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Estimation of Transmissivity and Permeability in Swedish Bedrock

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Leif Carlsson and Anders Carlstedt
Chalmers University of Technology, Sweden, and
Geological Survey of Sweden

Statistical analysis of pumping-test data from wells have been used to calculate average values of transmissivity and permeability in different Swedish rocks. The influence of the well-loss on the calculations is discussed. The highest values of transmissivity and permeability of the investigated rocks are found in the sandstones of Algonkian and Cambrian age. The Archean crystalline rocks show a wide range of results, and of the investigated rocks the gneisses seem to be more permeable than the granites. However, the degree of tectonization affects the hydraulic properties of the rocks considerably.

Introduction

Pumping-test data are a useful tool in the comparison and evaluation of the ground water conditions in different areas. However, proper pumping-tests are usually not available. For the calculations or estimations of the hydraulic properties only the results from a simple form of pumping-test carried out by well drillers are available. The results obtained from such a pumping test are the capacity and the drawdown in the pumped well.

In the comparison of water-yielding properties of different areas, usually only the capacity values of the wells are used. A more adequate method is to use the specific capacity, which is defined as the ratio between the capacity and the drawdown.

The specific capacity of a well is not an exact criterion of the water-yielding properties of an aquifer because of other factors affecting the well production. Thus the depth of penetration into the aquifer, the diameter of the well, and the length of the open section are major factors influencing the drawdown and, consequently, the specific capacity.

The length of the open section of the well has to be considered when comparing specific capacities of different wells if the diameters of the wells are assumed constant. From the values of specific capacity or the specific capacity per meter of open section of a well the coefficients of transmissivity and permeability can be obtained if some rough assumptions are made regarding the rock-media.

Theory

According to Jacob (1947) the drawdown s_w of a well in a homogeneous and isotropic aquifer can be described as

$$s_w \equiv BQ + CQ^2 \tag{1}$$

BQ is the drawdown within the aquifer just outside the well and CQ^2 is the well-loss. At a constant pumpage rate the factor BQ can be described as (Theis 1935)

$$BQ \equiv \frac{W(u)}{4\pi T} \cdot Q \tag{2}$$

where $W(u)$ = the well function, which according to Theis can be expressed as

$$W(u) = \int_u^\infty \frac{e^{-u}}{u}\, du \tag{3}$$

$$u = \frac{r^2 \cdot S}{4t\ T} \tag{4}$$

T = aquifer transmissivity, m^2/s
S = storage coefficient
t = time since pumping started, min
r = here the radius of the well, m

For $u < 0.02$ i.e. small values of r or long pumping time t, the well function can be described as (Jacob 1940)

$$W(u) \equiv 2.30 \cdot \log\left(\frac{2.25\ T \cdot t}{r^2 \cdot S}\right) \tag{5}$$

The specific capacity of a well is given by the following equation

$$q = \frac{Q}{s_w} = \frac{1}{\frac{0.183}{T}\log\left(\frac{2.25\cdot T\cdot t}{S\cdot r^2}\right) + CQ} \qquad (6)$$

As a simplification the well-loss is neglected and Eq. (6) can be rewritten as

$$q \equiv \frac{T}{0.183\cdot\log\left(\frac{2.25\, T\cdot t}{S\cdot r^2}\right)} \qquad (7)$$

Thus the specific capacity is proportional to the transmissivity. According to Mucha (1972), the transmissivity estimated from a partially penetrated well can be written as the permeability multiplied by the length of the open section of the well h^1. Thus the relation between the permeability k, the length of the open section of the well, and the specific capacity can be written

$$q = \alpha\cdot k\cdot h^1 = \alpha\cdot T \qquad (8)$$

In Fig. 1 the relation between α-value, pumping time, well diameter and the hydraulic diffusivity T/S of the aquifer is shown.

