Squeeze Parameters in Reheating

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The difference in the properties of particle production for parametric resonance in reheating (preheating) and an expanding universe was investigated by calculating squeeze parameter values. It is shown that the parametric resonance has the characteristic behavior of squeeze parameters.

§1. Introduction

Reheating after inflation is an interesting phenomenon, and many investigations have been conducted concerning it. In particular, Kofman, Linde and Starobinsky 1) showed that parametric resonance occurs in reheating and that many particles are produced in some modes. A question arises regarding the difference in particle production for parametric resonance and for an expanding universe. Using the theory of squeezed quantum states we investigate this problem.

Since the theory of squeezed states was first used by Grishchuk and Sidorov 2) to calculate the power spectrum of primordial gravitational waves, inflationary cosmology has been investigated in terms of quantum squeezing. 2) - 4) Albrecht, et al. 5) investigated the time evolution of states in the case of a cosmological perturbation, and found that each mode of the perturbed field evolves as a squeezed state during the inflationary period.

To this time, it has been necessary to solve differential equations to obtain the values of the squeeze parameters of states. That is to say, the evolution operator which governs the time development of the Heisenberg operators can be written as a product of squeeze and rotation operators. The Heisenberg equations are differential equations with squeeze parameters as the variables. These differential equations are difficult to solve exactly, except for special cases. Therefore, numerical calculations and approximate methods have been used. In this paper a new method for calculating squeeze parameter values is developed. The interesting features of this method are that one does not need to solve such differential equations and that the squeeze parameter values can be calculated almost exactly. Furthermore, this method is applicable to many interesting cases, for example, inflation, radiation-dominated and matter-dominated epochs, and parametric resonance (preheating), and to any initial vacuum states.

In order to clarify the difference in the properties of particle production for parametric resonance in reheating (preheating) and an expanding universe, we investigate

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which squeeze quantum states appear when particle production occurs. Here we examine three cases. The first involves the model of parametric resonance used by Kofman et al.\textsuperscript{1}) The second case involves a model in which parametric resonance does not occur in reheating, and this serves to elucidate the contribution of the expanding universe. In the third case, as a typical example of an expanding universe, inflation is considered. The squeeze parameter values in the case of the expanding universe can be analytically calculated using the method developed here, but in the case of reheating, they cannot be analytically calculated. For this reason they have been numerically calculated. The results demonstrate the difference in squeeze states for the case of an expanding universe and the case of reheating.

In §2, the squeeze quantum states are explained, and a new method for obtaining the squeeze parameter values is developed. In §3, squeeze parameter values are calculated in three cases, parametric resonance (preheating), reheating (where parametric resonance does not occur) and inflation. In §4, we discuss the results.

§2. Squeeze parameters

Two methods for obtaining squeeze parameter values are described here. One is the ordinary method of solving the differential equations for squeeze parameters, and the other is the method presented here, which does not involve solving the differential equations for squeeze parameters. If the equation of motion for a scalar field can be solved exactly, the squeeze parameter values can be calculated almost exactly.

Here, a real scalar field is considered. Its Lagrangian density $\mathcal{L}$ is given by

$$\mathcal{L} = -\frac{1}{2} \sqrt{-g} \{ g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + V\phi^2 \}. \tag{1}$$

As the background metric, the Friedmann-Robertson-Walker universe, which is spatially flat, and described by the metric

$$ds^2 = -dt^2 + a^2(t) \{ dx^2 + dy^2 + dz^2 \}, \tag{2}$$

is considered.

