problems with nonholonomic constraints it is very important to consider carefully the physical aspects of the problem. Otherwise, one may find oneself solving a mathematical problem that does not have a sound physical basis.

The new formulation of the Lagrange method that was recently presented by Rosen and Edelstein, seems to offer a unified variational approach to both holonomic and nonholonomic constraints. For a simple example that was present above the new formulation gave results that were identical to the results of a Newtonian approach, while the "regular" approach to nonholonomic constraints failed to do that. The authors obtained similar results in other examples (Rosen and Edelstein, 1997b). Yet, the authors by no means feel that the difficulty of solving nonholonomic problems is over. The new formulation seems to be promising, but many more careful examinations of it are necessary in order to gain more confidence in its capabilities, further improve its performance, and knowing its limitations. The authors invite other researchers to test, extend, and improve the new formulation. For example, Eq. (18) can be extended to deal with Lagrangians that include derivatives higher than the second. If there exist problems that include high derivatives, the results of such an extension may be interesting.

Before closing, the authors would like to again thank Professor Hagedorn for his interesting and stimulating discussion.

References


Modal Decoupling of Systems Described by Three Linear Operators

W. Kliem*. The author describes an interesting modal approach for linearized forced dynamic systems not necessarily having symmetric system operators (e.g., matrices). In this way a series of well-known characteristics of conventional modal analysis for symmetric systems is preserved. A crucial assumption in the authors paper is that the system of eigenfunctions (eigenvectors) is complete. Nevertheless, a few changes in the modal matrices using generalized eigenvectors and more general normalization relations allow an extension of the theory to the case with an insufficient number of eigenvectors (see Kliem, 1980).

For the sake of simplicity we demonstrate this generalization by dealing with a two-dimensional matrix differential equation:

\[ M \ddot{u} + C \dot{u} + K u = F(t), \quad (M \text{ nonsingular}). \]  

(1)

We assume a simple eigenvalue \( \mu \) and an eigenvalue \( \lambda \) with algebraic multiplicity three and geometric multiplicity one. This means that there corresponds only one eigenvector \( x_0 \) to the multiple eigenvalue \( \lambda \). Such a case is, e.g., represented by the system matrices

\[ M = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad K = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}, \quad F(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}. \]

We introduce \( L(\lambda) = x_0^T M + \lambda C + K, L^{(1)}(\lambda) = 2x_0^T M + C, \) and \( L^{(2)}(\lambda) = M \) and define in the usual way a right eigenvector \( \lambda_0 \) and generalized eigenvectors \( \lambda_1, \lambda_2 \) by

\[ L(\lambda) x_0 = \lambda_0 x_0, \quad L^{(1)}(\lambda) x_0 + L(\lambda) x_1 = 0, \quad L^{(2)}(\lambda) x_0 + L^{(1)}(\lambda) x_1 + L(\lambda) x_2 = 0 \]

and similar left eigenvectors by

\[ y_0^T L(\lambda) = 0, \quad y_0^T L^{(1)}(\lambda) + y_1^T L(\lambda) = 0, \quad y_0^T L^{(2)}(\lambda) + y_1^T L^{(1)}(\lambda) + y_2^T L(\lambda) = 0. \]

For the eigenvalue \( \mu \) there exists one right and one left eigenvector

\[ L(\mu) u_1 = 0, \quad w_1^T L(\mu) = 0. \]

With these vectors we define

\[ v = x_0 + x_0 + x_0 T, \quad v = x_0, \quad w = y_0 - y_0 - y_0 T, \quad w = y_0. \]

and right and left modal matrices

\[ R, L = \begin{bmatrix} w_1^T & w_2^T & \ldots & w_n^T \end{bmatrix}. \]

\[ R \text{ contains four arbitrary constants and so does } L. \text{ The complete solution of the homogeneous part of (1) } (F(t) = 0) \text{ is given by } u = Re^{\Lambda t}, \text{ where } \Lambda = \text{diag} \{ \mu, \lambda, \lambda, \lambda \}, \text{ compare the author’s Eq. (7).} \]

To receive a solution of the inhomogeneous system (1), we normalize the two modal matrices by

\[ RL = 0 \quad \text{and} \quad RAL + RL = M^{-1}. \]

These Eqs. (2) substitute the authors orthogonality relations (6), (7), and (8). System (1) now has a solution

\[ u = RD, \quad D = [d_1(t) \ldots d_4(t)]^T, \]

with the restriction \( RD = RAD, \)

(3)

which is equivalent to the authors Eqs. (12) and (13).

The matrix \( D \) can be found by solving the decoupled first-order system

\[ \dot{D} - AD = LF(t), \]

which is analogous to the authors Eq. (14).

In the case of harmonic excitation \( F(t) = \Phi e^{i\omega t}, \) where \( \Phi \) is a constant column, the steady-state solution (3) becomes

\[ u = R \left( (i\omega I - \Lambda)^{-1} L + (i\omega I - \Lambda)^{-2} L \right) \Phi e^{i\omega t} \]

(4)

or explicitly

\[ u = \left\{ \begin{array}{l}
\frac{v_1 w_1^T}{i\omega - \mu} + \frac{v_1 w_1^T + v_1 (-w_1^T) + w_1^T}{i\omega - \lambda} \\
\frac{v_1 w_1^T}{i\omega - \lambda} + \frac{v_1 (-w_1^T) + w_1^T}{(i\omega - \lambda)^2} + \frac{v_1 w_1^T}{(i\omega - \lambda)^3} \end{array} \right\} \Phi e^{i\omega t}, \]

which resembles the well-known solution received by modal analysis for conservative systems. In the case of a complete set of eigenvectors, e.g., if all eigenvalues are distinct, we have \( L = I = 0 \) and (4) is equivalent to the result of Wahed and Bishop (1976).

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Another example with two double eigenvalues where each possesses only one eigenvector (modeling a double pendulum with a follower force and an external torque) can be found in Kliem (1980).

This generalization was carried out in the discrete system analysis of Wahed and Bishop (1976), but can also be formulated in the operator analysis of the author.

References
