

# A fuzzy approach for considering uncertainty in transient analysis of pipe networks

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## ABSTRACT

Uncertain parameters in the transient analysis of pipe networks lead to uncertain responses. Typical uncertainties are nodal demand, pipe friction coefficient and wave speed, which not only are imprecise in nature but also change significantly over time. Exploiting the fuzzy set theory and a simple scheme of the simulated annealing method, a conceptual model is developed. It can take into account the uncertainties of conventional transient analysis. This model helps designers of pipe systems in finding out the extent to which uncertainties in the inputs can spread to the transient highest and lowest pressures. A real piping system is analyzed herein as the case study. The results show that the transient extreme pressures can be highly affected by the uncertainties.

**Key words** | fuzzy sets, pipe networks, transient analysis, uncertainties

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## NOTATION

$A$	cross sectional area of pipe
$a$	wave speed
$C$	objective function
$D$	diameter of pipe
$f$	Darcy–Weisbach friction factor
$g$	gravitational acceleration
$H$	piezometric head
$m$	number of cuts
$n$	number of junctions
$P$	pressure head
$Q$	instantaneous discharge
$q$	consumption discharge
$t$	time
$x$	distance along pipe   a variable
$\alpha$	cut level
$\mu$	membership function
$\Delta a$	uncertainty in wave speed
$\Delta f$	uncertainty in friction factor
$\Delta q$	uncertainty in consumption discharge

## INTRODUCTION

In designing a new pipe network or assessing an existing one, transient analysis is often required. In this regard, the most important criterion needs to be fulfilled is relation to the pressure variations in the system. The highest and lowest pressures are checked at critical locations to be within permissible limits. For a safe design, recognizing the most severe transient flow in the system is a major concern for engineers. A transient condition is initiated from a disturbance in the governing steady state flow. In water supply networks, sudden changes in water demands, operation of valves and pumps are the most common events that generate a transient flow. Many references and manuals can be named which represent practical recommendations to identify the worst transient conditions in pressurized pipelines (Chaudhry 1987; Wylie & Streeter 1993; Thorley 2004). However, a pipe network consists of several links, consumption nodes, pumps, and valves etc., that hydraulically interact with each other. Furthermore, a transient flow usually initiates from a combination of the aforementioned

excitations. These issues make it hard to identify the worst transient state in the simulations. In general, engineering judgments and experiences are very helpful in solving this problem. Recently, with the aid of optimization methods, some good systematic approaches have been guided to simulate serious transients in piping systems. In this context, Jung & Karney (2009) assumed that severe transients in pipe networks are generated by the fluctuating nature of nodal demands. They combined a genetic algorithm (and also particle swarm optimization) with transient analysis to identify the worst-combination of loads resulting in the most severe transient in the system. In that research, the loading scenarios are decision variables while the highest and lowest pressure heads are objective functions. Then, the optimization method was applied to optimize the transient protection strategy and facilities. With a similar logic, Haghighi & Shamloo (2011) applied a simple genetic algorithm to a transient analysis model to find the most effective transient for leak detection and calibration of piping systems. In that work, the valve maneuver considered as the transient excitation in the network was optimized for a severe enough transient for the cited purpose.

Besides the above issues, this work aims to open a new discussion that can also be important to responses of a transient analysis. It is related to the problem uncertainties that frequently appear in simulating all physical phenomena. Through hydraulic modeling of a piping system, it is easy to identify some imprecise parameters for which there is not enough information with a sensible degree of reliability. For instance, the pipe friction factor is inherently imprecise since it is a function of pipe roughness and flow specifications both of which change over time and make all estimations more uncertain. Another example is the nodal demands. It is quite impossible to determine specific design water demands since they are continuously fluctuating through users' consumption habits. The pipe diameter and wall thickness may also be uncertain since they are constantly exposed to erosion and sedimentation. These cases introduce uncertainties to the results of both steady and unsteady flows. However, one of the most uncertain characteristics in transients is the pressure wave speed of pipes. This crucial parameter is a function of other uncertainties like pipe diameter, wall thickness, module of elasticity, fluid specifications and support constraints.

Obviously, introducing such uncertain parameters to a transient analysis solver leads to uncertain responses like the pressure head variations. Without the uncertain transient analysis, a designer cannot be sure of the critical answers even if the most severe transient scenario is simulated. Therefore, a systematic way is required to consider uncertain input values in the model to see how the results (as output values) are influenced by them. For this purpose, the current paper intends to introduce a quantitative notional model for uncertain transient analysis in pipe networks with the aid of fuzzy sets theory and optimization.

The fuzzy logic was developed to tackle the fuzzy (non-crisp) values in simulations and is stated against the binary logic that can only deal with crisp sets. This theory was originally introduced by Zadeh (1965). Since then, it was gradually extended by him and his colleagues until its foundation was formally established. For instance, other influential researchers in this theory are Mamdani & Assilian (1975), Sugeno (1985), Zimmermann (1985), Buckley (1987), Pedrycz (1989), Kandel (1992) Bit *et al.* (1992), and Bardossy & Duckstein (1995).

