odd-degree perturbations.

The present study attempts to deal with these problems by developing a method for measuring normal mode amplitudes and restricting attention to fundamental spheroidal mode amplitudes, for several reasons. Normal mode frequency shifts are, with the exception of spheroidal–toroidal coupling, largely insensitive to the above effects. Smith & Masters (1989) show that generally good agreement exists among the frequency studies pertaining to the even-degree components of earth structure. Thus, the problem lies with the determination of the odd-degree structure. Mochizuki (1986), Romanowicz (1987), and Park (1987) have shown that normal mode amplitudes depend on both the odd- and even-degree structure, and we shall find that amplitudes are more sensitive to odd-degree structure than even-degree structure. Although the frequency range of this study is more limited than that of previous studies, the principal advantages of using normal mode amplitudes as the primary data source are that the determinations of the odd- and even-degree components are essentially decoupled, and the problems facing the recovery of odd-degree earth structure may be better addressed. Park (1987) has shown that the effects of off-path scattering and focusing can, in many cases, have a significant effect on the theoretical normal mode amplitudes. We believe that seismic records which do not contain the first surface wave arrival \( R_1 \), do not effectively constrain odd-degree earth structure, and use of such records in previous investigations has been possible at the expense of employing the resonance peak approximation for multiplet spectra (Woodhouse & Dziewonski 1984; their equation 16). This approximation has been justified as a consequence of the geometrical optics approximation, but its application is questionable since the second and higher moments of the multiplet spectra are essentially ignored (Pollitz 1989). The effects of off-path scattering and focusing may potentially be strongly manifested in the higher multiplet moments, even when the geometrical optics approximation is adequate for the modelling of the zeroth and first moments. As applied in the waveform inversion studies, the ignorance of the higher multiplet moments leads to an unmodelled signal that varies non-linearly with respect to perturbations in the earth model, even near the correct earth model. The severity of the approximation ultimately depends on the smoothness (roughness) of the actual earth. Synthetic experiments on fully coupled aspherical earth seismograms need to be done in order to test the feasibility of performing earth structure inversions on data sets dominated by incomplete records (in which only arrivals \( R_2 \) and beyond are available).

The problem of better determination of the odd-degree earth structure depends upon minimizing all of these corrupting effects through an unbiased amplitude estimation procedure. In this study, following Park (1987), we shall model the normal mode amplitudes by including only along-branch coupling effects. The amplitude estimation procedure is designed to recover the amplitudes of each unresolvably split multiplet without any assumption concerning its spectral shape in the frequency domain. By
restricting attention to the fundamental spheroidal mode dispersion branch between degrees \( l = 26 \) and \( l = 43 \), we avoid the \( S_{l-1}S_{l+1} \) interference in the \( l = 10-20 \) band and are justified in using the asymptotic and geometrical optics approximations of Park (1987). The \( S_{l-1}S_{l-1} \) crossover point near \( l = 32 \) (Masters et al. 1983) does not present a practical problem and can be eliminated by culling nearby modal observations if necessary.

We shall first review the theory of normal mode amplitudes and discuss the simplifications which we have adopted in this study. We then present a new method for measuring the amplitudes of unresolvably split normal multiplets and discuss its application to long-period recordings from the IDA and GEOSCOPE networks. Because of the inherent limitations of this procedure, the number of recordings which yield coherent amplitude estimates is far less (about 100) than the number used in large-scale studies such as that of Woodhouse (1984), which employed more than 1000 recordings. Nevertheless, the global data set is of high quality and is sufficiently distributed to be inverted for earth structure coefficients. Because of the poor depth resolution of our data set, we shall be limited to models of local eigenfrequency perturbation \( \delta \omega_{\text{local}} \). Recovery of depth dependent structure will depend upon the inclusion of lower frequency fundamental modes and higher order modes in future studies. We shall demonstrate consistent differences between the new model and model M84A (Woodhouse & Dziewonski 1984).

2 THEORY

A normal mode multiplet of degree \( l \) consists of \( 2l+1 \) distinct hybrid singlets whose resonance functions are centred about the unperturbed degenerate frequency \( \omega_0 \) (Park 1987). In the absence of rotational coupling, the contribution to the vertical component seismogram from the spheroidal mode multiplet may be written in the form

\[
s(t) = \sum_{j=1}^{2l+1} a_j \cos[(\omega_0 + \delta \omega_j)t]e^{-\alpha t}.
\]

The amplitude \( A_i \) of the multiplet is defined to be

\[
A_i = \sum_{j=1}^{2l+1} a_j.
\]

Assuming that the dominant perturbing effects arise from along-branch coupling, neglecting attenuation differences among the modes on the fundamental dispersion branch, and assuming the geometrical optics approximation, we may evaluate \( A_i \) (Park 1987):

\[
A_i = [(A_0)_i + (\delta A_0)_i](\sin \Delta)^{-1/2} \times \cos \left( k \Delta - \frac{\pi}{4} + \delta \Phi - \Phi_i \right) + O \left( \frac{1}{\Delta^2} \right).
\]

Here \( A_0 \) is the unperturbed amplitude of \( S_0 \) on the spherical earth, \( k = l + 1/2 \), \( \Delta \) is the epicentral distance, \( \Phi_i \) is the source phase shift and is a function of the moment tensor and source-receiver geometry (Dahlen 1980), \( \delta A_0 \) is an apparent change in \( A_0 \) due to focusing effects, and \( \Phi \) is the modal phase shift

\[
S_k = \frac{1}{U} \int_{0}^{\infty} [\delta \omega_{\text{local}}(\xi) - \delta \omega] d\xi.
\]

In (4), \( U \) is the group velocity in radian s\(^{-1} \), \( \delta \omega_{\text{local}}(\xi) \) is the local eigenfrequency perturbation, \( \delta \omega \) is the average eigenfrequency perturbation along the source-receiver great circle, and the integration is taken along the source-receiver minor arc. On the aspherical earth, the source phase shift \( \Phi_i \) depends on the apparent source-receiver azimuth, which may depart from the true source-receiver azimuth by an amount \( \Phi_e \) which depends on the transverse gradient of \( \delta \omega_{\text{local}} \) along the major and minor arcs. \( |\Phi_e| < 3^\circ \) for the examples considered by Park (1987) on the M84A earth. Since we shall also confine ourselves to smooth earth models, the model \( \Phi_i \) is of the same order as the typical error in moment tensor geometry and is therefore neglected.

Since the local eigenfrequency perturbation \( \delta \omega_{\text{local}}(\xi) \) for a mode of degree \( l \) is a slowly varying function of \( \xi \), we may approximate a set of observations \( A_i \) (\( l_{\text{min}} \leq l \leq l_{\text{max}} \)) by replacing \( \delta \omega_{\text{local}}(\xi) \) and \( \Phi_i \) with constants \( \delta \omega_0 \) and \( \Phi \), where a suitable choice for \( \Phi \) is the average of \( \Phi_i \) over the set of observations and a similar choice for \( \delta \omega_0 \). The distinction between this and other choices for \( \Phi \) and \( \delta \omega_0 \) is negligible for smooth earth models (Pollitz, Park & Dahlen 1987). Thus we may replace (3) with

\[
A_i = [(A_0)_i + \delta A_0](\sin \Delta)^{-1/2} \cos \left( k \Delta - \frac{\pi}{4} + \delta \Phi_i - \Phi \right).
\]

For a given seismic record, we may recover a set of amplitude observations which may then be inverted for the best-fitting \( \delta A_0 \) and \( \Phi \) in a least-squares sense. In the formulation of Woodhouse & Dziewonski (1984), the focusing term \( \delta A_0 \) is not accounted for explicitly. Adjustment of the elements of the moment tensor, as is done in that study, is equivalent to the inclusion of this term for a single record but is not sufficient for the modelling of many records from the same seismic event. Focusing effects on the earth may be very large (Niazi & Kanamori 1981; Lay & Kanamori 1985), warranting its explicit inclusion in the theoretical expression (5). In this study we will be concerned only with the information contained in the modal phase shift \( \Phi \), since this quantity depends explicitly on the earth structure and is not highly sensitive to errors in the moment tensor scaling or geometry.

3 AMPLITUDE ESTIMATION

Previous processing of long-period records have primarily employed the resonance peak fit method (Masters et al. 1982), whereby an isolated normal mode multiplet is approximated as a single resonance peak centred on a suitably defined centre frequency. This method yields the amplitude, frequency, and initial temporal phase of the multiplet. Several studies have employed this method to compile a frequency shift data set and have interpreted the frequency shifts using the theory of Jordan (1978). Davis & Henson (1986) and Romanowicz & Roult (1986) have shown that the frequency estimates are generally in excellent agreement with the theoretically predicted frequencies, particularly when additional terms in the
asymptotic expression for the multiplet location parameter are accounted for. The resonance peak fit method is not accurate enough, however, to recover the amplitudes of normal mode multiplets. Pollitz et al. (1987) point out that spurious initial temporal phase, away from the theoretically predicted 0° or 180° (Gilbert & Dziewonski 1975), is a serious drawback which severely limits the number of amplitudes which may be recovered. Qualitatively, this occurs because on an aspherical earth, the isolated normal mode multiplet of degree \( l \) is composed of 2\( l + 1 \) singlets that are well separated in the frequency domain. For example, \( \alpha S_{26} \) has a total splitting width of 24 μHz on the model M84A earth. This leads to destructive interference and cancellation among the singlets when a small-bandwidth taper such as the Hanning taper is applied. In order to show how this affects the amplitude estimation, we constructed synthetic seismograms for the 1983 334 Chagos event at all IDA stations, using the algorithm of Henson (1987), which incorporates self-coupling effects from aspherical earth model M84A. The seismograms contain all fundamental spheroidal modes of frequency less than 7 mHz. The amplitudes of modes \( \alpha S_{26} - \alpha S_{43} \) were determined with the resonance peak fit method using the Hanning taper whose spectrum is shown in Fig. 1(a). The amplitudes (real part) are compared with the theoretical amplitudes in Fig. 1(b). Compared with the method to be described, the scatter is considerable and of the same order as the expected amplitude perturbation itself, the quantity we wish to model. In practice, the use of the resonance peak fit method for the recovery of normal mode amplitudes within the 3.5-5 mHz band is even less accurate than that indicated in Fig. 1. Although part of this results from the presence of nearby overtones, in all likelihood the poor accuracy of this method results from rough earth structure which splits the isolated multiplets to a greater extent than that predicted by the smooth earth model M84A.