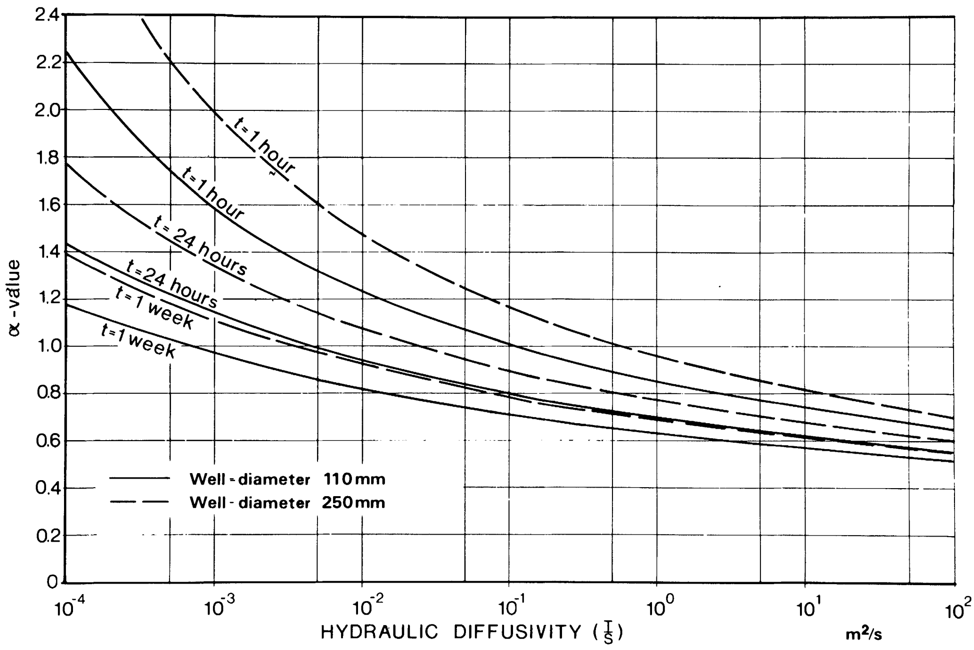

Fig. 1. The relation between α -value, pumping time, and hydraulic diffusivity of two different diameters.

For regional studies Jetel (1964) and Jetel and Krásný (1968) have introduced the parameters Y and Z. When the specific capacity is expressed in m³/s. m and the length of the open section h' in m, the following equations define the parameters Y and Z.

$$Y = \log(10^9 \cdot q) \tag{9}$$

$$Z = \log(10 \cdot \frac{q}{h'}) \tag{10}$$

Thus the Y- and Z-parameters give the values of transmissivity and permeability in a logarithmic form. Jetel (1972) has shown by statistical treatment that the parameters Y and Z are normally distributed within specified geohydrological regions. Median values and standard deviations from such analyses make it possible to compare different geological areas with regard to transmissivities and permeabilities. To transform the Z-value to permeability and the T-value to transmissivity, Jetel assumes a steady state condition or »quasi-steady« state condition to be prevailing, which gives the following expressions

$$k = 10^{Z-c} + \log(\log \frac{R}{r_w}) \tag{11}$$

$$T = 10^{Y-c} + \log(\log \frac{R}{r_w}) \tag{12}$$

where c = constant 9.436
R = radius of influence
r = radius of the well

If a non-steady state is assumed to be prevailing, the expressions for the transmissivity and permeability can be written

$$T = \frac{10^{(Y-9)}}{\alpha} \tag{13}$$

$$k = \frac{10^{(Z-9)}}{\alpha} \tag{14}$$

The α-value is obtained from Fig. 1. Since most of the wells in Swedish bedrock are drilled with a diameter of 110 mm and the duration of the pumping tests made by the drillers usually does not exceed one day, the α-value can be estimated to range from 0.9 to 1.1.