The notion behind the fuzzy sets theory makes it very useful in interpreting ambiguities in a wide range of engineering problems. Up to now, much research in water engineering has exploited the fuzzy sets in various fields like hydrology (Pesti *et al.* 1996; Pongracza *et al.* 1999; Ozelkan & Duckstein 2001; Bardossy *et al.* 2002; Srinivas *et al.* 2008; Srivastava *et al.* 2010), water quality (Sasikumar & Mujumdar 1998; Nasiri *et al.* 2007; Ghosh & Mujumdar 2010), ground water (Guan & Aral 2004; Dixon 2005; Kurtulus & Razack 2010), urban flood management (Chang *et al.* 2008; Fu *et al.* 2011), reservoir operations (Saad *et al.* 1996; Cheng & Chau 2001; Karaboga *et al.* 2004), river engineering (Mujumdar & Subbarao 2004; Ozger 2009; Kişi 2010) and pipe hydraulics (Revelli & Ridolfi 2002; Bhawe & Gupta 2004).

Among the cited works, an outstanding investigation was guided by Revelli & Ridolfi (2002) especially for pipe networks. In that work, the imprecise parameters in piping systems were introduced and a fuzzy-based approach was developed for analyzing the steady state flow. This work was the most motivating cause in developing the current research.

The present paper intends to utilize the fuzzy theory in the transient analysis of pipe networks to see how the

input uncertainties spread on the responses. On this basis, a conceptual tool is suited to the problem as a combination of a transient hydraulic model, fuzzy sets theory and an optimization solver. For the crisp (precise) input values, the governing equations of transient flow are solved using the method of characteristics (MOC). However, the fuzzy input data, defined as uncertainty intervals, results in a system of fuzzy equations. In this situation, to find the critical unknowns of interest, e.g. the highest and lowest nodal pressures, a nonlinear programming (NLP) problem appears. To solve that, a well-known meta-heuristic, the method of simulated annealing (SA), is then applied. A real case study of the Baghmalek pipe network is also taken into account to be analyzed by the proposed approach through which the method is discussed further.

## TRANSIENT HYDRAULICS

For most piping systems the maximum and minimum operating pressures occur during transient flows. In pressurized pipes, a transient state results from the generation and propagation of pressure waves initiated from a disturbance in the system such as valves and pump operations or consumption fluctuations. Apart from how and from where a transient condition is initiated, the network layout and the pipes and junction specifications play a remarkable role in the severity of the flow variations produced. When an excitation occurs, a pressure wave starts to travel along the pipe from the excitation location at the speed of the sonic velocity. While the wave is affected by the pipe features, e.g. minor and friction losses, it reaches the other pipe's end where a reservoir, a pump or a junction may exist. The wave is highly influenced by these components, particularly the role of junctions in pipe networks. When the wave encounters a junction, it is transmitted into the other connecting pipes and reflected back into the original pipe, producing a new set of pressure waves. This issue becomes more significant when the junction involves fluctuating consumption. Consequently, the wave propagation and reflection in the original pipe are under the influence of other pipes as well as the consumption discharge at the junction. After this, other connecting pipes also experience transient condition and new pressure waves. The same

story holds for these pipes and their relevant junctions until the whole network is covered by the transient flow variations. Afterwards, the most critical combinations of wave interactions determine the extreme transient responses at each location. By solving the governing equations against a certain transient excitation, one can numerically estimate the response-time histories. However, when the basic input data include uncertainties and are defined as intervals, finding the most critical combination of wave interactions is not that easy. Handling the uncertainties in the transient models and looking for the specific wave interactions in a pipe network is beyond the hydraulic simulations alone. To this end, other techniques should be called to help; those are fuzzy sets theory and optimization in this work.

## Governing equations

Two conservative rules of mass continuity and momentum govern transient flows in pressurized pipes. The major uncertainties in the transient analysis are considered to be the friction factor ( $f \pm \Delta f$ ), wave speed ( $a \pm \Delta a$ ) in pipes and consumption discharge ( $q \pm \Delta q$ ) at junctions. With this assumption,  $f$ ,  $a$  and  $q$  are the crisp values of the parameters while  $\Delta f$ ,  $\Delta a$  and  $\Delta q$  indicate the uncertainties which may change over the system lifetime.

By rewriting the basic compatibility equations (from Wylie & Streeter 1993) with respect to the aforementioned uncertain variables, the following two fuzzy partial differential equations are yielded:

$$\frac{\partial \tilde{H}}{\partial t} + \frac{\tilde{a}^2}{gA} \frac{\partial \tilde{Q}}{\partial x} = 0 \quad (1)$$

$$\frac{\partial \tilde{Q}}{\partial t} + gA \frac{\partial \tilde{H}}{\partial x} + \frac{\tilde{f} \tilde{Q} |\tilde{Q}|}{2DA} = 0 \quad (2)$$

in which the crisp quantities are:  $x$  = distance along pipe;  $t$  = time;  $g$  = gravitational acceleration;  $A$  = cross-sectional area of pipe;  $D$  = pipe diameter, and the fuzzy variables indicated by the tilde superscription are:  $\tilde{a}$  = wave speed;  $\tilde{Q}$  = instantaneous discharge;  $\tilde{H}$  = instantaneous piezometric head; and  $\tilde{f}$  = Darcy-Weisbach friction factor. It must be also mentioned that the other uncertainties in the analysis

are the nodal demands which are independent variables similar to the pipe wave speed and friction factor. The pipe discharge and the nodal heads are dependent fuzzy variables. Furthermore, in some applications one may assume that other parameters like the pipe length and diameter are fuzzy too.