In the new method, we do not fit the frequency spectrum in the neighbourhood of the multiplet according to a pre-determined spectral shape function, but rather we determine the sum of the singlet amplitudes directly. More generally, rewriting (1) as

\[
s(t) = \Re \left( e^{i\omega_0} \sum_{j=1}^{2l+1} a_j e^{i\delta\omega_0} \right) e^{-\alpha_0^2}
\]

we define the \( k \)th moment to be

\[
\mu_k = \sum_{j=1}^{2l+1} a_j (\delta\omega_j)^k.
\]

This definition differs from that of Jordan (1978) by a factor of \( \pi/2 \). In particular, the zeroth moment \( \mu_0 \) is simply equal to the amplitude \( A_0 \) of equation (2).

The Fourier transform of a tapered time series of length \( T \) evaluated at the fiducial frequency \( \omega_0 \) has the form

\[
F[s(t)e^{i\omega_0}]_{\omega=\omega_0} = \frac{1}{T} \sum_{j=1}^{2l+1} a_j \left[ \int_0^T dt w(t)e^{i\delta\omega_0} \right].
\]

If we could design a taper \( w(t) \) such that \( F[w(t)] \) had the shape of a boxcar of half-width \( |\delta\omega_0|_{\max} \) (Fig. 2), then we

Figure 1. Aspherical earth synthetic seismograms containing fundamental spheroidal modes \( \alpha S_{26} - \alpha S_{43} \) were generated for the 1983 334 Chagos event using the algorithm of Henson (1987), with record length of 24 hr. All modes \( \alpha S_{26} - \alpha S_{43} \) were fit for amplitude and phase using the resonance peak fit method (Masters et al. 1982), with the Hanning taper being employed. (a) Spectrum of Hanning taper. Solid line indicates real-part spectrum, and dotted line indicates imaginary part. (b) Scatter plot of fit versus exact real-part amplitudes.

Figure 2. The spectrum of the ideal taper for the recovery of amplitudes of split multiplets. The real-part spectrum is uniformly flat within the splitting interval \((-|\delta\omega_0|_{\max}, |\delta\omega_0|_{\max})\), and zero outside the interval. The imaginary-part spectrum is identically zero. In practice large imaginary sidelobes must be present in the spectrum of a time limited taper.
would have the estimated amplitude

\[ A_{est} = 2F[s(t)w(t)e^{i\omega t}]|_{\omega = \omega_0} / F[w(t)]|_{\omega = \omega_0} = \sum_{j=1}^{2l+1} a_j, \]

(9)
since each individual singlet would then contribute to the sum in (8) with equal weight. The boxcar shape would also remove interference from any modes having frequencies outside the band

\[(\omega_0 - |\delta \omega|_{\text{max}}, \omega_0 + |\delta \omega|_{\text{max}}).\]

We now show that it is possible to design such a taper with the boxcar property. Assuming that the temporal attenuation parameter \(\alpha_0\) is known, we seek a taper \(w(t)\) such that

\[ f(w(t)| t = 1, \ldots , T) = \int_{-\infty}^{\infty} |F[w(t)]|_{\omega} - H(\omega, \delta \omega)|^2 \, d\omega \]

(10)
is minimized, where \(\delta \omega\) is a pre-specified half-bandwidth and

\[ H(\omega, \delta \omega) = \begin{cases} 1 & |\omega| \leq \delta \omega, \\ 0 & |\omega| > \delta \omega. \end{cases} \]

However, the solution of (10) is the sinc function \(w(t) = \sin(\delta \omega t)/\pi t\). The real-part Fourier transform of the function is in general not flat near \(\omega = 0\), as shown in Fig. 3(a). The result (9) would then not be applicable because the singlet amplitudes would not be given equal weight. In order to improve the flatness of the real-part spectrum near \(\omega = 0\), we choose to include an additional term in (10) which minimizes the integrated second derivative of \(F[w(t)]|_{\omega}\) within the bandwidth \((-\delta \omega, \delta \omega)\). The problem then becomes the minimization of

\[ f[w(t)] = \int_{-\infty}^{\infty} |F[w(t)]|_{\omega} - H(\omega, \delta \omega)|^2 \, d\omega \]

+ \(\eta\int_{-\delta \omega}^{\delta \omega} \frac{d^2}{d\omega^2} F[w(t)]|_{\omega}^2 \, d\omega \]

(11)

where \(\eta > 0\).

\[ \text{spectrum of } \sin(\delta \omega t)/\pi t \]

\[ \text{spectrum of } \sin(\delta \omega t)/\pi t \]

(a) (b)

\[ \text{Aspherical Earth Scatter Plot} \]

\[ \text{Aspherical Earth Scatter Plot} \]

(c) (d)

Figure 3. (a) Spectrum of sinc function taper \(\sin(\delta \omega t)/\pi t\), with record length \(T = 24\) hr and \(\delta \omega = 1.1 \omega_R\), where \(\omega_R = 2\pi/T\). (b) Spectrum of sinc function taper with \(\delta \omega = 0.5 \omega_R\). (c) Scatter plot of fit versus exact amplitudes using the same synthetic seismograms as in Fig. 1, fit with equation (9) and the taper shown in (a). (d) Scatter plot generated using the taper shown in (b).
Figure 4. The tapers resulting after applying the bandlimitedness and flatness criteria of equation (11). The tapers are shown in both the time domain and frequency domain. With $T = 24$ hr (divided into 512 time points), the first taper (a-b) was generated using the 12 highest $4\pi$-prolate eigentapers, $\eta = 7/T^4$ and $\delta\omega = 1.1\omega_R$. The second taper (c-d) was generated from (11) using the 13 highest $4\pi$-prolate eigentapers, $\eta = 9/T^4$, and $\delta\omega = 1.3\omega_R$. Solid line: real-part spectrum. Dotted line: imaginary-part spectrum.

An elegant way of finding the required $w(t)$ is to expand it in terms of $p\pi$ prolate tapers $w_n(t)$, which are eigenfunctions of an integral equation (Slepian 1983)

$$\int_0^T \sin(\Omega(t-t')) w_n(t') dt = \lambda_n w_n(t')$$

(12)

where $\Omega = 2\pi p/T$. Each taper $w_n(t)$ has an associated eigenvalue $\lambda_n$, with $\lambda_1 > \lambda_2 > \lambda_3 > \ldots$. Each successive taper leads to a stationary (maximum) value in the signal power in the band $(-\Omega, \Omega)$ relative to the power in the band $(-\infty, \infty)$, and this ratio is equal to the eigenvalue (Park, Lindberg & Thomson 1987). The set $\{w_n(t)\}$ are complete in the time domain. Letting

$$w(t) = \sum_{n=1}^{N} b_n w_n(t)$$

(13)

we may solve for the weight coefficients $b_n$ from the minimization problem (11). Provided that $\Omega > \delta\omega$, it can be shown that $w(t)$ of (13) converges to $w(t)$ with error of order $O(\lambda_n^{-2})$. This result greatly reduces the computational effort needed to find $w(t)$. Fig. 4(a-b) shows the taper and its spectrum derived for the specific case $\delta\omega = 1.1\omega_R$, where $\omega_R = 2\pi/T$ is the Rayleigh frequency, and $T = 24$ hr. The Lagrangian factor $\eta$ has been scaled by $T^4$. The taper changes negligibly with respect to an increase in the number of tapers in the basis set, which consists of the 12 lowest order $4\pi$ prolate eigentapers. Fig. 4(c-d) shows the result for the case $\delta\omega = 1.3\omega_R$. Each taper has the desirable properties of uniform real part spectrum near $\omega = 0$ and small real-part sidelobes. Also, the beginning of the record is given much greater weight then the end of the record, where the signal-to-noise ratio is expected to be smaller.
To test the utility of the derived tapers, synthetic seismograms were generated on the spherical earth and the aspherical earth with the 1.1$\omega_R$ taper (Fig. 4) and equation (9). Each seismogram contains only fundamental spheroidal modes with frequency less than 7 mHz, with a record length of 24 hr. Amplitude estimates for each mode $\omega_{S_6^+} = \omega_{S_6^-}$ were found by application of the 1.1$\omega_R$ taper of Fig. 4(b), using equation (9). The estimated amplitudes are compared with the exact synthetic amplitudes in Fig. 5(a–b). There is little difference between the results for the spherical and aspherical earth fits. In particular, the comparison on the aspherical earth is superior to that obtained from the resonance peak-fit method (Fig. 1). In order to demonstrate that a sufficiently wide and flat taper is necessary for the proper amplitude estimation, the aspherical earth synthetics are also fit with a sinc-function taper whose spectrum is shown in Fig. 3(b). Because the width of the peak of the real-part spectrum is only 0.5$\omega_R$, we do not expect the amplitude estimation to be as accurate, and this is seen in Fig. 3(d) where the scatter is comparable with that of the resonance peak-fit method. Even a sinc taper of sufficient width (Fig. 3a) generates less accurate amplitude estimates (Fig. 3c) because its taper spectrum is not flat near $\omega = 0$.