First, a Hamiltonian is constructed in which a canonical variable $\chi \equiv a^{3/2} \phi$ is adopted. The canonical Hamiltonian is written as

$$H = \int \left\{ \frac{1}{2} (\pi)^2 + \frac{3\dot{a}}{2a} \chi \pi + \frac{1}{2a^2} (\partial_i \chi)^2 + \frac{V}{2} \chi^2 \right\} d^3x, \tag{3}$$

where overdots represent derivatives with respect to $t$. The Heisenberg equations of motion are given as

$$\frac{d\chi}{dt} = \pi + \frac{3\dot{a}}{2a} \chi, \tag{4}$$

$$\frac{d\pi}{dt} = -\frac{3\dot{a}}{2a} \pi + \frac{\partial^2 \chi}{a^2} - V\chi, \tag{5}$$

where the canonical commutations $[\chi(t, x), \pi(t, y)] = i\delta(x - y),...$ are used.
We now consider the time evolution of the operators. The unitary operator $U(t)$ is the time translation operator which connects the Heisenberg and Schrödinger pictures. The relations between operators at $t$ and at $t_1$ can be written as

\[ \chi(t, x) = U(t)^\dagger \chi(t_1, x) U(t), \quad \pi(t, x) = U(t)^\dagger \pi(t_1, x) U(t). \]

The operator $U(t)$ can be written as a product of the squeeze and rotation operators:5,6)

\[ U(t) = S(r, \phi) R(\theta), \]

where

\[ S(r, \phi) = \exp \left\{ \frac{r}{2} (e^{-2i\phi} a_k a_{-k} - e^{2i\phi} a_k^\dagger a_{-k}^\dagger) \right\}, \]

\[ R(\theta) = \exp \left\{ -\frac{i\theta}{2} (a_k^\dagger a_k + a_{-k}^\dagger a_{-k}) \right\}. \]

To study the time evolution of the operators, one has to define the initial conditions of the quantum field theory. At $t = t_1$, the operators $\pi$ and $\chi$ are expressed by the creation and annihilation operators $a_k^\dagger$ and $a_k$ as:

\[ \chi(t_1, x) = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ u_k(t_1) a_k + a_k^\dagger(t_1) \}, \]

\[ \pi(t_1, x) = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ w_k(t_1) a_k + a_k^\dagger(t_1) \}, \]

where $w_k(t) = \dot{u}_k(t) - \frac{3a}{2} u_k(t)$. From Eqs. (4) and (5) a differential equation is derived as

\[ \ddot{f}_k + \left( k^2 - \frac{3\dot{a} - 3a^2}{2a} + V \right) f_k = 0, \]

where $f_k$ satisfies the normalization condition $f_k \dot{f}_k^* - f_k^* \dot{f}_k = i$. A general, normalized solution to Eq. (13) is

\[ u_k(t) = C_1 f_k(t) + C_2 f_k^*(t). \]

If an initial state is fixed such as $\chi(t_1, x)$ and $\pi(t_1, x)$, from Eqs. (6) and (7), $\chi(t, x)$ and $\pi(t, x)$ are given as

\[ \chi(t, x) = U(t)^\dagger \chi(t_1, x) U(t) = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ u_k(t) a_k + a_k^\dagger(t) \} \]

\[ = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ (e^{-i\theta} \cosh r u_k(t_1) - e^{-i(\theta + 2\phi)} \sinh r u_k(t_1)) a_k \}
+ \{ e^{i\theta} \cosh r u_k^* (t_1) \}, \]

\[ \pi(t, x) = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ w_k(t) a_k + a_k^\dagger(t) \} \]

\[ = \frac{1}{(2\pi)^{3/2}} \int d^3 k e^{ikx} \{ (e^{-i\theta} \cosh r w_k(t_1) - e^{-i(\theta + 2\phi)} \sinh r w_k(t_1)) a_k \}
+ \{ e^{i\theta} \cosh r w_k^* (t_1) \}. \]
In terms of the squeeze parameters \( r(t) \), \( \phi(t) \) and \( \theta(t) \), Eqs. (4) and (5) can be written as

\[
\frac{dr}{dt} = |\sigma| \sin(2\phi + \alpha),
\]
(17)

\[
\frac{d\phi}{dt} = -\omega + |\sigma| \cos(2\phi + \alpha) \coth 2r,
\]
(18)