### Method of solution

Equations (1) and (2) in conjunction with the appropriate initial and boundary conditions form the mathematical model. The steady flow provides the initial conditions to the model. The initial steady flow specifications, nodal pressures and pipe velocities, are calculated from solving the system of equations of energy and continuity developed in the pipe network. For this purpose, EPANET is linked with the main unsteady model. In this study, a standard transient analysis model for looped pipe systems is exploited which has been developed using the method of characteristics. It accepts only crisp values for input variables ( $f$ ,  $a$  and  $q$ ). This model has been already used by [Shamloo & Haghighi \(2010\)](#) to solve the inverse transient analysis problem in pipe networks. The model is now intended to be extended for including the system uncertainties as it is described in the next section.

## UNCERTAINTY IN TRANSIENT ANALYSIS

In a transient analysis, it is clear that uncertainty in the input variables like  $f$ ,  $a$  and  $q$  results in uncertainty in the responses. This section intends to answer the question of how and to what extent the input uncertainties are spread over the results. The key tool in handling the imprecise data in this work is the fuzzy sets theory which is briefly described as follows. Afterwards, the proposed analysis method is introduced.

### Fuzzy sets theory

The basic concept of fuzzy theory is inferred from the fuzzy sets which may also be referred to as fuzzy numbers. A fuzzy number  $N$  is a set defined on a universe of real numbers  $N \in R$ . For each variable  $x \in N$ ,  $\mu_N(x) \in [0, 1]$  is called the

grade of membership of  $x$  in  $N$ . If  $\mu_N(x) = 0$ , then  $x$  is called 'not included' in the fuzzy set and if  $\mu_N(x) = 1$ , then  $x$  is called fully included, and if  $0 < \mu_N(x) < 1$ ,  $x$  is called fuzzy member. A so-called  $\alpha$ -cut operation,  $\alpha \in \mu_N$ , denoted by  $N_\alpha$  is so applied to fuzzy numbers that each  $N_\alpha$  is a crisp interval defined as  $[x_{a,\alpha}, x_{b,\alpha}]$  as shown in [Figure 1](#). When  $\alpha = 0$ , the corresponding interval is called the 'support' of the fuzzy number which is indicated by the interval  $[x_a, x_b]$ . For  $\alpha = 1$ , and when the membership function is triangular (for example), the interval reduces to one crisp value only,  $x_c$ , that is, the 'most likely' value of  $N$ . This definition allows for identifying any crisp interval existing within the fuzzy set as a specific  $\alpha$ -cut if the membership function  $\mu_N$  is continuous and the fuzzy set is normalized and convex. The normalization condition implies that the maximum membership value is 1:

$$\exists x \in R, \quad \mu_N(x) = 1 \quad (3)$$

The convexity condition indicates that two arbitrary  $\alpha$ - and  $\alpha'$ -cut intervals satisfy the flowing relation ( $N_\alpha = [x_{a,\alpha}, x_{b,\alpha}]$ ):

$$\alpha' < \alpha \Rightarrow x_{a,\alpha'} < x_{a,\alpha}, \quad x_{b,\alpha'} > x_{b,\alpha} \quad (4)$$

which means that  $N_\alpha$  is a subset of  $N_{\alpha'}$ .

The real advantage of the fuzzy approach is to translate the qualitative available information into the mathematical language. As a result, it provides a tool to quantitatively process the imprecise data given as qualitative information.

According to the conditions described for the membership functions, two types of function are often utilized:

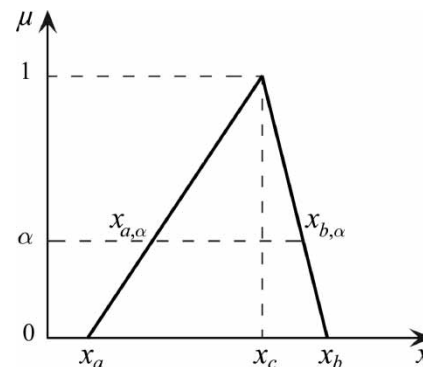


Figure 1 | Triangular membership function.

triangular and trapezoidal. In this context, the conceptual terms ‘belong’ to an interval or ‘close’ to a quantity are considered based on the knowledge that one has about the data. For instance, the imprecise input data like ‘the variable belongs to  $[x_a, x_b]$  and is close to  $x_c$ ’ can now be translated to a triangular membership function for that variable as seen in Figure 1. The uncertainties of a phenomenon explained as fuzzy numbers are then applied to the analysis model. Accordingly, the model provides the unknowns of the problem as fuzzy numbers. The membership functions corresponding to the unknowns help designers make qualitative decisions on the system responses.

### Fuzzy analysis approach

While the solution of the standard transient models, which work with crisp quantities, is time histories of pressure and discharge, the solution of the fuzzy-transient equations is the corresponding fuzzy time histories. Instead of defining a certain value for each of  $f$ ,  $a$  and  $q$ , these parameters are considered to be fuzzy in this study with a triangular membership function as seen in Figure 2. Herein, a transient analysis model based on the MOC is available which is intended to be utilized for the fuzzy transient analysis. The proposed approach for this purpose is described in the following paragraphs.