The synthetic seismograms used above contain only self-coupling interactions and therefore the amplitudes of the multiplets are identical with the aspherical earth amplitudes for the corresponding modes. It is of interest to test the method on fully coupled aspherical earth synthetic seismograms (Park 1987; Dahlen 1987) which contain the perturbations in the multiplet amplitudes which we shall actually be modelling. Fully coupled seismograms for the same Chagos records have been generated using the subspace projection algorithm described by Dahlen (1987), and the amplitudes estimated as before using equation (9). In Fig. 5(c), these estimates are compared with the amplitudes predicted by the asymptotic formula (5). Only those records for which $25^\circ < \Delta < 155^\circ$ are retained in the

Figure 5. Scatter plots of fit versus exact amplitudes after fitting the 1983 334 Chagos synthetics using the 1.1$\omega_R$ taper (Fig. 4) and equation (9). The results are shown for (a) spherical earth synthetics, and (b) aspherical earth synthetics. The latter comparison is a marked improvement over Fig. 1(b) and Fig. 3(b). (c)–(d) Scatter plots after fitting fully coupled aspherical earth synthetics with the 1.1$\omega_R$ taper and equation (9). Exact values on the aspherical earth are generated using the asymptotic result (5). Records are culled according to $25^\circ < \Delta < 155^\circ$.
plot since this is the approximate range of validity of the asymptotic theory. The amplitude estimates are clearly in excellent agreement with the asymptotic values, and this is to be contrasted with the comparison with the spherical earth values in Fig. 5(d). The scatter in the latter plot gives an indication of the average size (20 per cent) of the amplitude anomalies to be found in the data.

If the available seismic recording begins a time $\delta t$ after the origin time of the seismic event, then the amplitude of the target multiplet with fiducial frequency $\omega_0$, assuming an initial temporal phase of 0° or 180°, is

$$A = \mathcal{R}\left[\int_0^T s(t)w(t)e^{i \omega_0 t + \delta t}e^{-i \omega t} dt e^{i \omega_\delta t} dt \right] \bigg| \omega = 0.$$  \hspace{1cm} (14)

Errors in the initial multiplet phase or event origin time will be spuriously mapped into the amplitude estimate because of the large imaginary sidelobes of the taper spectrum (Fig. 4b,d). For example, a multiplet phase $\Phi_1$ which departs from 0° or 180° may arise from mode-mode rotational coupling, and a phase error $\Phi_2 = \omega_0 \delta t$ may arise from an error $\delta t$ in the origin time. Letting $\Phi = \Phi_1 + \Phi_2$, the spurious amplitude arising from a signal of amplitude $|A|$, phase $\Phi$, and frequency $\omega_1$ is

$$-|A| \sin \Phi \mathcal{R}[F[w(t)]|_{\omega = \omega_0 - \omega_1}.$$  \hspace{1cm} (15)

In practice, errors in the published origin time are generally small (less than 5 s). Since $\mathcal{R}[F[w(t)]|_{\omega = 0} = 0$, the principal errors stemming from (15) are those arising from the presence of nearby overtones as well as those arising from rotational self-coupling (which creates two complex resonance peaks with slightly different frequencies). Overtone interferences can be identified either visually or through calculation of the appropriate excitation coefficients, and we shall derive a theoretical correction for the effect of rotational self-coupling.

Figure 6. The spectra of various time offset tapers designed for application to records in which a time $\delta t$ is missing from the beginning of the record. The real and imaginary parts of $F[w(t)]|_{\omega = 0} e^{-i \omega \delta t}$ are plotted as a function of $\omega$ for (a) $\delta t = 900$ s, (b) $\delta t = 1800$ s, (c) $\delta t = 2700$ s, and (d) $\delta t = 3600$ s. The real-part sidelobes become large for $\delta t \geq 1800$ s.
A severe limitation on the records which can yield accurate amplitudes is imposed by the starting time of the record. If the first point in the data record corresponds to the origin time of the earthquake, then \( \delta t = 0 \) and no correction of the estimated amplitude is necessary. However, if the first point corresponds to a non-zero time after the origin time, then a phase correction becomes necessary. More precisely, if \( \delta t > 0 \), then the available time series corresponding to the target multiplet has the form

\[
s(t) = \Re \left[ e^{i \omega_0 (t + \delta t)} \sum_{j=1}^{2l+1} a_j e^{i \delta \omega_j (t + \delta t)} \right] e^{-i \omega_0 \delta t}
\]

and the estimated amplitude is

\[
A_{est} = 2F[s(t)e^{i \omega_0 (t + \delta t)}]_{\omega=\omega_0} e^{-i \omega_0 \delta t}/F[w(t)]_{\omega=0}.
\]

Comparison of (16) and (17) shows that \( A_{est} = \sum_{j=1}^{2l+1} a_j \) only if \( F[w(t)]_{\omega} e^{-i \omega \delta t} \) behaves like \( H(\omega, \delta \omega) \) for \( |\omega| < |\delta \omega|_{max} \) (i.e. has the boxcar-like shape). Neglecting edge effects, the required taper has the form \( w_{\delta \omega}(t) = w_{\delta \omega}(t + \delta \omega) \), i.e. the taper for the offset record is shifted by the same amount \( \delta \omega \).

We may solve for the time-offset dependent taper \( w(t) \) by incorporating an extra phase factor into equation (11), simply by replacing \( F[w(t)]_{\omega} \) with \( F[w(t)]_{\omega} e^{-i \omega \delta t} \). The resulting minimization problem may be solved, as before, with the substitution \( w(t) = \sum_{j=1}^{N} b_j w_{\delta \omega}(t) \). The tapers and their spectra have been found for \( \delta \omega = 1.1 \omega_0 \) using the 12 lowest \( 4\pi \)-prolate tapers, for the cases (a) \( \delta t = 900 \) s (Fig. 6a), (b) \( \delta t = 1800 \) s (Fig. 6b), (c) \( \delta t = 2700 \) s (Fig. 6c), and (d) \( \delta t = 3600 \) s (Fig. 6d). It is seen that for records starting more than 30 min after the earthquake, the resulting spectral interference is large and such records cannot be used. Removal of the first surface wave packet from a long-period record thus prohibits the use of the record for obtaining amplitude observations for individual multiplets. Qualitatively, this occurs because the determination of the zeroth moment depends on the summing up of many singlets with the same initial temporal phase. When a fraction of the

Figure 7. Scatter plot of fit versus exact amplitudes after fitting the time offset Chagos aspherical earth synthetics with equation (17) using the tapers with respective time offsets (a) \( \delta t = 900 \) s, (b) \( \delta t = 1800 \) s, (c) \( \delta t = 2700 \) s, and (d) \( \delta t = 3600 \) s. The scatter for the last three cases is clearly large compared with that resulting from the zero offset taper (Fig. 5b).
record has been removed at the start, the split singlets are observed with different temporal phases and the available seismogram can be deconvolved to obtain the in-phase sum only at the expense of spectral leakage from outside the band of interest. From a travelling wave viewpoint, most of the information about modal amplitudes (and odd-degree earth structure) is contained in the first surface wave arrival $R_1$. Subsequent arrivals have an increasingly difficult time distinguishing between phase anomalies caused by aspherical earth structure and those caused by aliasing in the frequency domain.

Synthetic tests have been performed on time offset records which are identical to those used in the previous tests except that specified portions (900, 1800, 2700, or 3600 s) have been removed from the beginning of the record. The amplitude estimation is made with the time offset tapers of Fig. 6(a–d), using equation (17), and the comparison with the exact amplitude is shown in Fig. 7(a–d). Clearly, the scatter is large compared with the results for the zero offset records (Fig. 5b). If overtones were added to the synthetics, the scatter for the finite time offset tapers would become prohibitively large. In practice, about 80 per cent of the potential data set in this study must be discarded because of removal of the first surface wave arrival $R_1$.

The inversions performed in this study do not incorporate corrections for the effect of rotational self-coupling. Pollitz (1990) (his equation A2) gives a form for the displacement seismogram on a slowly rotating earth which is characterized by an apparent time-dependent amplitude. For each isolated multiplet, the seismogram can, in fact, be decomposed into a sum of two distinct resonance peaks with complex valued amplitudes. When (9) is used to estimate the amplitude of the isolated multiplet, it is found that the self-coupling effect leads to an additional modal phase shift $\Phi_m$, given by

$$\Phi_m = -\frac{d}{d\omega} \left. \Re F[w(t)] \right|_{\omega=0},$$

$$\Phi_m = -\frac{\alpha_s \cos \Theta}{\omega} \left. \Re F[w(t)] \right|_{\omega=0},$$

where $\alpha_s$ is a coupling parameter which depends on the radial and longitudinal spherical earth eigenfunctions, and $\Theta$ is the colatitude of the source–receiver great circle, on the left going from source to receiver. This modal phase shift must be subtracted from that derived from the amplitude pattern in order to make the proper correction. This effect is equivalent to a shift $\delta \Phi_s$ in the source longitude given by

$$\delta \Phi_s = \frac{\alpha_s \cos \Theta}{\omega} \left. \Re F[w(t)] \right|_{\omega=0}.$$

Thus, in principle, correction of amplitude estimates using (18) is not necessary for records corresponding to relocated events, since $\delta \Phi_s$ would already be implicitly present in the source location. It is found that the contribution of $\Phi_m$ for modes $\partial S_{2n}$ and $\partial S_{2n}$ is about 2 per cent of that expected from lateral heterogeneity. The predicted source longitude shift for mode $\partial S_4$ (assuming a record length of 13.5 hr) is $-0.009^\circ$. We regard the signal arising from $\Phi_m$ to be small enough that no explicit correction for the effect is necessary.