\[
\frac{d\theta}{dt} = \omega - |\sigma| \cos(2\phi + \alpha) \tanh r,
\]
(19)

where

\[
\sigma(t) = w_k^2(t_1) + \frac{3\dot{a}(t)}{a(t)} w_k(t_1) u_k(t_1) + \left( \frac{k^2}{a^2(t)} + V \right) u_k^2(t_1)
\]
\[
= |\sigma| e^{i\alpha},
\]
(20)

\[
\omega(t) = |w_k(t_1)|^2 + \frac{3\dot{a}(t)}{2a(t)} (w_k(t_1) u_k^*(t_1) + w_k^*(t_1) u_k(t_1))
\]
\[
+ \left( \frac{k^2}{a^2(t)} + V \right) |u_k(t_1)|^2.
\]
(21)

These equations are in the same form of an equation derived by Grishchuk, \(^3,5\) but the coefficients are different. For different initial vacuum states are adopted, the differential equations (17)–(19) differ, and then the solutions to the equations may also differ.

Here, another method for obtaining squeeze parameter values, which does not involve solving the differential equations (17)–(19), is proposed. First, \( A_k(t) \) and \( B_k(t) \) are defined as

\[
A_k(t) = e^{-i\theta} \cosh r,
\]
(22)

\[
B_k(t) = e^{-i(\theta + 2\phi)} \sinh r.
\]
(23)

From Eqs. (15) and (16), \( u_k(t) \) and \( w_k(t) \) can be written as

\[
u_k(t) = A_k(t) u_k(t_1) - B_k(t) u_k^*(t_1),
\]
(24)

\[
w_k(t) = A_k(t) w_k(t_1) - B_k(t) w_k^*(t_1),
\]
(25)

and from Eqs. (4) and (5), relations are given as

\[
\frac{du}{dt} = A_k \left\{ w_k(t_1) + \frac{3\dot{a}}{2a} u_k(t_1) \right\} - B_k \left\{ w_k^*(t_1) + \frac{3\dot{a}}{2a} u_k^*(t_1) \right\},
\]
(26)

\[
\frac{dw}{dt} = -A_k \left\{ \left( \frac{k^2}{a^2} + V \right) u_k(t_1) + \frac{3\dot{a}}{2a} w_k(t_1) \right\} + B_k \left\{ \left( \frac{k^2}{a^2} + V \right) u_k^*(t_1) + \frac{3\dot{a}}{2a} w_k^*(t_1) \right\}.
\]
(27)

On the other hand, as \( u_k(t) (= C_1 f_k(t) + C_2 f_k^*(t)) \) is the solution of Eq. (13), when one fixes the coefficients \( C_1 \) and \( C_2 \) at the initial time \( t_1 \), the coefficients \( C_1 \) and \( C_2 \)
do not change after that. If one can solve Eq. (13) exactly, the mode functions \( u_k(t), \dot{u}_k(t), w_k(t) \) and \( \dot{w}_k(t) \) can be calculated exactly. From Eqs. (26) and (27), \( A_k \) and \( B_k \) are given as

\[
A_k(t) = \left\{ u_k^*(t_1)\dot{u}_k(t) - \left( u_k^*(t_1) + \frac{3\dot{a}(t)}{2a(t)}u_k^*(t_1) \right) u_k(t) \right\} \frac{1}{d},
\]

\[
B_k(t) = \left\{ u_k(t_1)\dot{u}_k(t) - \left( u_k(t_1) + \frac{3\dot{a}(t)}{2a(t)}u_k(t_1) \right) u_k(t) \right\} \frac{1}{d},
\]

where \( d = u_k^*(t_1)w_k(t_1) - u_k(t_1)w_k^*(t_1) \). The squeeze parameters \( r, \theta \) and \( \phi \) can be written in terms of \( A_k \) and \( B_k \) as

\[
r = \text{arccosh} \ |A_k| = \text{arcsinh} \ |B_k|,
\]

\[
e^{-i\theta} = \frac{A_k}{|A_k|},
\]

\[
e^{-2i\phi} = \frac{B_k A_k^*}{\sinh r \cosh r}.
\]