Using a limited number of  $\alpha$ -cuts, the continuous membership functions of uncertainties are so discretized that for each  $\alpha$ -cut, an interval is obtained. At each cut, the problem is treated as a normal transient analysis such that the input variables  $f$ ,  $a$  and  $q$  can adopt any value from their relevant intervals. Obviously, each combination of  $f$  and  $a$  for pipes and  $q$  for nodes results in a specific response. In this

study, the responses of interest are the maximum and minimum pressure heads at each node  $i$ , denoted by  $P_{\max,i}$  and  $P_{\min,i}$ , respectively. At each  $\alpha$ -cut, all the model input parameters except the fuzzy ones are considered to be crisp. The responses are then:

$$C_{1,i} = P_{\min,i}(\tilde{a}_\alpha, \tilde{f}_\alpha, \tilde{q}_\alpha) \tag{5}$$

$$C_{2,i} = P_{\max,i}(\tilde{a}_\alpha, \tilde{f}_\alpha, \tilde{q}_\alpha) \tag{6}$$

where  $C_{1,i}$  and  $C_{2,i}$  are the objective functions related to node  $i$ . In order to find the corresponding intervals for  $P_{\max,i}$  and  $P_{\min,i}$  at a certain  $\alpha$ -cut, the pressures indicated by Equations (5) and (6) are treated as optimization objective functions in which the decision variables are the fuzzy  $f$ ,  $a$  and  $q$ . Accordingly, to find the bound of variation of  $P_{\max,i}$  and  $P_{\min,i}$  due to the uncertainties, two optimization problems must be solved for each of (5) and (6) at each  $\alpha$ -cut. These problems consist of maximizing and minimizing both  $P_{\max,i}$  and  $P_{\min,i}$  with respect to the fuzzy variable intervals. As a result, to find the aforementioned unknown intervals for each node  $i$  in the network, the optimization solver, SA in this case, is called four times.

Through each optimization, the SA searches in the fuzzy intervals as the decision space to find the critical combinations of  $f$ ,  $a$  and  $q$  that makes the objective functions (5) and (6) maximum or minimum. For this purpose, the SA is run with an initial feasible solution, i.e. a combination of crisp values of fuzzy variables from their  $\alpha$ -cut intervals. With the selected values, the standard transient analysis model can be easily used to calculate the flow specifications and return the value of objective function. The SA

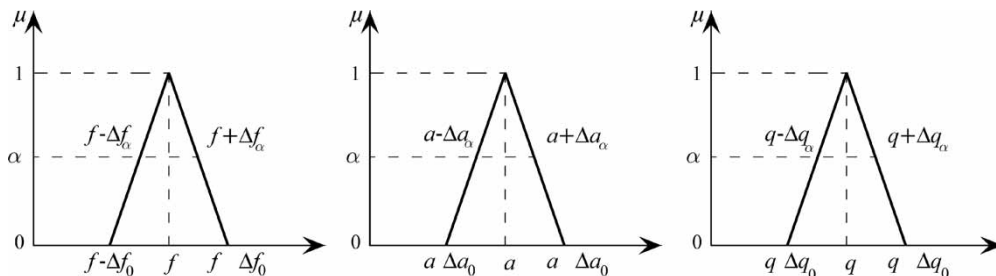


Figure 2 | Membership function of transient fuzzy variables.

procedure is then continued until the optimum set of the decision variables with respect to the desired objective function, is achieved.

When all the optimizations (four separate ones) were performed for a node, two sets of decision variables are yielded for each objective function. One set corresponds to the upper and the other corresponds to the lower bound of the unknown interval. In fact, the optimum values of (5) and (6) against their optimum pair sets of decision variables indicate their bound of variations at the node and the  $\alpha$ -cut at hand.

This algorithm is applied to the other locations of interest (other junctions) and is then repeated for the next cuts. As such, corresponding to the fuzzy input variables shown in Figure 2 there will be fuzzy results with specific membership functions which show how the uncertainties reflect on the responses. For more clarification, Figure 3 schematically describes the proposed algorithm.

### Optimization technique

In the proposed approach, a mathematical programming problem is formed. The objective functions (5) and (6) are both maximized and minimized for each junction. The decision variables are the model's uncertain input data including the pipe friction factor, wave speed and nodal consumptions (Figure 2), whereas the bound of each variable is its  $\alpha$ -cut interval. To solve the optimization problems herein, the method of simulated annealing (SA) is utilized.

Simulated annealing was originally introduced by Kirkpatrick *et al.* (1983) as a search algorithm capable of escaping from local solutions. Its ease of implementation and convergence properties and its use of hill-climbing moves to escape local optima have made it a popular technique over the past 2 decades (Gendreau & Potvin 2010).

Simply speaking, the meaning of annealing in industry is the cooling of a metal so slowly that it gets close to a crystalline state. In high temperatures, the particles in the metal can freely move around and change the structure of the metal and its behaviour. Through the annealing process, the temperature is lowered very