The value of the konstant C in Eq. (1) is depenent on the transmissivity and the diameter of the well. Results obtained from step-drawdown tests (Gustafson 1974 and Hörnsten et al 1974, unpublished report) give the following relation between transmissivity, well radius r_w, and C (Fig. 2).

$$C \cdot r_w \cdot T^{1.25} \doteq 1 \tag{15}$$

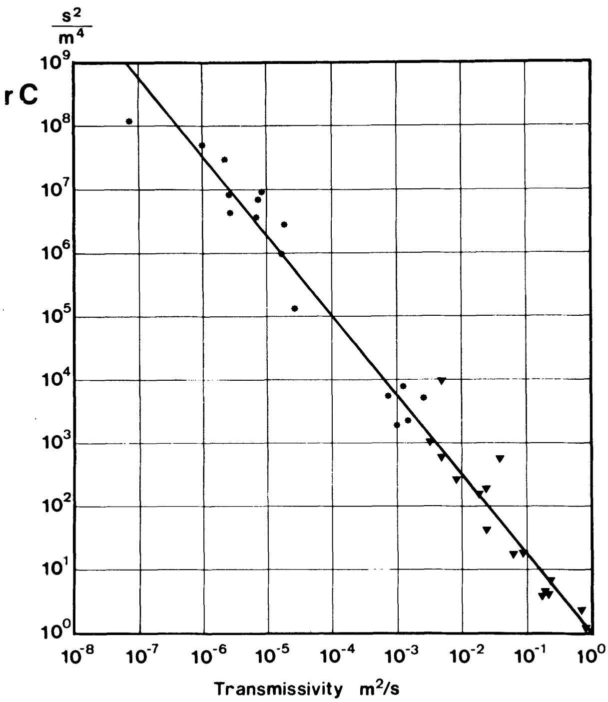

Fig. 2. The relation between transmissivity and the product of well-radius and well-loss. (Values from Gustafson 1974 and Hörnsten et. al. 1974, unpublished report).

Thus Eq. (6) can be rewritten as:

$$\frac{1}{q} = \frac{1}{\alpha \cdot T} + \frac{Q}{r_w \cdot T^{1.25}} \qquad (16)$$

From Fig. 3 the relation between drawdown and capacity for different values of the transmissivity T can be obtained. The well-loss (CQ^2) constitutes more than 50% of the total drawdown in a 110 mm well when the pumpage rate is high, the drawdown is great and the transmissivity high.

Pumpage from drilled wells (diameter 110 mm) in Swedish bedrock is rarely greater than 7,000 l/h, and the drawdowns usually do not exceed 80 m. Theoretically this means that less than 25% of the drawdown depends on the well-loss. Thus a deviation of 75% for the transmissivity can be expected as a maximum when neglecting the well-loss in Eq. (16).

The occurrence and movement of ground water in Swedish bedrock is determined by cracks, joints, fissures, etc. The basic theory, however, describes the ground water conditions in homogeneous and isotropic conditions. A theoretical analysis of wells in