Finally, we note that the mean number of produced quanta, \( n_k \), can be written as

\[
n_k = \sinh^2 r.
\]

### §3. Squeeze parameters

In order to clarify the difference in the properties of particle production for parametric resonance in reheating (preheating) and an expanding universe, we investigated what squeeze quantum states appear when particle production occurs. Here, we consider three cases: the case of parametric resonance (preheating), the case in which parametric resonance does not occur in reheating, and the case of inflation.

#### 3.1. The case of parametric resonance (preheating)

Here, we obtain the squeeze parameter values in the case of parametric resonance in reheating (preheating) after inflation. The scalar field for parametric resonance from Kofman et al.\(^1\) obeys the equation

\[
\ddot{f}_k + \left( \frac{k^2}{a^2} + g^2\Phi^2 \sin^2 mt + m^2 - \frac{3\dot{a}}{a} - \frac{3\dot{a}^2}{4a^2} - \xi R \right) f_k = 0.
\]

In our case, the potential term for the scalar field is fixed as \( V = g^2\Phi^2 \sin^2 mt + m^2 - \xi R \) and \( \Phi^2 = M_P^2/(3\pi m^2 t^2) \). The scale factor in reheating is \( a(t) = (t/t_1)^{2/3} \), where \( t_1 = \pi/(2m) \). We consider the case in which \( \xi = 0 \), and the time variable \( t \) changes with \( x \) according to \( t = 2\pi x/m \). Then Eq.(34) can be written as

\[
\frac{d^2 f_k(x)}{dx^2} + \left( 4\pi^2 k^2 m^2 \left( \frac{x_1}{x} \right)^{4/3} + \frac{g^2 M_P^2}{3\pi m^2 x^2} \sin^2 2\pi x + \frac{4\pi^2 m^2}{m^2} \right) f_k(x) = 0,
\]
where $M_P$ is the Planck mass. From Eq. (35), one may note that when $g^2M_P^2/m^2$ is larger than $4\pi^2k^2/m^2$, i.e., when $k$ is much smaller than $m$, the solution of Eq. (35) depends only very weakly on $k$. Thus, the squeeze parameter values change very little as functions of $k$ in the case where $0 \leq k < m$. Such a property can be confirmed through a numerical estimate. As an initial condition ($x = x_1 = 1/4$), the positive-frequency solution $f_k(1/4) = e^{-i\pi\omega/2m}\sqrt{2\omega}, \frac{df_k(1/4)}{dt} = -i\sqrt{2m}e^{-i\pi\omega/2m}$ is taken, where

$$\omega = k\left(1 + \frac{4g^2M_P^2}{3\pi^4k^2} + \frac{m^2}{k^2}\right)^{1/2}.$$ 

If solutions of Eq. (35) are calculated, we can derive $A_k$ and $B_k$ from the relations of (28) and (29), and using Eqs. (30) and (32), the squeeze parameter values are obtained. Unfortunately, Eq. (35) cannot be solved analytically, and therefore we solved it numerically. First, we display the evolution of the squeeze factor $r$ and the squeeze angle $2\phi$ in the case (a), where $m_X^2 = 0$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 0.1m$, in Fig. 1, and in the case (b), where $m_X^2 = 0$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 4m$, in Fig. 2. Even in the case where $k = 0$, the squeeze parameters $r$ and $\phi$ can be expressed in Fig. 1. Next, when $\chi$ has mass, the squeeze parameter values can be derived, as in the massless case. These are shown in Fig. 3 in the case that $m_X^2 = m^2$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 4m$. If the mass of $\chi$ $m_X$ is taken to be $10m$, it seems that parametric resonance is almost non-existent (see Fig. 4). The behavior of $r$ is not basically different from that of $\ln n_k$ (see Ref. 1)), but the shape of the distribution of $r$ is different from that of $\ln n_k$. The squeeze angle $\phi$ displays characteristic behavior. That is, from Figs. 1 and 2, the value of $\phi$ remains fixed at most times, but periodically it changes greatly in a $mt/2$ cycle in the massless case. However, in the massive case it seems that this feature changes slightly as seen in Fig. 3. Of course, the squeeze parameters $r$ and $\phi$ can be derived directly by solving Eqs. (17) and (18), and the same results are obtained.