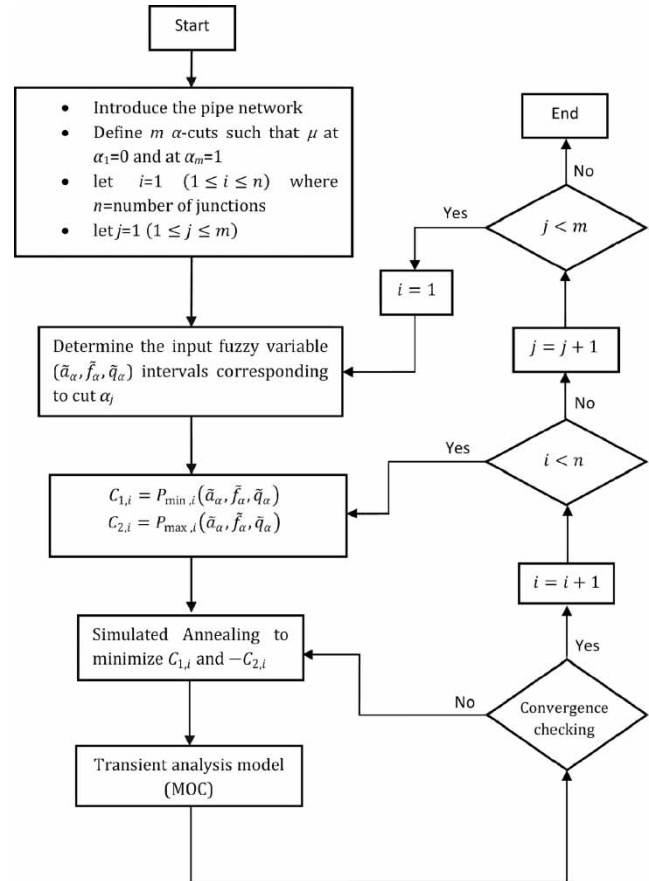


Figure 3 | Fuzzy transient analysis algorithm.

gradually in order to limit the domain of particles movements until the crystalline position is achieved. This procedure is what inspired simulated annealing in solving an optimization problem. With an initial solution, the method starts to search in the decision space within which the decision variables can freely move around. In each iteration, the current solution is disturbed in some manner in its neighbourhood to create a new solution. The energy of the state of the two solutions at hand is evaluated using the objective function based on which the new state is then selected. If the new solution has a better fitness, the procedure is moved there otherwise it may be moved with a probability depending on the current temperature and the difference in fitness. In fact, a solution has the chance of being accepted if it is slightly worse than the previous state or if the temperature is very high. This helps the SA not to be trapped in local

optima. After a large enough number of iterations, one finds that the solution state does not change and the temperature becomes zero. This condition indicates that the problem is solved. A simple program following the basic ingredients of the SA from Gendreau & Potvin (2010) was developed herein to solve the problems.

## CASE STUDY

To investigate the effects of uncertainties on the results of a transient analysis and how the proposed model works, the following example was used. The case study is based on Baghmalek water distribution network, a city in southwest of Iran (Figure 4). The network consists of 37 steel pipes, one of which (numbered 37) transmits water to a new area

which is constructed for a specific purpose in the northeast of the system. The network also has 28 nodes while a reservoir at node 1 gravitationally feeds the system. There is a valve immediately at the upstream of node 2 on pipe 37 that regulates the flow discharged to the new area.

Consider that with the exception of pipe 37, the rest of the system is about 15 years old and the operators are mostly aware of the network components and of the way to operate them safely. However, when pipe 37 was later constructed to serve the new area with a  $76.7 \text{ l s}^{-1}$  flow rate, the client raised a concern. The operator was curious to know what happens to the system when the regulating valve located at the downstream end of pipe 37 is maneuvered. For this purpose, a scenario was defined so that the valve is rapidly closed in 5 s while all the demand nodes are experiencing their maximum consumptions. The most likely (crisp) values of the

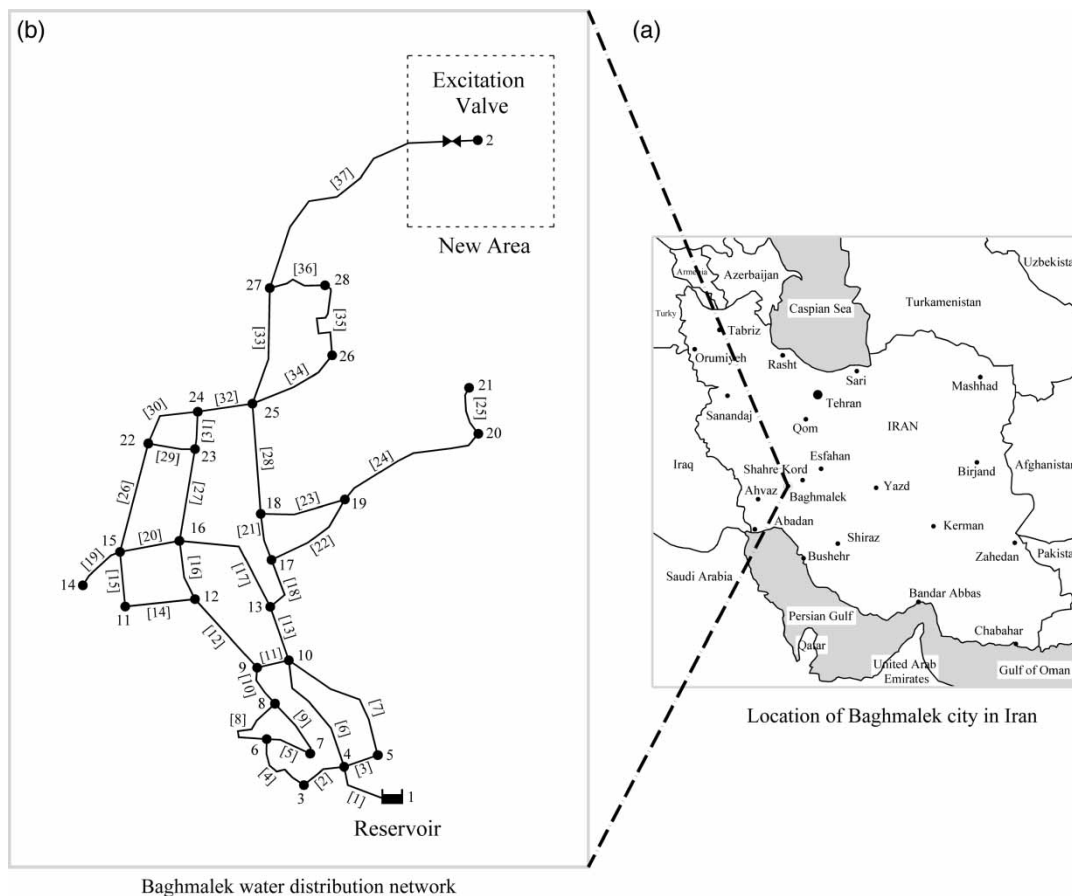


Figure 4 | Baghmalek city and its water distribution network.