### 4 DATA ANALYSIS

The initial data set consisted of 1005 edited IDA recordings, spanning the period 1977–April 1985, and 98 vertical component GEOSCOPE recordings spanning the period 1983–June 1986. Each record was tapered with the $1.1\omega_R$ taper of Fig. 4(a) and the real-part spectrum plotted. Only those records with acceptable spectra were kept. The remainder, the majority of the records, were discarded for several reasons. Most commonly, the first surface wave arrival $R_1$ from moderate and strong events suffered from non-linear instrument response. As shown in the previous section, the amplitude estimates from such records are severely corrupted by spectral leakage from outside the band of interest. A number of records from the weaker seismic events were judged to have poor signal-to-noise ratio and were discarded. Other records had poorly resolved spectral peaks arising from strong overtone interference or a suspected complicated source time function. These effects combine to reduce the potential data set from 1103 to 172 source–receiver pairs. The acceptable records were tapered with the $1.1\omega_R$ taper (Fig. 4), with record length 13.5 hr. This choice of record length minimizes the variance of the amplitude estimates when they are estimated with (9) in the 3–5 mHz band.

A typical real-part spectrum (1984 March 6 recorded at IDA station NNA) is shown in Fig. 8(a) by the solid line, with the real-part spectrum of the corresponding spherical earth synthetic shown by the dotted line. Comparison of the data and spherical earth spectra clearly shows that the amplitude pattern for this record has a periodicity of approximately 5 in wavenumber. This is predicted from equation (5) for the source–receiver distance $\Delta = 142.5^\circ$. For example, the triplets $\partial S_{2n,3n}$ and $\partial S_{4n}$ each exhibit the same amplitude anomaly patterns. This record, as with all of the data records, the fiducial frequency $\omega_R$ was chosen for each mode at approximately the midpoint of each well-defined spectral peak, amplitudes were estimated using (9), and error estimates were assigned subjectively based on the signal-to-noise power in the spectrum. Fifteen amplitudes within the band $l = 26–43$ were recovered from this record, and best-fitting $\delta A_0$ and modal phase shift $\Phi$ were derived from least-squares inversion of (5) over the entire band. In this study, all moment tensor solutions are taken from the published CMT catalogue. After applying a correction for ellipticity (Dahlen 1975), the modal phase shift $\Phi$ due to aspherical earth structure is found to be $-0.152$ rad, compared with the predicted $-0.095$ rad on the M84A earth for mode $\partial S_4$. The data $\Phi$ computed here is actually based on a relocated source position for this event. We shall discuss later the details of the source relocations.

Another example is shown in Fig. 8(b) for the 1986 May 7 event recorded at GEOSCOPE station DRV. The amplitude pattern in this record has a periodicity of 3, consistent with the pattern expected for the source–receiver distance $\Delta = 122.4^\circ$. After applying an ellipticity correction, the modal phase shift $\Phi$ (over the entire band) is found to be $-0.314$ rad, compared with the predicted $-0.031$ rad on the M84A earth for mode $\partial S_4$. This is one of the largest disagreements between the data and model M84A that we have encountered in this data set. In addition, the amplitude...
Inversion of normal mode amplitudes

Figure 8. Data and spherical earth synthetic records have been tapered with the 1.1 wavelength of Fig. 4. (a) 1984 66 event recorded at IDA station NNA. Plotted is the real-part spectrum for both the data seismogram (solid line) and the corresponding spherical earth synthetic seismogram (dotted line). The spherical earth synthetic spectrum is out of phase with respect to the data spectrum with a periodicity of 5 in wavenumber. Note that the same patterns are present for the $oS_{38}-oS_{43}$ triplet or the $oS_{37}-oS_{40}$ triplet. The overall pattern is correctly predicted from equation (5) with $\Delta = 142.3^\circ$ and $\Phi = -0.152$ rad ($-8.7^\circ$). (b) 1986 127 event recorded at GEOSCOPE station DRV. The amplitude pattern has a periodicity of 3 in wavenumber. The overall pattern is predicted from equation (5) with $\Delta = 122.4^\circ$ and $\Phi = -0.134$ rad ($-16.7^\circ$), but the amplitude anomalies are clearly larger at frequencies above 4 mHz.

pattern in the data record changes abruptly near about 4 mHz ($oS_{32}$). This is best seen by noting the magnitudes of the amplitude anomalies for the modes $oS_{28}$, $oS_{31}$, $oS_{34}$, $oS_{37}$, $oS_{40}$, and $oS_{43}$. Only small amplitude anomalies are seen for the modes $oS_{28}$ and $oS_{31}$, but systematically larger anomalies are seen for the higher frequency modes. This reflects different sensitivities of the normal modes to the aspherical earth structure being sampled along the source-receiver great circle.

In order to test how reliably the modal phase shifts are recovered from the data records, we computed synthetic amplitudes ($l = 26-43$) on the M84A earth for nine events at all IDA stations using the subspace projection method (Park 1987). For each record, these amplitudes were inverted for best-fitting $\delta \lambda_0$ and $\Phi$ in the same manner as the data. The resulting $\Phi$ is compared in Fig. 9 with the exact $\Phi$, computed from equation (4) for mode $oS_{34}$, which lies at the centre of the band of observations. Only source-receiver pairs with $155^\circ > \Delta > 25^\circ$ are retained in the plot, since near the source and antipode the asymptotic approximation to the Legendre function used by Park (1987) breaks down. The agreement in Fig. 9 is generally good and indicates that, with the same restrictions, similar results may be expected from the data. The exact $\Phi$ are not obtained from the fits because of the limitations of the geometric optics approximation implicit in (4) as well as the process of approximating $\Phi$, within the entire mode band with the constant $\Phi$. 
Figure 9. Synthetic amplitudes $\phi_{S_4-O_{S_4}}$ for nine selected events at all IDA stations have been generated using the subspace projection method (Park 1987), using aspherical earth model M84A (Woodhouse & Dziewonski 1984). These records have each been fit for $\delta A_0$ and $\Phi$ from equation (5) using all amplitudes within the band $\phi_{S_4-O_{S_4}}$. The derived $\Phi$ are corrected for the effect of ellipticity and then compared with the exact $\Phi$ for mode $O_{S_4}$ computed from (4) using model M84A.

All 172 data records yielded amplitudes which were then inverted for $\delta A_0$ and $\Phi$, followed by application of an ellipticity correction. The quality of the record fits is defined by the variance reduction, equal to

$$1 - \frac{\sum_{i} \left( \frac{\phi_i - \phi_{th}}{\sigma_i} \right)^2}{\sum_{i} \left( \frac{\phi_i}{\sigma_i} \right)^2}$$

where $\phi_i$ is the observed amplitude of $O_{S_4}$, $A_i$ is the theoretical amplitude (5), and $\sigma_i$ is the standard deviation of the amplitude observation. The quality of the resulting inversions for earth structure is defined by the $\chi^2$ value, equal to

$$\chi^2 = \sum_{n=1}^{N} \frac{(\Phi)_{n} - (\phi_{th})_{n}}{\sigma_{\phi}}$$

where $(\Phi)_{n}$ is the derived $\Phi$ for the $n$th record, $(\phi_{th})_{n}$ is the theoretical $\Phi$ calculated from (4) at the centre of the employed frequency band, $(\sigma_{\phi})_{n}$ is the standard deviation of $(\Phi)_{n}$, and $N$ is the total number of records used. The variance reduction $\Delta \sigma^2$ is equal to $1 - \chi^2/V$, where $V$ is the variance on the spherical earth.

In order to ensure that only high-quality records were kept, a number of criteria were used to cull away bad records. Records were discarded if: (i) the focusing term $\delta A_0$ was more than 50 per cent of the average scaling amplitude $(A_0)$; (ii) the variance reduction over the range of amplitude measurements was less than 85 per cent; (iii) the receiver location was within $25^\circ$ of the source or antipode; or (iv) fewer than six amplitudes were available for fitting with (5). This reduced the data set to 109 source-receiver pairs. The resulting data set of modal phase shifts $\Phi$ is compared with the exact $\Phi$ expected on the M84A earth (Fig. 10a). The scatter in this plot is large, and the scatter between the data and model predictions is clearly large. (b) The same set of data $\Phi$ is compared with the predicted $\Phi$ on the model M84A earth after correcting for the epicentre locations. Source relocations were actually performed on the wide-band degree 1–6 earth model. The scatter between the data and model predictions is substantially reduced. A data cut-off $|\Phi - \Phi_{M84A}| < 0.33$ rad is subsequently used to cull the greatest outliers from the data set.
The derived data and predicted \( \Phi \) on the M84A earth are compared for two selected events: (a) 1984 289 event, and (b) 1984 138 event (Table 1). The scatter is large but is markedly reduced after allowance is made for epicentre relocations (squares). The magnitude of the relocations is large for most of the events for which at least three records (modal phase observations) had been recovered (Table 1). In most cases, four or more records are available for the source relocations. In order to perform these corrections without biasing the final earth model, these corrections must be recomputed for each successive revised earth model, and the process continued until convergence in both the earth model and source location is achieved. We found that only three iterations of this process were necessary to obtain satisfactory convergence for the inversion which was performed. In order to make use of the maximum amount of data in constraining the source locations, the entire mode band \( l = 26-43 \) was used for the iterative inversion of earth structure and source locations, and we shall refer to the resulting earth model as the wide-band model.