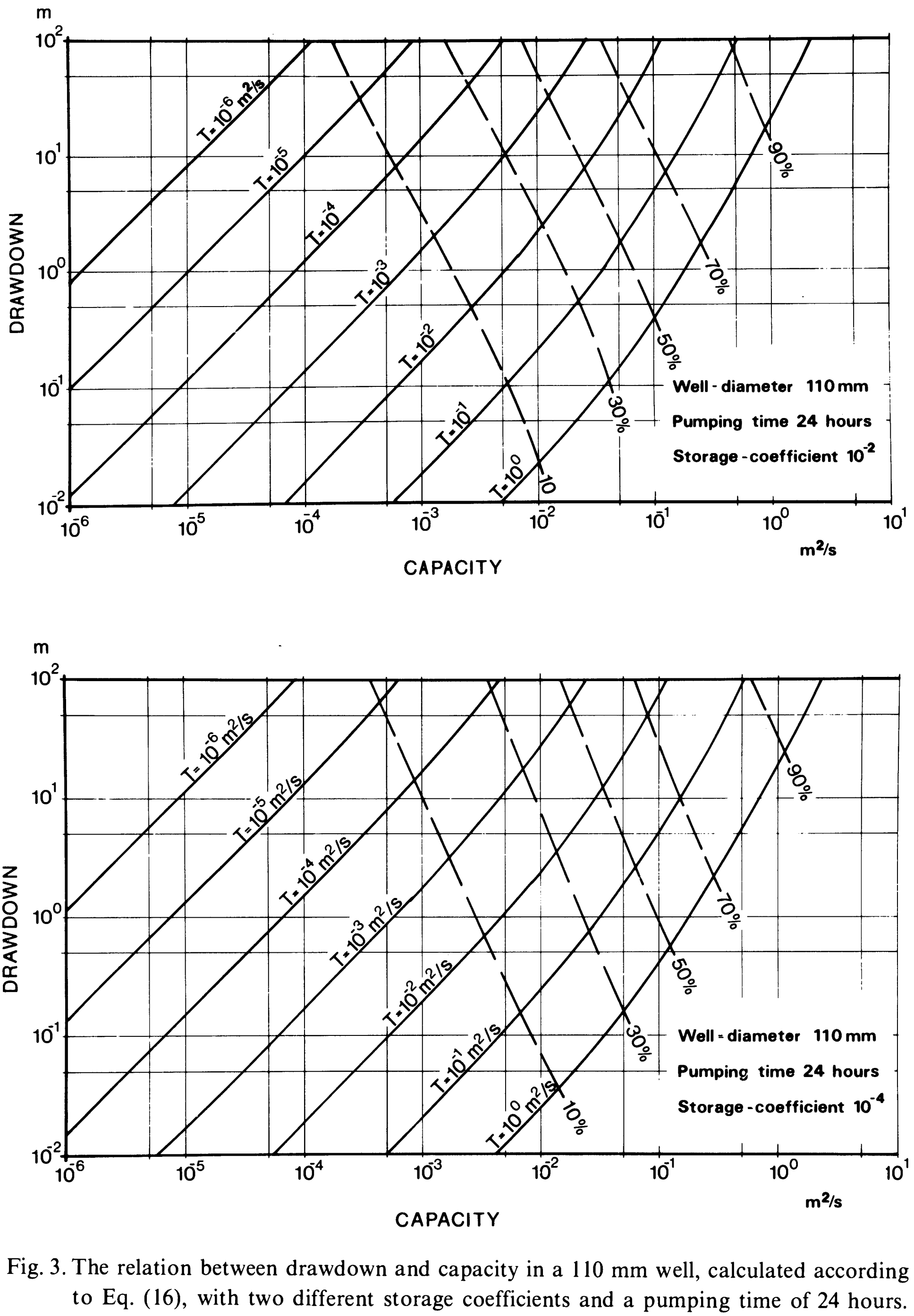

Fig. 3. The relation between drawdown and capacity in a 110 mm well, calculated according to Eq. (16), with two different storage coefficients and a pumping time of 24 hours. The influence of well-loss is indicated in percent of the total drawdown.

aquifers with horizontal and vertical fractures has been carried out by Gringarten and Ramsey (1974 a and b). As pointed out by Gringarten and Witherspoon (1972) the divergences from Theis' well function are considerable.

Estimated Values of Transmissivities and Permeabilities in Some Swedish Rocks

The areas investigated, which all are situated in southern Sweden, are shown in Fig. 4. In Table 1 a brief review of each area is given.