**Table 1** | Nodal specifications

Node	Elevation (m)	$q_c$ (l/s)	Node	Elevation (m)	$q_c$ (l/s)
1	750.00	0.00	15	700.79	5.56
2	690.58	76.76	16	706.02	6.77
3	730.42	3.73	17	708.30	2.66
4	736.59	0.99	18	705.95	1.32
5	730.39	8.15	19	694.51	10.65
6	728.61	12.00	20	699.75	15.00
7	727.32	15.00	21	699.93	5.50
8	721.85	4.89	22	694.39	17.40
9	720.55	10.10	23	696.77	15.00
10	721.60	4.43	24	694.24	4.65
11	705.12	15.00	25	697.58	9.84
12	711.62	8.00	26	687.61	4.40
13	713.41	5.59	27	673.41	5.21
14	699.42	1.22	28	674.86	14.30

consumptions  $q_c$  in this scenario, together with the nodal elevations have been reported in Table 1. Furthermore, Table 2 shows the most likely quantities of the friction

**Table 2** | Pipes specifications

Pipe	Length (m)	$a_c$ (m/s)	$f_c$	$D$ (mm)	Pipe	Length (m)	$a_c$ (m/s)	$f_c$	$D$ (mm)
1	84.30	1,100	0.024	500	20	171.00	1,100	0.030	150
2	176.00	1,100	0.031	150	21	258.00	1,100	0.025	400
3	166.00	1,100	0.029	400	22	365.64	1,100	0.029	150
4	268.20	1,100	0.031	100	23	443.00	1,100	0.033	150
5	180.10	1,100	0.034	100	24	615.00	1,100	0.030	150
6	430.00	1,100	0.025	500	25	182.00	1,100	0.034	200
7	463.00	1,100	0.029	300	26	611.00	1,100	0.033	200
8	346.23	1,100	0.033	100	27	440.00	1,100	0.032	100
9	254.01	1,100	0.032	100	28	430.23	1,100	0.025	300
10	169.53	1,100	0.030	150	29	178.84	1,100	0.037	200
11	93.00	1,100	0.028	200	30	270.00	1,100	0.033	150
12	275.00	1,100	0.031	100	31	173.00	1,100	0.032	100
13	258.00	1,100	0.025	500	32	170.00	1,100	0.030	150
14	168.00	1,100	0.032	300	33	358.00	1,100	0.026	300
15	175.00	1,100	0.034	200	34	267.00	1,100	0.032	100
16	260.00	1,100	0.034	200	35	354.95	1,100	0.035	100
17	523.00	1,100	0.028	200	36	260.00	1,100	0.032	100
18	258.00	1,100	0.024	350	37	1,230.00	1,100	0.022	350
19	180.00	1,100	0.043	100					

factor,  $f_c$ , and wave speed,  $a_c$ , of pipes estimated based on the design time information and the engineering judgments made by the operators. In the notation, index  $c$  indicates the crisp values defined for each parameter.

As can be seen, the quantities of the nodal demands, the wave speed and friction factor of pipes are not precise in this problem. In fact, they include uncertainty in their input data, thus the problem needs to be solved using the developed fuzzy approach. To this end, the aforementioned uncertainties are treated as the fuzzy values with a triangular membership function as shown in Figure 2. It is estimated that the most likely (crisp) consumptions  $q_c$  may have  $\pm 20\%$  uncertainty. In addition, for the wave speed,  $a_c$ , and the friction factor,  $f_c$  of the pipes,  $\pm 10\%$  and  $\pm 20\%$  uncertainties are considered, respectively. Referring to Figure 2, this means that for each consumption node  $\Delta q_0 = 0.2q_c$ , and for each pipe  $\Delta a_0 = 0.1a_c$  and  $\Delta f_0 = 0.2f_c$  at the support fuzzy numbers where  $\mu = 0$ .

Now, given the membership function of fuzzy quantities, we aim to identify the membership function of  $P_{\min}$  and  $P_{\max}$  at each node. The calculations are carried out for the



time duration of 100 s in which the transient flow covers the whole system and fluctuates for a few periods. Following the algorithm shown in Figure 3, the network specifications are introduced to the transient solver assuming that all the parameters are initially crisp. Then, three  $\alpha$ -cuts are taken into account including  $\alpha_1 = 0$  (the support),  $\alpha_2 = 0.5$  and  $\alpha_3 = 1$  (the crisp). Then, at each  $\alpha$ -cut and for each node, using a two-loop calculation the corresponding uncertainties for  $P_{\min}$  and  $P_{\max}$  are obtained.