Physically, the apparent source locations inferred from long-period seismic data differ from those derived by body waves because of the finite length and duration of the rupture process (Kanamori & Given 1981). Generally, the apparent source epicentres in this study differ from the body

| Table 1. Original and final source locations of relocated events. |
|---|---|---|---|---|---|---|---|---|---|
| year | month | day | \( \phi^\circ \) | \( \phi^\circ \) | \( \phi^\circ \) | \( \phi^\circ \) | \( \phi^\circ \) | \( \phi^\circ \) | \# stations |
| 1978 | 3 | 7 | 31.95 | 31.60 | 0.329 | 0.182 | 3 |
| 1970 | 5 | 21 | -15.70 | -15.36 | -0.073 | 0.112 | 5 |
| 1982 | 6 | 22 | 7.28 | 135.09 | 0.100 | 3 |
| 1983 | 5 | 15 | -19.09 | -15.70 | -0.100 | 0.144 | 4 |
| 1984 | 5 | 17 | -36.44 | -35.34 | 0.074 | 0.074 | 7 |
| 1986 | 10 | 15 | -15.81 | -17.39 | 0.103 | 0.080 | 5 |
| 1986 | 4 | 14 | 1.77 | 136.56 | 0.148 | 0.080 | 4 |
| 1988 | 9 | 21 | 17.57 | 17.74 | 0.107 | 0.088 | 5 |
| 1980 | 5 | 7 | 51.33 | 51.01 | 0.061 | 0.088 | 4 |

* CMT source location after relocation
* \( \sigma_\phi \) and \( \sigma_\phi \) are the standard deviations in relocated latitude and longitude.

Inversion of normal mode amplitudes

Inversion of normal mode amplitudes is performed in this study allow for small shifts in both the source latitude and longitude. These are determined such that the difference between the data \( \Phi \) and theoretical \( \Phi \) is minimized in a least-squares sense over all of the records corresponding to a given event. The source relocations are model-dependent, since the theoretical \( \Phi \) depend on the earth model which is chosen. For a mode such as \( \phi_{M84A} \), a 1° shift in source location leads to a 0.6 rad error in apparent \( \Phi \). This is best seen in two examples from the present data set. The 1984 October 15 event (depth 119 km) is represented by seven stations, and the derived \( \Phi \) for each record is compared with the model M84A expected \( \Phi \) in Fig. 11(a). The scatter is large but is substantially reduced if the event is relocated, as shown by the solid circles. Although this event was actually relocated on the degree-6 earth model to be discussed, it is clear that approximately a 1° shift in source location is needed to adequately predict the amplitude pattern.

A large (0.4°) shift in source location is also needed to explain the amplitude pattern for the 1984 May 17 event (depth 10 km) as observed at five stations (Fig. 11b). In order to account for the necessary shifts in source location without introducing too many additional degrees of freedom, source location shifts were computed for those events for which at least three records (modal phase observations) had been recovered (Table 1). In most cases, four or more records are available for the source relocations. In order to perform these corrections without biasing the final earth model, these corrections must be recomputed for each successive revised earth model, and the process continued until convergence in both the earth model and source location is achieved. We found that only three iterations of this process were necessary to obtain satisfactory convergence for the inversion which was performed. In order to make use of the maximum amount of data in constraining the source locations, the entire mode band \( l = 26-43 \) was used for the iterative inversion of earth structure and source locations, and we shall refer to the resulting earth model as the wide-band model.

Physical...
wave hypocentres, which reflect the place of initiation of an earthquake. Large earthquakes, such as those employed in this study, tend to have characteristic rupture lengths of 20–200 km, and we shall find that the apparent source locations differ from the body wave source locations by comparable distances. The CMT moment tensor solutions employed in this study are generally based on both body wave and surface wave data (Dziewonski & Woodhouse 1983). It appears to us that, even with the inclusion of the surface wave data, the CMT solutions in many cases are dominated by the body wave information.

In general, the inferred rupture directions for the events listed in Table 1 have a large component normal to the slip directions found from the moment tensor inversions. In the case of the 1984 October 15 event, the horizontal slip azimuth along the preferred fault plane is 71° (Dziewonski, Franzen & Woodhouse 1985) or 80° (Person 1984), while the rupture direction (based on the difference between the surface wave epicentre and body wave epicentre) is 162°. Thus the two directions differ by a full 90°. The body wave epicentre (Person 1984) at -15.86°S, -173.64°E differs from the CMT-determined epicentre by 0.24° but differs from the relocated epicentre (Table 1) by 0.90°. The inferred dimension of the fault plane (200 km) is consistent with the large half-duration (13 s) for the earthquake. Another relocated event which shows evidence of large surface wave directivity is the 1985 April 13 event with a body wave epicentre of 1.62°N, 126.41°E (Person 1986). Depending on the choice of fault plane, the inferred rupture direction differs from the horizontal slip direction by 80°–90°. In addition to these cases, we note that for the well studied 1979 Colombian earthquake (Beck & Ruff 1984), the relative slip direction and the rupture direction (based on body wave and surface wave directivity) differ by about 50°. Finally, the epicentres of the relocated 1979 May 21 and 1984 May 17 events disagree significantly with the CMT epicentres (0.4°–0.5°), yet the PDE body wave epicentres for these two events agree with the relocated epicentres to within 0.1°. This suggests that while the CMT solutions may be dominated by body wave information for some events (for example, 1984 October 15 and 1985 April 13), they may be biased by additional effects (such as inaccurate path corrections) in other cases.

After correcting for the new source locations for each eligible event, the scatter between the measured modal phase and the predicted M84A Φ, shown in Fig. 10(b), is reduced. In order to prevent the final wide band model from being biased by a few inaccurate data points, a data cut-off was established to remove points from the data set for which |Φdata − ΦM84A| > 0.33 rad. This cut-off is large enough that potential differences between the present model and model M84A will be manifested, but also small enough to prevent significant bias from a small number of data points. Seven points are removed in this fashion, reducing the data set from 109 to 102 source–receiver pairs. Each of these seven points belongs to an even which has not been relocated and, given Fig. 11(a–b), may be reasonably suspected of possessing significant errors in the published location. In addition, the χ² reduction achieved by removing these points (comparing inversions both with and without these points) is 72.2, far greater than the number of points removed. Approximately 50 per cent of the remaining data set contains source–receiver pairs belonging to events which have not been relocated, and the question arises whether any of the remaining scatter is due to inaccurate locations for these events. Though this is a possibility, the scatter remaining in Fig. 10b is roughly evenly divided between the relocated and unrellocated events, suggesting that differences between the derived wide-band model and model M84A are real.

Equation (4) defines an overdetermined linear inverse problem for earth structure coefficients δω', which yield δωlocal(θ, φ) in a truncated spherical harmonic expansion (we employ the conventions of Edmonds 1960).

\[ \delta \omega_{\text{local}}(\theta, \phi) = \sum_{s=1}^{max} \sum_{t=-s}^{s} \delta \omega_s Y_s^t(\theta, \phi) \]  

(19)

where \( \delta \omega_{s} = \delta \omega_{s}' \). Letting \( \delta \omega_s' = A_s' + iB_s' \), (19) becomes

\[ \delta \omega_{\text{local}}(\theta, \phi) = \sum_{s=1}^{max} \sum_{t=-s}^{s} [A_s' \cos(t\phi) - B_s' \sin(t\phi)]X_s^t(\theta) \]  

(20)

where \( X_s^t(\theta) = Y_s^t(\theta, 0) \).

The coverage of the data set is shown in Fig. 12, where the final 102 source–receiver great circles used in inversion

![Figure 12. The distribution of source–receiver great circles used in the final wide band inversion.](https://academic.oup.com/gji/article-abstract/102/2/313/655444/102-2-3136554444 by guest on 28 January 2019)
Inversion of normal mode amplitudes

for the wide-band model have been plotted. This represents the distribution of information on the earth for which odd-degree structure may be recovered from the present data set. The coverage is poorest in northern Asia and the SE Pacific, and we do not expect the model to be able to predict local earth structure in these regions. In order to recover higher degree earth structure with the small data set, the even-degree earth structure must be constrained, and we have chosen to do this by fixing the $s = 2$, 4, and 6 degree coefficients according to the average of the values obtained by Smith & Masters (1989) over the band $l = 26-43$. The even degree components of $\delta \omega_{\text{local}}(\theta, \phi)$ tend to cancel out in the phase integral (5), while the odd-degree components essentially integrate constructively, leading to much smaller data kernels for the even-degree components. Thus the amplitude data set is expected to be more sensitive to odd-degree structure than even-degree structure. This will be verified, in the examples to follow, by the pattern of uncertainties in the earth structure coefficients when degree-2 coefficients are included in the inversion. In this sense, the amplitude data complements the normal mode frequency data, which is sensitive to only even-degree structure.