Table 1 - A brief description of areas used in estimation of permeability and transmissivity of the bedrock (according to Magnusson et. al. 1960)

Area	Type of bedrock	Physical character and age in millions of years	Groundwater occurrence
1. County of Halland	a. Mainly Pre-Gothian gneisses	Crystalline, metamorphosed, more than 1500 mill. years	In fissures and cracks
2. Billingen-area	b. Pre-Gothian gneisses	Crystalline, metamorphosed, more than 1500 mill. years	In fissures and cracks
	f. Lower Cambrian sandstone	Thick-banked pure fine-grained strongly cemented quarz-sand interbedded with thin layers of clay. 550-600 mill. years	In joints and primary pore-spaces
	g. Lower and middle Ordovician limestone	Banked pure dense limestone interbedded with thin layers of marl. 460-480 mill. years	In vertical joints
3. Area west of lake Vättern	c. Gothian granites	Crystalline, magmatic, strongly tectonized. 1500 mill. years	In fissures and cracks
	e. Algonkian sandstones	Loose to hard cemented sand of various colour, strongly tectonized. 1000 mill. years	In fissures, cracks and primary pore-spaces
4. Uppsala area	d. Svecofennian granites	Crystalline, magmatic. 1850-1900 mill. years	In fissures and cracks

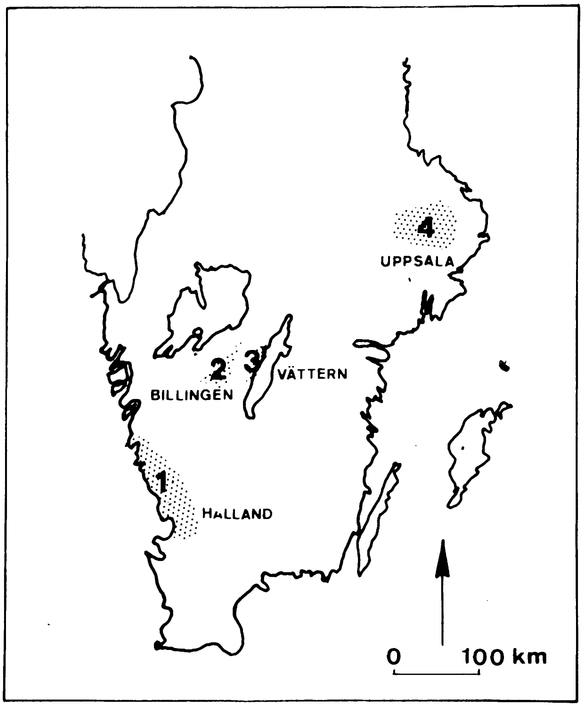

Fig. 4. Map of southern Sweden showing the areas from which data have been used for the calculation of permeability and transmissivity of the bedrock.

From the data available the *Y*- and *Z*-values have been evaluated in accordance with Jetel (1964). The results obtained are shown in Figs. 5 and 6 in normal distribution diagrams. The hydrogeological conditions of the geological formations within the areas are described by the values in the 40 - 60% interval of the normal distribution diagrams. In Tables 2 and 3 these values are presented together with the values of the transmissivity and the permeability, respectively, calculated from an α-value of 0.85. In Figs. 7 and 8 the transmissivities and permeabilities in the 40 - 60% interval of the normal distribution are illustrated.

In some cases drilled wells with a very low capacity are not reported by the drillers. These wells are abandoned and new ones drilled. Thus the values of the permeability of the Ordovician limestone exceed those obtained from step-drawdown tests. On the other hand, the mean value of the Cambrian sandstone is lower than the permeability estimated from pumping-tests during unsteady-state conditions (Carlsson and Carlstedt 1975).