Figures 5 and 6 show the fuzzy transient responses for  $P_{\min}$  and  $P_{\max}$ , respectively, against the fuzzy input variables and the excitation considered in this scenario. Figure 7 also shows the maximum, minimum and mean fuzzy pressures which occur at the network nodes at cut  $\alpha_1 = 0$ . In general, the nodes far away from the reservoir (node 1) and close to the excitation valve (node 2) are much more affected by the uncertainties. However, the nodes near to the reservoir are not that sensitive. According to the obtained membership functions for the responses, even a small variation in the input variables can result in a huge uncertainty in the

transient pressures. This issue is worse with the minimum pressures. For instance, the fuzzy analysis shows that for node 2 there is more than  $\pm 200\%$  uncertainty in  $P_{\min}$  with respect to the results of the crisp analysis while the maximum range of uncertainty in the inputs is only  $\pm 20\%$ . When the most likely values for the input variables are taken into account, the most likely minimum pressure at node 2 is obtained about 4.4 m which is a safe value from the cavitation or column separation view point. However, because of the input uncertainties, the minimum pressure at this node is evaluated to be uncertain belonging to a fuzzy interval from  $-7$  to 14 m. The lower bound in this interval,  $-7$  m, is a serious threat for the safety of any piping system. A reasonable decision for the designer is to pay attention to this value for designing the system protection facilities. The same comparisons and discussions can be made for the other nodes. This example clearly confirms that the fuzzy transient analysis is essential in many pipe networks especially for those with old components and imprecise information for modeling.

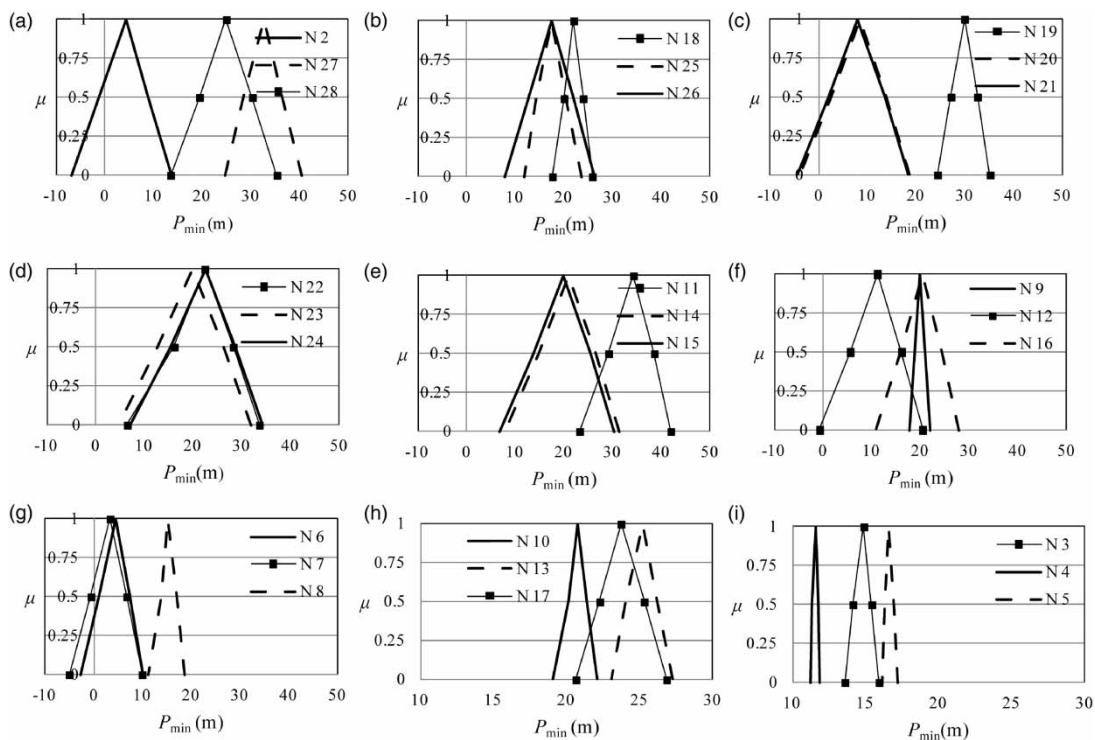


Figure 5 | Membership function of transient minimum nodal pressures for the case study.