Two inversions were performed. First, the best fitting degree-6 model was obtained by fixing the degree $s = 2$, 4, and 6 coefficients and inverting for the $s = 1$, 3, and 5 coefficients. These odd-degree structure coefficients are obtained by iteratively computing the new source locations on the degree-1-6 earth and inverting again for the odd components of the best fitting degree-6 model. The original (CMT) and new source locations for the relocated events are listed in Table 1. The derived odd-degree coefficients are plotted and compared with the M84A coefficients in Fig. 13. The agreement for degrees 1 and 3 is very good, but there is significant disagreement for degree 5, particularly $A_5^2$ and $B_5^2$. The wide-band degree 1-6 earth model is plotted in Fig. 14, with the contour interval equal to $5 \mu$Hz. The corresponding 95 per cent error map in $\delta \omega_{\text{local}}(\theta, \phi)$ computed from the covariance matrix of the structure coefficient estimates, is shown in Fig. 15. The errors are distributed among values between 7 and $17 \mu$Hz. As expected, the uncertainties in the earth structure have local maxima in NE Asia and the SE Pacific, where the coverage is the poorest. The general pattern of the derived model is in agreement with previous earth models, i.e. slow regions over western North America, the Gulf of Aden, the SE Indian Ocean, and the Pacific–Antarctic Rise; fast regions over old ocean crust in the western Pacific and southern Atlantic oceans. Although a possible fast region in NW Asia (and a slow region in NE Asia) is not detected in this study, the errors obtained for the other areas are small, further reinforcing the structure estimates for these regions. Overall, the errors in local earth structure obtained here are about a factor of 6 smaller than those shown by Tanimoto (1985) for 250 s Rayleigh waves. Since Tanimoto's model also includes those errors arising from the determination of the even-degree structure, a proper comparison with our results would need to take this into account but would still leave about a factor of 3 improvement in the present model errors (which arise only from the odd-degree components). Also, the model errors would be larger if errors associated with the relocation of earthquakes were taken into account, but this contribution to the overall errors is not included in the analysis of Tanimoto (1985) either. The variance reduction obtained here (Table 1) is higher than that of model M84A and roughly equal to that reported by Tanimoto.

![Earth Structure Coefficients](https://academic.oup.com/gji/article-abstract/102/2/313/655444/figw)

**Figure 13.** The earth structure coefficients resulting after inversion for the odd-degree coefficients over the wide band. The odd-degree coefficients corresponding to the expansion (19) are shown for the derived model (circles with 95 per cent error bars) and model M84A (hexagons).
Figure 14. Earth structure contours of $\delta\omega_{\text{local}}(\theta, \phi)$ after performing the wide-band inversion. Contour interval is 5 $\mu$Hz. The structure shown represents an average over the mode band employed ($S_{25}$-$S_{43}$). Degrees 1–6 in the expansion (20) are represented, with the odd-degree coefficients given in Fig. 13 and the even-degree coefficients fixed according to the values given by Smith & Masters (1989).

Figure 15. The 95 per cent error contours of $\delta\omega_{\text{local}}(\theta, \phi)$ (approximately two standard deviations) are plotted for the wide-band model depicted in Fig. 14. The errors are distributed over values spanning 7–17 $\mu$Hz.

& Anderson (1985) (54.5 per cent for 250 s Rayleigh waves). Thus we believe that the small model error estimates obtained here are not due to small data errors or the truncation of the model at degree 6, but reflect the advantages of using amplitudes rather than surface wave phase anomalies as observational data.

We have explored whether some of the disagreement between the derived model and model M84A is an artifact of the data distribution. To test this possibility, we performed the same inversion for $s = 1, 3, \text{ and } 5$ coefficients using the theoretical M84A path integrals as the ‘data’, substituting them for the actual data and assigning them the actual data errors. In this scheme, the degree $s = 2, 4, \text{ and } 6$ coefficients are fixed at their respective values on the M84A earth. The odd-degree structure coefficients derived from this inversion are compared with the exact M84A values in Fig. 16(a). It is clear that while the degree-1 and -3 coefficients are reproduced accurately, some of the degree-5 coefficients show sizable excursions. If the input model were truncated at degree-6, then we would find that this inversion recovers the input model exactly. Since the full degree 1–8 model M84A is used in this experiment, all of the misfit results from the mapping of degree-7 and -8 structure into the degree-1–6 model space of the inversion. Since unmodelled (degree-7 and higher order) structure is potentially significant on the real earth, we can not rule out the possibility that most of the disagreement for degree-5 is caused by the mapping of unmodelled structure into these coefficients. Nevertheless, the global distribution of $\delta\omega_{\text{local}}(\theta, \phi)$ derived from this synthetic inversion (Fig. 16b) does not differ sharply from that of model M84A (Fig. 16c). The differences are that the structural perturbations in certain regions (such as southern Africa) are not completely recovered; in particular, the magnitude of the structural perturbations in Asia (positive in the northwest, negative in the northeast) are less pronounced after the synthetic inversion. This effect is much more severe in the data inversion map (Fig. 14), suggesting that additional earth structure (not accounted for by model M84A) is being mapped in unpredictable ways into the degree-6 model space.

Precisely how higher ($s \approx 7$) earth structure can lead to
Inversion of normal mode amplitudes

Figure 16. (a) The earth structure coefficients derived from M84A path integrals ($\omega_{S_{n}}$) for odd-degree structure, using the same source–receiver pairs and errors as in the wide-band inversion. Hexagons are model M84A values for mode $\omega_{S_{n}}$. (b) Earth structure contours of $\delta \omega_{local}(\theta, \phi)$ for the model shown in (a). (c) Earth structure contours of $\delta \omega_{local}(\theta, \phi)$ on the M84A earth for mode $\omega_{S_{n}}$. The model is truncated at degree 6.
in two ways. First, it is possible to increase the model space to include higher degrees (shorter wavelengths) and attenuate the contribution of the higher degree structure by incorporating a smoothness constraint in the inversion. Park (1989) has shown that this procedure can lead to more accurate recovery of long wavelength ($s \leq 6$) structure. We have applied this idea to the amplitude data set by performing inversions for the smoothest degree-1–10 models satisfying the data set (over the wide band $0.5 < s < 3$) at a given level of variance. In Fig. 17 we show the variation of model coherence and gain (Park 1989) with variance reduction, with the reference model being the wide-band truncated degree-1–6 model (Figs 13–15). Coherence and gain values of unity correspond to exact duplication of the reference model. From Fig. 17 it is clear that the even-degree ($s = 2, 4$, and 6) coefficients are not accurately recovered from the amplitude data set [assuming the Smith & Masters (1989) values to be correct], but the odd-degree coefficients do not differ greatly from those obtained in the truncated degree-1–6 inversion. At 75 per cent variance reduction, the odd-degree components of the smoothest model have an overall coherence of 0.773 and gain of 0.885 compared with the reference model. This indicates that the recovery of the odd-degree coefficients in the truncated inversion is not being significantly corrupted by aliasing from higher degree earth structure.

Next, we have tested the sensitivities of the derived earth structure coefficients to higher degree structure through the following experiment. Using the same source-receiver and error distributions as in the data inversion, we have performed inversions for degree-1–10 structure. The degree-4 and -6 structure are held fixed in this procedure since, among the even-degree coefficients, we wish to focus on degree-2 sensitivity. The ‘determined’ earth structure is interpreted as being of the form

$$\delta \omega_{\text{local}}(\theta, \phi) = \sum_{s=1}^{10} \sum_{i=1}^{s} \delta \omega_s^i [\delta \omega_s^i + \eta_s^i] Y_s^i(\theta, \phi)$$

where $\delta \omega_s^i$ is the deterministic component of the (st) coefficient, and $\eta_s^i$ is the random component. From standard inverse theory, we know that the covariance matrix for the $\eta_s^i$ is such that the data covariance distribution resulting from it is identical with that of the actual data study. We are thus assuming that all of the data variance is due to the presence of the random components $\eta_s^i$, including their indirect effect on the model space through the model covariance matrix. It is a simple matter to show that the marginal distribution function for $\eta_s^i$ and $\eta_s^i$ is dominated by their cross-correlation for any fixed $\chi^2$ value (Fig. 18a), and that the error induced in the estimated $\delta \omega_s^i$ coefficient ($i \geq 6$) from a unit perturbation in the $\delta \omega_s^i$ coefficient ($s \geq 7$) in a truncated degree-1–6 inversion would be equal to

$$\sigma_{s(i)sr} \sigma_{sr}$$

Here, $\sigma_{s(i)sr}$ is the covariance of the structure coefficients $\delta \omega_s^i$ (real or imaginary part) and $\delta \omega_s^i$ (real or imaginary part). Such induced errors add linearly for each (st) component. More precisely, the expected value of the estimated $(ij)$ coefficient for particular values of the $\eta_s^i$ is given by

$$\langle \delta \omega_s^i + \eta_s^i \rangle = \delta \omega_s^i + \sum_{s'=7}^{10} \sum_{i'=7}^{s'} \frac{\sigma_{s(i)sr}}{\sigma_{sr}} \eta_s^i.$$  

This form of the expectation value for the $(ij)$ coefficient is known in statistics as the regression of $\eta_s^i$ with respect to each of the $\eta_s^i$ (Hamilton 1964). The perturbations which simulate a truncated degree-1–6 inversion correspond to $\eta_s^i = -\delta \omega_s^i$. Since the actual sizes of the higher ($s = 7$) $\delta \omega_s^i$ coefficients are unknown, we quantify the sensitivity to higher degree earth structure by defining a root-mean-square sensitivity:

$$\sigma_{(sr)} = \left[ \frac{1}{s} \sum_{s=7}^{10} \frac{\sigma_{s(i)sr}^2}{\sigma_{sr}} \right]^{1/2}.$$  

A high sensitivity to degree-7 and higher earth structure is reflected by high $\sigma_{(sr)}$ values. A useful property of the $\sigma$ is that they are independent of the data values themselves and only reflect how the data distribution is coupled with the structure coefficient recovery. Fig. 18(b) shows the derived average sensitivities for degrees 1, 2, 3, and 5. This figure indicates that large errors in the $A_s^5$ coefficient may be expected from aliasing of degree $s \geq 7$ earth structure. This may explain why the $A_s^5$ coefficient is generally poorly recovered when degree 2 is included in the inversions in this
Inversion of normal mode amplitudes

Figure 18. (a) Marginal distribution of $\eta_i$ and $\eta_i'$ for a given $\chi^2$ level. The change in the expected value of $\eta_i'$ from a change in $\eta_i$ is controlled by the slope of the tangent to the $\chi^2$ contour at $\eta_i' = 0$. (b) Average correlation coefficients of selected long-wavelength ($s \leq 6$) model parameters with short-wavelength ($s \approx 7$) model parameters.

study. On the other hand, the degree-1 structure is largely insensitive to aliasing of higher degree earth structure, as is also evident from Fig. 17.