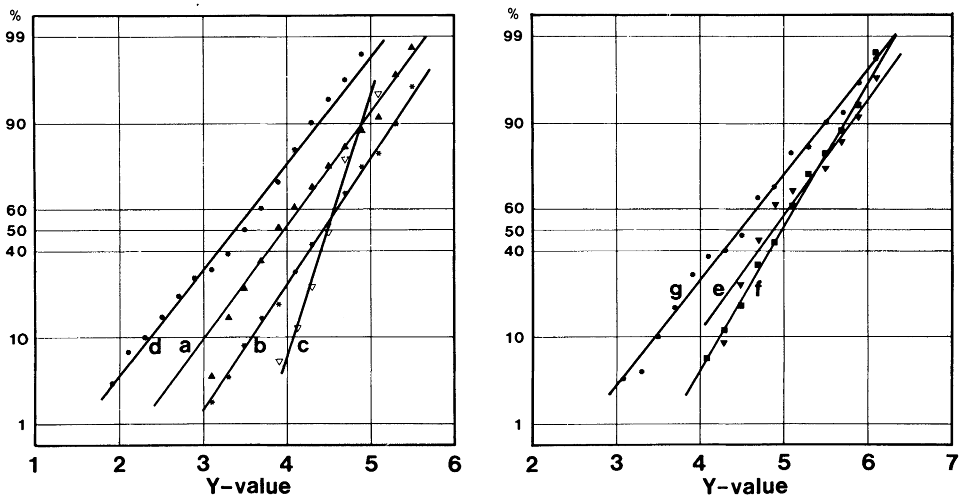

Fig. 5. Relative cumulative frequencies of the *Y*-values from crystalline bedrock (a = gneisses of Halland, b = gneisses of the Billingen area, c = granites west of lake Vättern, d= granites of the Uppsala area) and sedimentary bedrock (e = Algonkian sandstone west of lake Vättern, f = Cambrian sandstone of the Billingen area, g = Ordovician limestone of the Billingen area).

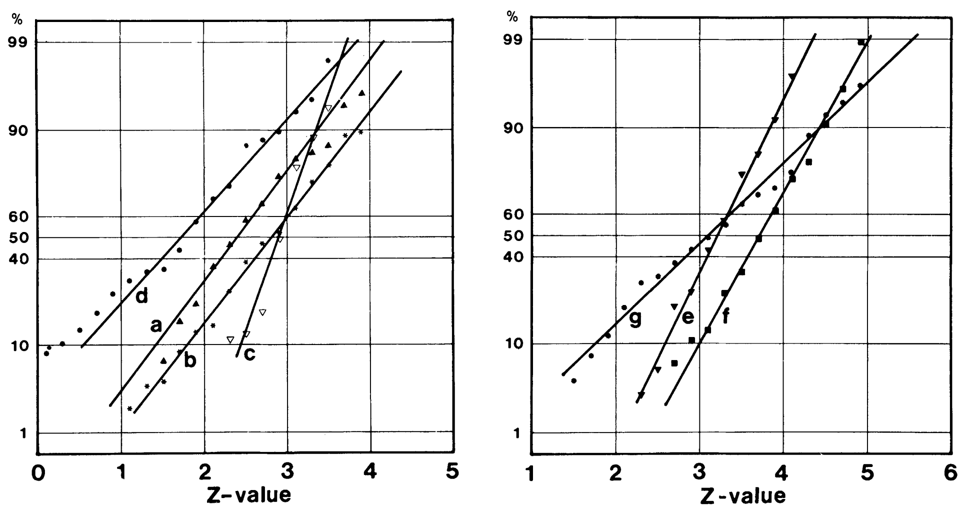

Fig. 6. Relative cumulative frequencies of the *Z*-value from crystalline bedrock and sedimentary bedrock (explanation see Fig. 5).

Table 2 - Y-values in the 40-60%-interval of the normal distribution diagrams and the corresponding transmissivities of the investigated areas.