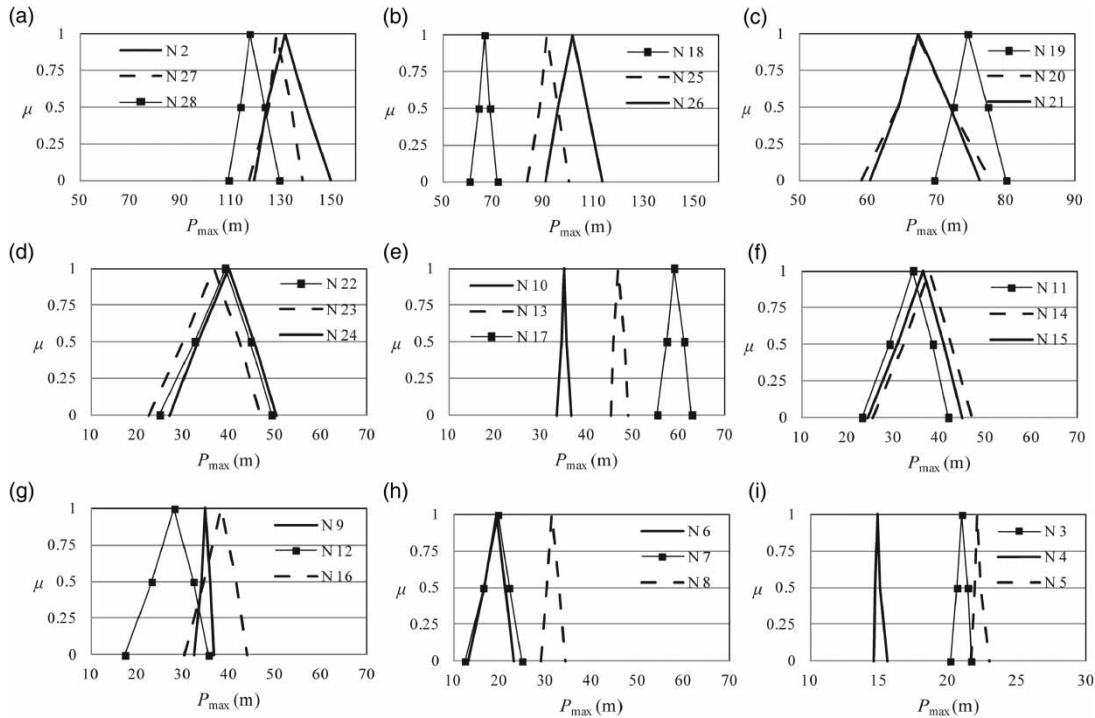


Figure 6 | Membership function of transient maximum nodal pressures for the case study.

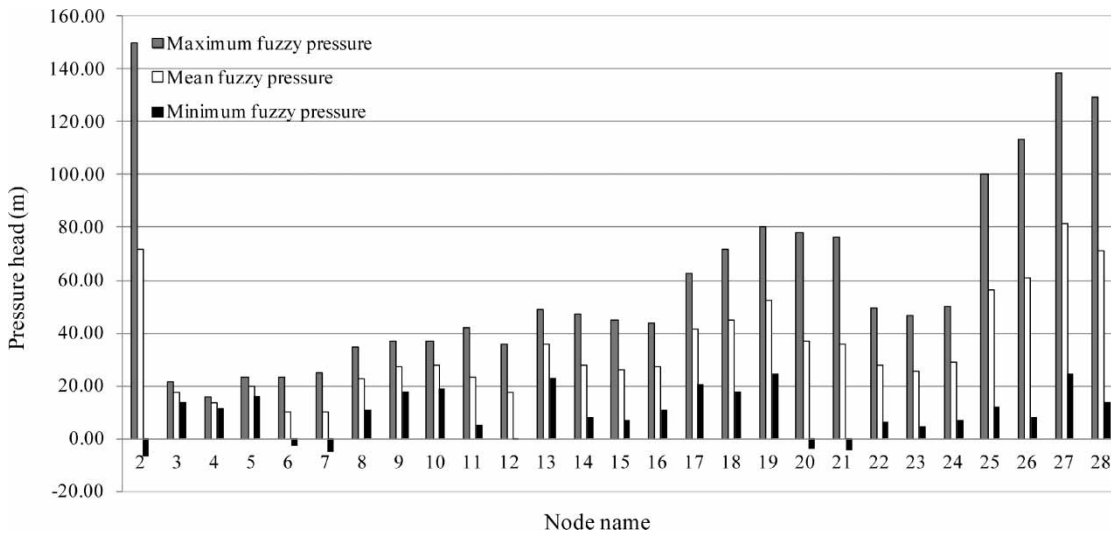


Figure 7 | Fuzzy maximum, minimum and mean pressures at the network's nodes.

## SUMMARY AND CONCLUSIONS

In the hydraulic design of pipe systems, there are some decisive criteria related to the results of transient analysis

that should be closely observed. Nevertheless, these results are extremely subject to change due to the system uncertainties. Actually, in a transient flow analysis, it is inevitable to face with imprecise input data including

friction coefficient and wave speed of pipes and discharge of consumptions. Thus empirically, an interval instead of a crisp quantity may be measured within which the input variables likely exist. There are some specific combinations of data quantities within those intervals which can result in devastating situations due to large positive or negative fluid pressures.

In this research, a conceptual model was introduced for uncertain transient analysis in piping systems with the aid of fuzzy sets theory and optimization. The uncertainties are recognized in the system and interpreted as fuzzy numbers with a specified membership function, being triangular in this work. A limited number of  $\alpha$ -cuts are considered to break the problem in a sequence of optimization sub-problems. For each cut the fuzzy input variables are introduced to the model as an interval. In this condition, the transient analysis includes an optimization problem in which the bounds of variations of nodal maximum and minimum pressures are objective functions. The simulated annealing method searches within the intervals corresponding to each cut and finds the optimum combination of variables with respect to each objective function. For this purpose, the optimization solver is joined with a standard transient analysis model developed based on the method of characteristics. This model returns the value of objective function against the crisp input variables fed by the simulated annealing. Through the algorithm depicted in Figure 3, the whole network is analyzed and the nodal membership functions for the critical pressure heads are obtained.

This approach was applied to a real pipe network as the case study. It was demonstrated that short data intervals (small uncertainties) can lead to long output intervals which can violate the allowable quantities given in design manuals. Accordingly, the system may appear safe if only crisp data quantities are applied but the safety fails considerably once the data uncertainties are taken into account. This research opens up a new and important consideration about pipe-systems design and introduces a tool to more accurately measure their safety factor. Here, only the three imprecise quantities are considered as the sources of uncertainty in the results. Nevertheless, some other effects of transient flows in pipe systems, e.g. unsteady friction factors, fluid-structure interaction, viscoelasticity, liquid column separation, dissolved gas and pipe-wall thickness were not considered

in this work. These effects, sometimes, can hardly be fully eliminated in reality and need to be quantitatively studied likewise to see how important each one is in a system.

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