In order to test the consistency of the amplitude data set with the degree-2 structure (which was fixed in the degree-1–6 inversion), an inversion for $s = 1, 2, 3$, and 5 structure was performed with the $s = 4$ and 6 coefficients held fixed and employing the final event locations of the previous inversion. This is necessary because the small data kernels for the even-degree coefficients do not allow for simultaneous inversion of the earth model and source locations in a stable fashion. In Fig. 19, the resulting earth structure coefficients are plotted and compared with the corresponding M84A values for mode $\ell = 4$ and the average Smith & Masters (1989) degree-2 values. The errors associated with the degree-2 structure coefficients are clearly large, as a result of their small data kernels. The derived odd-degree structure coefficients show little change from those derived in the odd-degree inversion (Fig. 13). The first two columns of Table 2 list the number of parameters fit and
the variance reductions corresponding to the wide band inversions.

5 NARROW BAND INVERSIONS

Here we wish to obtain the average odd-degree earth structure within the bands \( l = 26-36 \), \( l = 29-39 \), and \( l = 31-43 \). These bands are centred, respectively, on \( l = 31 \), \( l = 34 \), and \( l = 37 \). The variations in the obtained earth structure, as well as the quality of the resulting fits within each band, will lead us to identify important trends in both the determined and undetermined earth structure. Because the recovery of the modal phase shift \( \Phi \) requires a least-squares fit of the available amplitude observations via (5), the number of records yielding coherent amplitude patterns is expected to be reduced if the fits for \( \Delta A_0 \) and \( \Phi \) are done exclusively within the narrower mode bands. For example, we experimented with fitting amplitude observations \( \phi_{51} - \phi_{53} \) with (5) and inverting the phase shifts \( \Phi \) for \( s = 1, 3, \) and 5 structure coefficients using the final relocated events (Table 1). The number of usable source-receiver paths is reduced from 102 to 85, and \( \chi^2 \) is increased from 149.4 to 188.1. This result is unacceptable, and in order to improve the recovery of earth structure within the narrower frequency bands, we adopted a preferential weighting scheme. If \( \sigma_i \) is the standard deviation assigned to an amplitude observation \( A_i \), then we replace \( \sigma_i \) with \( 2\sigma_i \) for those observations which lie outside of the band. This effectively reduces the importance of observations outside of the band to 25 per cent compared with those within the band. Synthetic experiments indicate that very little bias is introduced in retaining outside observations in this fashion. We shall refer to this process as the \( 2\sigma \) weighting scheme. From a practical viewpoint, inclusion of the extra modal observations primarily stabilizes the determination of \( \Delta A_0 \) and better decouples it from the determination of \( \Phi \) in the least-squares fit of (5). Obviously, it would be preferable to avoid the process of narrow-band inversions using somewhat arbitrary constraints from outside the band of interest. The most straightforward procedure is to invert directly for depth-dependent earth structure (density, bulk and shear moduli) using all available modes simultaneously. More modes from the lower frequency bands (1.5–3.5 mHz) need to be included before this is possible. As discussed by Pollitz

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Figure 19. The earth structure coefficients after inverting for the \( s = 1, 2, 3, \) and 5 coefficients over the wide band. The data values are shown by the circles with the 95 per cent error bars, model M84A values are shown by hexagons, and Smith & Masters (1989) values (averaged over the wide band) for the degree-2 coefficients are shown by the diamonds.

Table 2. Fitness parameters of wide band and narrow band inversions.

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\( N \) is the total number of modal phase observations inverted, and \( M \) is the number of fitted parameters in the model. The narrow band results correspond to the \( 2\sigma \) weighting scheme. \( \Delta \sigma^2 \) is the variance reduction in percent.
Inversion of normal mode amplitudes

The residuals of amplitude fits

Figure 20. Compiled scaled residuals of data amplitudes after successively removing individual modes \(o_{S1}, \ldots, o_{S33}\) from the data records. Records have been fit for \(\delta A_0\) and \(\Phi\) from (3) using amplitude observations \(o_{S11-o_{S43}}\), with one mode removed in each fit. The residuals are compiled as a function of mode removed for each record, scaled to unity, and then plotted. The average scaled residual shown by the solid line appears to be smallest when mode \(oS_{33}\) is removed from the data set, indicating that systematic bias may be present in the amplitude observations of these modes.

Inversion of normal mode amplitudes (1990), complications arising from the effect of spheroidal–toroidal coupling must be overcome before this can be achieved.

We have examined the sensitivity of the modal phase determinations to removal of specific modes \(o_S\) from the data set. This provides a test of whether or not some modes possess systematic errors which may bias the earth structure inversions. In the experiment with \(o_{S11-o_{S43}}\) described above, we successively removed modes \(o_{S11}, o_{S21}, \ldots, o_{S43}\) from the data set. For each record, the average rms residuals after fitting for \(\delta A_0\) and \(\Phi\) are determined as a function of the mode removed, and then this set of quantities is normalized to unity. The compiled rms residuals for all of the records are shown in Fig. 20. It is clear that removal of either \(oS_{33}\) or \(oS_{41}\) leads to greater variance reduction for the record fits, and we are led to question whether the amplitude estimates of these modes possess systematic departures from the theoretical amplitudes arising from unmodelled effects. In the case of \(oS_{33}\) we strongly suspect that spheroidal–toroidal rotational coupling is responsible for its consistent misfit within the data set. The consistent misfit of \(oS_{41}\) is less easily explained and may be due to cross-branch coupling of the spheroidal and toroidal fundamental mode branches through lateral heterogeneity. In any case, we believe that the \(oS_{33}\) misfit is systematic and have removed \(oS_{33}\) from the data set in all subsequent narrow-band inversions.

For each narrow band inversion, the even-degree earth structure coefficients were fixed according to the average of the values obtained by Smith & Masters (1989) within each respective mode band. The source locations computed on the wide-band earth model (with event locations listed in Table 1) were employed for the narrow-band inversions. This is a reasonable procedure since the frequency dependence of apparent source locations is expected to be small (Kanamori & Given 1981). As with the wide band, two sets of inversions were performed. First, the \(s = 2, 4,\) and 6 coefficients were held fixed and the odd-degree coefficients \(s = 1, 3,\) and 5 were determined. The fitness parameters corresponding to this set of inversions are listed in Table 2. The data \(\Phi\) and M84A predicted \(\Phi\) within each respective mode band are shown in Fig. 21(a–c). It is clear that the disagreement between the data and M84A values increases with increasing band frequency. These departures are (with the exception of the greatest outliers) roughly evenly divided between the un relocated events (solid circles) and relocated events (diamonds). The derived earth structure coefficients are plotted and compared with the M84A coefficients in Fig. 22(a–c). It is apparent that agreement between degrees 1 and 3 is generally good, while there is substantial disagreement among the degree-5 coefficients, particularly for the \(o_{S31-o_{S43}}\) band. Given the fact that \(\chi^2/(N - M)\) increases as the average frequency of the band increases (Table 2), it is likely that unmodelled higher degree earth structure is responsible for the trend of the \(\chi^2\) residuals as well as the disagreement between the two sets of earth models.

In order to test whether the inclusion of modes from outside the band significantly biases the earth structure inversions, we performed inversions for the odd-degree coefficients assigning zero weight to the outside observations (restricted narrow-band scheme). We first fit each record for \(\delta A_0\) and \(\Phi\) over the entire mode band \((o_{S25-o_{S43}})\), and then we fit each record for \(\Phi\) in the narrow-band holding \(\delta A_0\) fixed. The earth structure coefficients determined from this set of inversions are plotted with the triangles in Fig. 22(a–c). There is little difference between the 2\(\sigma\) method and this method of inversion, and in reality the second inversion scheme is simply an alternative way of using outside information to constrain information within the narrow band.