Area and bedrock	Y_{40}-Y_{60}	T_{40}-T_{60} m^2/s	T mean m^2/s	amount of data
a. Gneisses of Halland	3.77-4.15	$0.7\text{-}1.7 \cdot 10^{-5}$	$1.1 \cdot 10^{-5}$	70
b. Gneisses of the Billingen-area	4.26-4.60	$2.1\text{-}4.7 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	50
c. Granites west of lake Vättern	4.40-4.57	$3.0\text{-}4.4 \cdot 10^{-5}$	$3.6 \cdot 10^{-5}$	15
d. Granites of the Uppsala-area	3.18-3.57	$1.8.\text{-}4.4 \cdot 10^{-6}$	$2.8 \cdot 10^{-6}$	59
e. Algonkian sandstone west of lake Vättern	4.70-5.06	$0.6\text{-}1.4 \cdot 10^{-4}$	$9.1 \cdot 10^{-5}$	34
f. Cambrian sandstone of the Billingen area	4.83-5.13	$0.8\text{-}1.6 \cdot 10^{-4}$	$1.1 \cdot 10^{-4}$	120
g. Ordovician limestone of the Billingen area	4.27-4.90	$2.2\text{-}9.3 \cdot 10^{-5}$	$3.5 \cdot 10^{-5}$	50

Discussion

The aquifer conditions are assumed to be isotropic and homogeneous when the specific capacity is transformed to transmissivity. By no means the bedrock areas dealt with can be described as being isotropic and homogeneous regarding their hydraulic properties. However, in regional comparative investigations the method described is applicable without any appreciable error as pointed out by Jetel and Krásný (1968) and Krásný (1975). The degree of homogeneity in the transmissivity and the permeability of the bedrock is reflected by the slope of the lines in Figs. 5 and 6, i.e. the steeper the slope, the more homogeneous the bedrock.

Thus, from the diagrams it seems as if the granites (c) and the Algonkian sandstones (e) west of lake Vättern show the greatest homogeneity. Those bedrocks build up an area which has been strongly tectonized. This tectonization has caused a cracking of the bedrocks, which has a favourable effect on the hydraulic conditions. When regarding the homogeneity of the Algonkian sandstone (e) and also the Cambrian sandstone (f), one has to take into account that these bedrocks are to a certain extent porous media. However, these sandstones are cemented, and the groundwater occurrence mainly determined by the fissures.

Table 3 - Z-values in the 40-60%-interval of the normal distribution diagrams and the corresponding permeabilities of the investigated areas.

Area and bedrock	Z_{40}-Z_{60}	k_{40}-k_{60} m/s	k mean m/s	amount of data
a. Gneisses of Halland	2.20-2.60	$1.9\text{-}4.7 \cdot 10^{-7}$	$2.6 \cdot 10^{-7}$	70
b. Gneisses of the Billingen area	2.62-3.02	$0.5\text{-}1.1 \cdot 10^{-6}$	$7.8 \cdot 10^{-7}$	50
c. Granites west of Lake Wättern	2.81-2.98	$0.6\text{-}1.0 \cdot 10^{-6}$	$9.4 \cdot 10^{-7}$	15
d. Granits of the Uppsala area	1.50-1.95	$0.4\text{-}1.1 \cdot 10^{-7}$	$6.3 \cdot 10^{-8}$	59
e. Algonkian sandstone west of lake Vättern	3.10-3.35	$1.5\text{-}2.6 \cdot 10^{-6}$	$2.0 \cdot 10^{-6}$	34
f. Cambrian sandstone of the Billingen area	3.58-3.86	$4.5\text{-}8.5 \cdot 10^{-6}$	$6.3 \cdot 10^{-6}$	120
g. Ordovician limestone of the Billingen area	2.84-3.37	$0.8\text{-}2.8 \cdot 10^{-6}$	$1.5 \cdot 10^{-6}$	50

The lowest degree of homogeneity is found in the Ordovician limestone (g). This is assumed to be caused by the jointing, which is mainly vertical, and therefore it is difficult to find the best location for a well. A more adequate well drilling technique, inclined drilling, is thought to improve the water-yielding capacity of the wells.

The fairly low degree of homogeneity of the gneisses of Halland (a) and the Billingen area (b) and the granites of the Uppsala area is supposed to be due to the comparatively low frequency of joints.

