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Contact Stresses in the Rolling of Metals[1]

J. H. Hitchcock.[2] The writer wishes to congratulate the authors on a splendid piece of work, characterized by meticulous and thorough care, in preparation of instrumentation for this ambitious project in measurement of contact stresses between material and rolls. It is extremely interesting to find so extensive application of modern electronics technique in this enterprise.

The most critical element in this equipment of course is the weigh bar which will be subjected to contact stresses. With the steps which the authors have taken to assure that the point of the weigh bar will match the roll surface in contour and in flexibility, their need be little misgiving that the presence of the weigh bar will affect the rolling process sufficiently to invalidate the results. In this respect it is believed that the construction adopted is admirable. There may be a possibility, however, that the minute clearance between the point of the weigh bar and the surrounding roll will become obstructed so that the stresses applied to the point of the weigh bar may not be transmitted fully to the strain-indicating instrument. This possibility is more likely to be troublesome in hot-rolling than in cold-rolling, and only repeated trials will demonstrate whether this feature will be troublesome or not.

Of considerable interest also are the load cells, located between the bearings and screws for measurement of bearing loads. Although no reference is made in the paper to provision of torsional restraint, it is believed the load cells are so constructed that any torsion, which may be applied to them by rotation of the screws, is transmitted through the load-cell housings to the bearings, leaving the load cells themselves completely free of torsional stresses and subject only to direct compression between the screws and the bearings. This is considered to be an essential feature of such devices.

It is natural to expect that a comprehensive research program, employing instrumentation as competently assembled as the equipment described, will produce results of inestimable value to rolling-mill engineers. The writer is confident that the results of this investigation will fulfill all such expectations.

Authors' Closure

The authors wish to thank Mr. Hitchcock for his kind comments on their work. The problem undertaken is indeed an ambitious one and it is hoped that some experimental data, obtained by the use of the equipment so developed, will soon become available.

The General Proof of the Principle of Maximum Plastic Resistance[1]

H. J. Greenberg.[2] The writer wishes to call attention to the paper by R. Hill[3] of the Cavendish Laboratory, Cambridge, England. Whereas the author of the paper under discussion assumes the finite stress-strain law, Equations [1], Hill assumes the incremental or differential stress-strain law of the Saint Venant-Lévy-Mises theory, which is obtained from Equation [1] by replacing each strain component by the corresponding increment. He proves in direct fashion that the plastic power or rate of work of the surface forces is an absolute maximum, the velocities being prescribed on the surface. The method of proof he employs applies equally well, however, if one assumes the stress-strain law, Equations [1] (an assumption which is justified only if during the deformation the principal axes remain fixed and the ratios of the principal stresses remain constant), in which case we shall show readily that the work done by the surface forces is an absolute maximum. This is a stronger result than that of the present author who shows only that the first variation of the work vanishes. It is unfortunate that Hill's results, which were obtained several years ago, could receive only limited attention because of the fact that they were originally presented in a wartime British restricted report (1945).

Let $\sigma_x^*, \sigma_y^*, \sigma_z^*, \tau_{xy}^*, \tau_{yz}^*, \tau_{zx}^*$ be any set of stress components which satisfy the equations of equilibrium and the plasticity condition, Equation [2] of the paper, i.e., $T_d^* = T_d = \text{const}$, where T_d^* is T_d evaluated for the starred stresses. By X_ν^*, Y_ν^*, Z_ν^* we denote the surface forces corresponding to these stresses. The original nomenclature of the author will be used without further definition. To prove that the work done by the actual surface forces is an absolute maximum, we have to prove that

$$\int_S (u_0 X_\nu + v_0 Y_\nu + w_0 Z_\nu) dS \geqslant \int_S (u_0 X_\nu^* + v_0 Y_\nu^* + w_0 Z_\nu^*) dS$$

Using Green's theorem for transformation from surface to volume integrals and the equilibrium equations, this reduces to showing that

$$\int_V [\epsilon_x(\sigma_x - \sigma_x^*) + \epsilon_y(\sigma_y - \sigma_y^*) + \epsilon_z(\sigma_z - \sigma_z^*) + \gamma_{xy}(\tau_{xy} - \tau_{xy}^*) + \gamma_{yz}(\tau_{yz} - \tau_{yz}^*) + \gamma_{zx}(\tau_{zx} - \tau_{zx}^*)] dV \geqslant 0$$

Taking into account the incompressibility condition $\epsilon_m = 0$, and writing for the sake of brevity

$$\begin{aligned} \sigma_x' &= \sigma_x - \sigma_m, \\ \sigma_x^{*\prime} &= \sigma_x^* - \sigma_m^*, \end{aligned} \quad (x, y, z)$$

where $\sigma_m^* = (\sigma_x^* + \sigma_y^* + \sigma_z^*)/3$, this is the same as showing that

$$\int_V [\epsilon_x(\sigma_x' - \sigma_x^{*\prime}) + \epsilon_y(\sigma_y' - \sigma_y^{*\prime}) + \epsilon_z(\sigma_z' - \sigma_z^{*\prime}) + \gamma_{xy}(\tau_{xy} - \tau_{xy}^*) + \gamma_{yz}(\tau_{yz} - \tau_{yz}^*) + \gamma_{zx}(\tau_{zx} - \tau_{zx}^*)] dV \geqslant 0$$

Substituting from Equation [1] of the paper for the strains, the integral becomes

$$\int_V \frac{1}{\lambda} [\sigma_x'(\sigma_x' - \sigma_x^{*\prime}) + \sigma_y'(\sigma_y' - \sigma_y^{*\prime}) + \sigma_z'(\sigma_z' - \sigma_z^{*\prime}) + 2\tau_{xy}(\tau_{xy} - \tau_{xy}^*) + 2\tau_{yz}(\tau_{yz} - \tau_{yz}^*) + 2\tau_{zx}(\tau_{zx} - \tau_{zx}^*)] dV$$

This, however, can be rewritten

$$\int_V \frac{1}{\lambda} [T_d^2 - \mathsf{T}_d \cdot \mathsf{T}_d^*] dV$$

where

$$\mathsf{T}_d \cdot \mathsf{T}_d^* = \sigma_x'\sigma_x^{*\prime} + \sigma_y'\sigma_y^{*\prime} + \sigma_z'\sigma_z^{*\prime} + 2\tau_{xy}\tau_{xy}^* + 2\tau_{yz}\tau_{yz}^* + 2\tau_{zx}\tau_{zx}^*$$

Since $T_d^2 = T_d^{*2}$ by the plasticity condition, it follows that

$$T_d^2 - \mathsf{T}_d \cdot \mathsf{T}_d^* \geqslant 0$$

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[1] By C. W. MacGregor and R. B. Palme, published in the September, 1948, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 70, pp. 297–302.

[2] Director of Research, Morgan Construction Company, Worcester, Mass. Mem. ASME.

[1] By A. H. Philippidis, published in the September, 1948, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 70, pp. 241–242.

[2] Assistant Professor of Applied Mathematics, Brown University, Providence, R. I.

[3] A Variational Principle of Maximum Plastic Work in Classical Plasticity, by R. Hill, *Quarterly Journal of Mechanics and Applied Mathematics*, vol. 1, 1948, pp. 18–28.

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