### 3.2. The case of non-parametric resonance

The next case considered is that in which parametric resonance does not occur in reheating. Here we make the contribution of the expanding universe clear. In this case, Eq. (34) becomes

$$\ddot{f_k} + \frac{k^2}{a^2}f_k = 0,$$

where it is assumed that $g^2\Phi^2 = 0$, $m_X^2 = 0$ and $\xi = 0$. The solution is given as

$$f_k(t) = \frac{1}{\sqrt{2k}}\left(\left(\frac{t}{t_1}\right)^{1/3} - \frac{i}{3k}\right)\exp(-3ikt_1^{2/3}t^{1/3}),$$

where $t_1 = \pi/2m$. If one takes the Bunch-Davies vacuum $C_1 = 1$ and $C_2 = 0$ as an initial state, with Eqs. (28) and (29), $A_k$ and $B_k$ can be calculated as

$$A_k(x) = \frac{1}{36\pi^3k^3x}\exp\left(-3ikt(1 + 2^{2/3}x^{1/3})\right)\frac{2m}{2m} \times \left(-4im^3 - 6km^2\pi + 2^{2/3}(6km^2\pi - 9ik^2m^2\pi)x^{1/3}\right).$$


\[ B_k(x) = -\frac{1}{36^3 k^3 x} \exp \left\{ -3i\pi (1 + 2^{2/3} x^{1/3}) \right\} \{ 4im^3 - 6km^2 \pi \\
+ 2^{2/3}(-6km^2 \pi - 9ikm^2 \pi)x^{1/3} + 2^{1/3}(-6ikm^2 \pi + 9km^2 \pi)x^{2/3} \\
+ (-16im^3 + 24km^2 \pi + 12ikm^2 \pi)x \\
+ 2^{2/3}(24km^2 \pi + 36ikm^2 \pi - 18km^2 \pi)x^{4/3} \}. \]

where \( x = mt/(2\pi) \). The squeeze parameters \( r \) and \( \phi \) can be derived from Eqs. (30) and (32). Thus, the squeeze parameters \( r \) and \( \phi \) are approximately given as

\[ r = \frac{1}{3} \log x + \frac{1}{2} \log \left\{ \frac{4(16m^4 + 12km^2 \pi^2 + 9k^2 \pi^4)}{9 \cdot 2^{2/3} k^4 \pi^4} \right\} \\
+ \frac{2}{81} \left( \frac{32m^6 + 24km^2 \pi^2 + 18m^2 \pi^4}{k^6 \pi^6} \right) x^{-2/3} + \cdots, \] (40)

\[ 2\phi = -\pi + \frac{3k \pi}{m} + \frac{12}{}(-4km^3 + 3k^3 m^2 \pi) + \cdots. \] (41)

The angle \( \phi \) becomes a fixed value. The behavior of the squeeze angle \( \phi \) is considerably different from that of the parametric resonance. If we choose a different initial state, the behavior of \( r \) changes slightly, and the value of the angle \( \phi \) comes close to a different fixed value.