From Czechoslovakia, calculations of the transmissivity of crystalline bedrock of southern Bohemia have been presented by Krásný (1975). In this investigation migmatitized gneisses were found to have a higher transmissivity, about 10^{-4} m^2/s, than magmatites, 3 - 7 x 10 $^{-5}$ m^2/s. Mica schists, mica schist gneisses, and paragneisses showed the lowest transmissivities, 2 - 4 x 10 $^{-5}$ m^2/s. Thus values obtained from calculations of the Swedish crystalline bedrocks are lower, which is a consequence of a low frequency of joints compared to the conditions in Czechoslovakia.

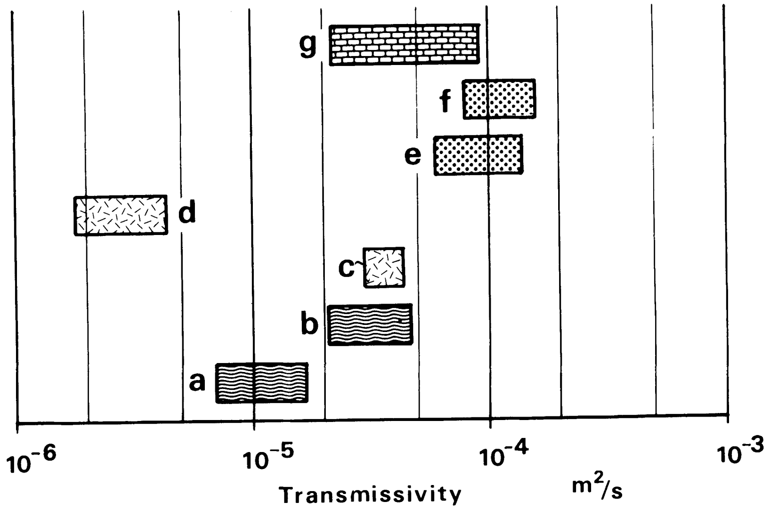

Fig. 7. Transmissivity in some Swedish bedrocks given as the 40-60%-interval of the normal distribution (explanation see Fig. 5).

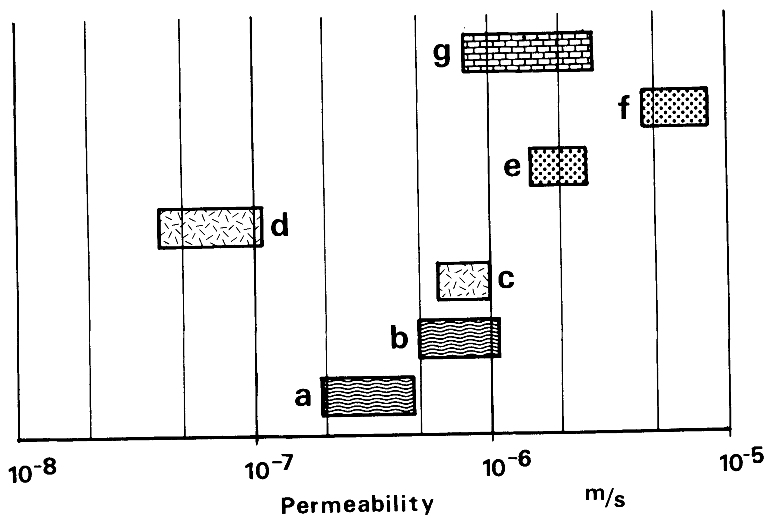

Fig. 8. Permeability in some Swedish bedrocks given as the 40-60%-interval of the normal distibution (explanation see Fig. 5).

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Address:
Leif Carlsson,
Chalmers Tekniska Högskola,
Inst. för Vattenförsörjnings- och Avloppsteknik,
Sven Hultins Gata 8,
Fack, S 402 20 Göteborg, Sweden.

Anders Carlstedt
SGU, Sveriges Geologiska Undersökning,
Box 2048,
S-18302 Täby,
Sweden.