In the second set of inversions, the \(s = 4\) and 6 coefficients were held fixed and inversion for the \(s = 1, 2, 3,\) and 5 coefficients was performed. These sets of inversions essentially test whether or not the amplitude data set is sensitive to the same degree-2 structure as the frequency data sets, as represented by the Smith & Masters (1989) \(s = 2\) coefficients. This set of inversions was performed using the 2\(\sigma\) weighting scheme. The derived earth structure coefficients are compared with the corresponding M84A coefficients as well as the Smith & Masters degree-2 coefficients (Fig. 23a–c). Comparing Fig. 23 and Fig. 22 it is clear that there is little variation in the derived odd-degree coefficients. The derived \(A^2_s\) coefficient does depart from the Smith & Masters values in the two higher frequency bands. We attribute this to aliasing of higher order (degree-7 and higher) earth structure into the model space. The \(A^2_s\) is particularly susceptible to such aliasing (Fig. 18b) because of its very small data kernel (note the large errors associated with its estimated values), an effect which is magnified by the preponderence of equatorial paths in the data.
Figure 21. Modal phase shifts $\Phi$ are derived from the data set within narrower bands and compared with the predicted M84A value $\Phi_r$ corresponding to the centre of the band: (a) $0S_{26}-0S_{36}$, $l = 31$, (b) $0S_{29}-0S_{39}$, $l = 34$, and (c) $0S_{31}-0S_{43}$, $l = 37$. Diamonds correspond to relocated events, and dots correspond to unrelocated events. It is clear that the scatter generally increases as the average frequency of the narrow band increases.

6 DISCUSSION

The ability to fit the data with a degree-1–6 earth model within the various narrow bands is shown in Fig. 24(a–c), where the data $\Phi$ are compared with the predicted $\Phi_r$ on each respective narrow-band model. For a proper comparison, only those points which were actually used in the inversion are plotted, these points which lie outside the data cut-off in Fig. 21(a–c) are not shown. From a comparison of Fig. 21 and Fig. 24 it is clear that the derived model does a significantly better job of predicting the data $\Phi$ than model M84A, despite the fact that the full degree-1–8 model was used to generate the M84A values. This supports the suggestion of Park (1988) that the degree-7 and -8 structure may not be well constrained in model M84A.

The global distribution of $\delta \omega_{\text{local}}(\theta, \phi)$ for each narrow-band inversion is shown in Fig. 25. Significant differences exist between each model, with the overall magnitude of the structural perturbations increasing as a function of band frequency. In all likelihood this reflects actual differences in the earth structure being sampled by the modes belonging to each band. This interpretation is supported by the fact that the $\chi^2$ residuals for each narrow-band inversion are much smaller than the $\chi^2$ value for the wide-band inversion (Table 2). Thus the wide-band inversion represents an imperfect average of earth structure over several distinct frequency bands. The increase in variance reduction with band frequency, even in the presence of a corresponding increase in $\chi^2$ residual, is due to the fact that the initial variance (on the spherical earth) is small in the $0S_{26}-0S_{36}$ band and increases to a high value in the $0S_{31}-0S_{43}$ band. This also supports the interpretation that the different mode bands are sampling different earth structure.

The fact that the relocated events are fit less well within the higher frequency bands in Figs 21 and 24 suggests that the effect of source finiteness contributes to the variance of $\Phi$ even after correcting for aspherical earth structure. This suspicion is based on the fact that apparent source locations depend roughly linearly on frequency and rupture length (Kanamori & Given 1981). Thus, for the earthquake sources with the largest fault dimensions used in this study (the 1984 October 15 and 1984 May 5 events), the ability to describe the long-period spectra in terms of a single apparent source location becomes less accurate at higher frequencies. The data point corresponding to the 1984 May 5 KIP record actually lies outside of the data cut-off in Fig. 21(c).
Inversion of normal mode amplitudes

Figure 22. Earth structure coefficients after fitting for the odd-degree coefficients $s = 1$, 3, and 5 within the bands (a) $0S_{26}$-$0S_{36}$, (b) $0S_{29}$-$0S_{39}$, and (c) $0S_{31}$-$0S_{43}$. The circles with 95 per cent error bars correspond to inversion using the $2\sigma$ weighting scheme (see text), the triangles correspond to inversion using the restricted narrow-band scheme (see text), and the hexagons are the model M84A values.

In light of the data distribution in Fig. 12 which shows distinct gaps in the coverage within NE Asia and the SE Pacific, it seems necessary to justify how the present data set constrains local earth structure as effectively as earlier global studies which have very dense data coverage (for example, Woodhouse & Dziewonski 1984; Tanimoto 1985). The explanation lies in the fact that the dense coverage of the earlier studies allows them to recover the even-degree earth structure quite accurately. As we have pointed out earlier, the majority of records used in previous earth structure inversions have suffered from inadequate sampling of $R_1$, the first arriving surface wavepacket. While such records still
place tight constraints on the average earth structure underlying the source-receiver great circle and thus contain ample information concerning even-degree earth structure, such records contain only limited information concerning odd-degree earth structure. If the global data sets employed in the previous studies were stripped of such incomplete records, we suspect that the distribution of source-receiver great circles would not appear much different from that shown in Fig. 12. The information concerning even-degree earth structure present in the majority of long-period
Inversion of normal mode amplitudes

Figure 24. The compiled data $\Phi$ are compared with the predicted $\Phi$, computed on the narrow-band earth models (Fig. 22) within the bands (a) $0^S_{26} - 0^S_{36}$, $l = 31$, (b) $0^S_{29} - 0^S_{39}$, $l = 34$, and (c) $0^S_{31} - 0^S_{43}$, $l = 37$.

Figure 25. Earth structure contours of $\delta\omega_{\theta\phi}(\theta, \phi)$ for the degree 1-6 models derived from the narrow-band inversions. (a) $0^S_{26} - 0^S_{36}$, (b) $0^S_{29} - 0^S_{39}$, and (c) $0^S_{31} - 0^S_{43}$. The odd-degree coefficients are those in Fig. 19, and the even-degree coefficients are fixed respectively at the average of the values obtained by Smith & Masters (1989) over the narrow band.

The compiled data are compared with the predicted $\Phi$, computed on the narrow-band earth models (Fig. 22) within the bands (a) $0^S_{26} - 0^S_{36}$, $l = 31$, (b) $0^S_{29} - 0^S_{39}$, $l = 34$, and (c) $0^S_{31} - 0^S_{43}$, $l = 37$.

seismic recordings is, in this study, implicitly contained in the even-degree earth structure coefficients fixed according to the results of the frequency studies.

To summarize, the distribution of information in the present study is divided between the roughly 100 employed records which constrain the odd-degree earth structure, and the even-degree earth structure coefficients provided by the frequency studies. The open questions which remain are whether the inclusion of several hundred incomplete records ($R$, not present) adds any significant amount of information concerning odd-degree earth structure, and whether source relocations [which this study and Woodhouse & Dziewonski (1984) have shown are necessary] may be adequately
performed using incomplete records. The answers to these questions will require detailed studies of fully coupled aspherical earth seismograms. We believe that limited information concerning odd-degree earth structure may be recovered from incomplete records, but that neither the surface wave approach (employing the resonance peak approximation) nor the present approach (requiring the construction of amplitude estimates for individual multiplets) appears to be capable of retrieving this information. Certain exceptional records, corresponding to extremely fast or extremely slow great circle paths, are amenable to the surface wave approach since the normal mode multiplets do behave as simple resonance peaks for such paths (Davis & Henson 1986). The vast majority of long-period seismic records, however, are composed of split normal mode multiplets. The incorporation of these records in global earth structure inversions shall likely involve waveform fitting of data seismograms with fully coupled aspherical earth seismograms (Dahlen 1987; Pollitz 1989). Since this procedure is highly non-linear, presumably an accurate starting model would be available in order to make this procedure feasible.

7 CONCLUSIONS

The principal results of this study are as follows.

(1) Normal mode amplitudes (zeroth moments) are sensitive to primarily odd-degree aspherical earth structure. The information in the amplitude pattern for an individual data record is contained almost exclusively in the modal phase shift $\Phi$ (equation 4).

(2) The amplitude estimation method that we employ accurately recovers the amplitudes of unresolvably split normal mode multiplets. The method requires that the first Rayleigh wavepacket $R_1$ be present in the data record. This restriction reduces the potential data set by approximately 80 per cent. From a spectral analysis viewpoint, the amplitudes of individual multiplets cannot be retrieved from incomplete records ($R_i$ not present), with the implication that such records provide only limited constraints on odd-degree earth structure. Conceivably, a more general inversion procedure not requiring the construction of zeroth moment estimates might retrieve limited odd-degree information from incomplete records.

(3) The recovery of $\Phi$ for individual source-receiver paths is strongly affected by mislocation of the assumed source hypocentre. Source relocations are necessary in order to adequately recover aspherical earth structure from amplitude observations. Apparent mislocations of several tenths of 1° are common and reach 0.9° in the severest case. Selected data records corresponding to un relocated events are removed from the data set in a systematic manner. After inversion for aspherical earth structure, the remaining variance is roughly evenly divided between the relocated and un relocated events.

(4) The distribution of gross earth data is dense for normal mode frequencies (which constrain even-degree earth structure) but relatively sparse for normal mode amplitudes (which constrain odd-degree earth structure).

(5) The derived odd-degree earth structure coefficients and those of model M84A are generally in agreement for degrees 1 and 3 but in substantial disagreement for degree 5.

(6) The structure coefficients derived from the truncated degree-1–6 inversions are moderately sensitive to aliasing from unmodelled higher degree earth structure, with the $A^0_2$ coefficient being the most sensitive.

(7) The $\chi^2$ residuals remaining after fitting for degree-1–6 earth structure increase as the average frequency of the employed mode band increases, suggesting that unmodelled (degree-7 and higher) earth structure becomes increasingly large for the higher frequency modes. The $\chi^2$ residuals after inverting for aspherical earth structure within restricted frequency bands are smaller than those resulting after inverting over the entire available frequency band, suggesting that differences among the various narrow band models are real.

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REFERENCES


Inversion of normal mode amplitudes