3.3. The case of inflation

Here, the squeeze parameter values in the case of inflation are calculated to investigate the difference in squeeze parameter values for the case of an expanding universe and that of parametric resonance. For the simple calculation the scale factor in inflation is given as \( a^I(t) = \exp(\frac{Ht}{2}) \), where the beginning of inflation is \( t = 0 \). A real massless scalar field that is minimally coupled is considered. In this case, we fix \( V = 0 \) in the Lagrangian (1). Then the solution of the scalar field is written as

\[ f^I_k(t) = \frac{1}{\sqrt{2k}} \left( e^{Ht/2} + \frac{iH}{k} e^{3Ht/2} \right) \exp \left( \frac{ik}{H} e^{-Ht} \right). \] (42)

A general, normalized solution for the case of inflation can be written as \( C_1 f^I_k(t) + C_2 f^I_k(t)^* \), where the constants \( C_1(k) \) and \( C_2(k) \) are functions of \( k \). The choice of \( C_1(k) \) and \( C_2(k) \) depends on the freedom of the vacuum. There is freedom to set an initial vacuum state, and here two initial vacuum states are considered.

Case 1. At \( t = 0 \) the Bunch-Davies vacuum \( C_1 = 1 \) and \( C_2 = 0 \) is adopted.

Case 2. The Normal vacuum state: \( v^I_k(0) \)

\[ u^I_k(0) = \frac{1}{\sqrt{2k}}, \quad w^I_k(0) = -\frac{1}{\sqrt{2k}}. \] (43)

In this case, \( C_1 \) and \( C_2 \) are given as

\[ C_1 = \left( 1 - \frac{iH}{2k} \right) e^{-ik/H}, \quad C_2 = \frac{iH}{2k} e^{ik/H}. \] (44)
The calculation in the normal vacuum case is complicated, but because the initial state is a minimum uncertainty state, we take it as an initial state. Using Eqs. (28) and (29), $A_k(t)$ and $B_k(t)$ can be calculated as

$$
A_k(t) = e^{3Ht/2} \left( \frac{iH \cos \varphi}{2k} - \frac{iH^2 \sin \varphi}{2k^2} + \frac{H \sin \varphi}{2k} \right) + e^{Ht/2} \left( \frac{\cos \varphi}{2} - \frac{iH \cos \varphi}{2k} - \frac{i \sin \varphi}{2} \right) + e^{-Ht/2} \left( \frac{\cos \varphi}{2} - \frac{H \sin \varphi}{2k} - \frac{i \sin \varphi}{2} \right),
$$

(45)

$$
B_k(t) = e^{3Ht/2} \left( -\frac{iH \cos \varphi}{2k} + \frac{iH^2 \sin \varphi}{2k^2} - \frac{H \sin \varphi}{2k} \right) + e^{Ht/2} \left( -\frac{\cos \varphi}{2} + \frac{iH \cos \varphi}{2k} + \frac{i \sin \varphi}{2} \right) + e^{-Ht/2} \left( \frac{\cos \varphi}{2} - \frac{H \sin \varphi}{2k} - \frac{i \sin \varphi}{2} \right),
$$

(46)

where $\varphi = \frac{k}{H} \left( 1 - e^{-Ht} \right)$. With $A_k(t)$ and $B_k(t)$, the values of the squeeze parameters $r$ and $\phi$ can be calculated as

$$
r = \frac{3Ht}{2} + \frac{1}{2} \log \left\{ \frac{H^4 + 2H^2 k^2 - H^4 \cos 2\varphi - 2H^3 k \sin 2\varphi}{2k^4} + e^{-Ht} - \frac{4H^2 k^2 \cos 2\varphi + 2H^3 k \sin 2\varphi}{2k^4} + \cdots \right\},
$$

(47)

$$
2\phi = \pi - \frac{2k}{H} e^{-2Ht} + \cdots.
$$

(48)

In this case of inflation, the value of the squeeze angle $\phi$ becomes $\pi/2$. It is worth noting that this behavior of the squeeze angle is different from that in the case of parametric resonance. Next, we give the results for the Bunch-Davies vacuum. The values of the squeeze parameters $r$ and $\phi$ can be calculated as

$$
r = \frac{3Ht}{2} + \frac{1}{2} \log \left\{ \frac{H^2}{k^2} + e^{-2Ht} \left( 1 - \frac{2H^2}{k^2} \right) + \cdots \right\},
$$

(49)

$$
2\phi = \pi - \frac{2k}{H} + \cdots.
$$

(50)

The difference between the initial conditions is not large, but the value of the squeeze angle becomes $\pi/2 - k/H$.

\section{Summary and discussion}

In this paper, the difference in the properties of particle production for parametric resonance in reheating (preheating) and an expanding universe was investigated by calculating squeeze parameter values. A new method for obtaining squeeze
Fig. 1. The evolution of the squeeze parameters $r$ and $2\phi$ as functions of $x (= m t / 2 \pi)$ in the case of parametric resonance. The squeeze parameters in the cases where $m_\chi^2 = 0$, $m = 10^{-6} M_P$, $g = 5 \times 10^{-4}$ and $k = 0.1 m$ are plotted.

Apparently, the behavior of $r$ and $\phi$ change very little for the range of values of $k$ satisfying $0 \leq k < m$.

parameter values without solving the corresponding differential equations was proposed. In particular, this method can be applied to many interesting cases, for example, parametric resonance (preheating), inflation, and radiation-dominated and matter-dominated epochs, and to any initial vacuum states.

Using this new method, we calculated the squeeze parameter values in some interesting cases. In the case of parametric resonance, we used the model of Kofman et al. The squeeze parameters $r$ and $\phi$ are plotted in Figs. 1 and 2 for the case in which the $\chi$ field is massless, and in Fig. 3 for the case in which the $\chi$ field is massive. The squeeze parameter $r$ has nearly the same form for any value of $k$ satisfying $0 \leq k < m$, but when $k$ is larger than $m$, it appears that the maximum value of $r$ is smaller. In the case of massive $\chi$, the maximum value of $r$ is smaller than that in the massless case, and when $m_\chi \approx 10 m$, it seems that parametric resonance does not occur (see Fig. 4). Next, we showed that the squeeze angle $\phi$ exhibits the
Fig. 2. The evolution of the squeeze parameters $r$ and $2\phi$ as functions of $x$ ($= mt/2\pi$) in the case of parametric resonance. (a) The squeeze parameters in the case where $m^2\chi = 0$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 4m$ are plotted, and, for reference, the squeeze factor $r$ in the case of non-parametric resonance ($r \approx \log x/3$) is plotted. (b) In order to illustrate in detail the behavior of the squeeze parameters $r$ and $2\phi$, they are plotted in the range $17 \leq x \leq 20$, as in (a).

nearly same behavior for all $k$ satisfying $0 \leq k < m$, and it has a peculiar form, i.e., the value of $\phi$ is a constant at most times but periodically changes greatly in a $mt/2$ cycle. But, in the case of a massive $\chi$ field, the periodic change in the value of $\phi$ becomes slightly irregular.

On the other hand, in the case of an expanding universe, the squeeze parameters can be derived analytically, but they have very complex forms. In such cases the squeeze parameters exhibit familiar and simple behavior. We may, therefore, reasonably conclude that a difference in the properties of created particles for parametric resonance in reheating (preheating) and an expanding universe exists. However, defining the physical implication of this difference is a problem to be investigated in the future.

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Fig. 3. The evolution of the squeeze parameters $r$ and $2\phi$ as functions of $x (= mt/2\pi)$ in the case of parametric resonance. The squeeze parameters in the case where $m_\chi^2 = m^2$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 4m$ are plotted.

Fig. 4. The evolution of the squeeze parameters $r$ and $2\phi$ as functions of $x (= mt/2\pi)$ in the case of parametric resonance. The squeeze parameters in the case where $m_\chi^2 = (10m)^2$, $m = 10^{-6}M_P$, $g = 5 \times 10^{-4}$ and $k = 4m$ are plotted. It seems that parametric resonance does not occur in this case.

References

2) L. P. Grishchuk and Y. V. Sidorov, Class. Quantum Grav. 6 (1989), L161.