A Model for the $^3$He$(\vec{d}, p)^4$He Reaction at Intermediate Energies

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Polarization correlation coefficients have been measured at RIKEN for the $^3$He$(\vec{d}, p)^4$He reaction at intermediate energies ($E_d = 270$ MeV). We propose a model for the $(\vec{d}, p)$ reaction mechanism using the $pd$ elastic scattering amplitude, which is rigorously determined with a Faddeev calculation and using modern NN forces. Our theoretical predictions for the deuteron polarization observables $A_y$, $A_{yy}$, $A_{xx}$ and $A_{xz}$ at $E_d = 140, 200$ and $270$ MeV are given. The $A_y$ observables agree qualitatively in shape with the new experimental data for the reaction $^3$He$(\vec{d}, p)^4$He.

Introduction Measurement$^1$ of the $^3$He$(\vec{d}, p)^4$He reaction for $E_d = 270$ MeV at RIKEN was carried out as an investigation of the high-momentum components of the deuteron wave function and the D-state admixture linked to them. High precision data resulted for the polarization observables $A_y$, $A_{yy}$, $A_{xx}$ and $A_{xz}$. From these, the linear combination $C_{||} = 1 + \frac{1}{4}(A_{yy} + A_{xx}) + \frac{3}{4}(C_{y,y} + C_{x,x})$ has been formed.$^1$ The Dubna and Saturne groups also obtained the polarization correlation coefficient $C_{||}$ built in this case from the measurements of $T_{20}$ and $\kappa_0$ in $d + p$ backward scattering$^2$ and from the inclusive deuteron breakup process.$^3$ The polarization correlation coefficient $C_{||}$ at forward angles of the outgoing proton is directly related to the ratio of deuteron wavefunction components if one uses the...
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Table I. The polarization correlation coefficient $C_\parallel$ given by Eq. (1) in a simple PWIA model for different NN potentials and the experimental value for the reaction $^3\text{He}(d,p)^4\text{He}$. We also show the corresponding deuteron D-state probabilities.

<table>
<thead>
<tr>
<th>Potential</th>
<th>$C_\parallel$(PWIA)</th>
<th>D-state Probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD-Bonn $^4$)</td>
<td>0.645</td>
<td>4.86</td>
</tr>
<tr>
<td>AV18 $^5$)</td>
<td>0.722</td>
<td>5.78</td>
</tr>
<tr>
<td>Nijmegen 93 $^6$)</td>
<td>0.710</td>
<td>5.76</td>
</tr>
<tr>
<td>Nijmegen I $^6$)</td>
<td>0.712</td>
<td>5.68</td>
</tr>
<tr>
<td>Nijmegen II $^6$)</td>
<td>0.726</td>
<td>5.65</td>
</tr>
<tr>
<td>exp. $^1$)</td>
<td>0.223 ± 0.044 (statistical) ± 0.037 (systematic)</td>
<td>–</td>
</tr>
</tbody>
</table>

plane wave impulse approximation (PWIA):

$$C_\parallel^\text{(PWIA)} \equiv \frac{9}{4} \frac{u^2(k_{pn})}{u^2(k_{pn}) + w^2(k_{pn})}.$$  

Here $u$ and $w$ are the S- and D-wave components of the deuteron wavefunction, and $k_{pn}$ is the kinematically fixed relative momentum of the $pn$ pair. For the reaction of Ref. 1), $k_{pn} = 1.19$ fm$^{-1}$. These PWIA calculations are in very poor with the data. $^1$ This is shown in Table I for $C_\parallel$. There we also exhibit the different D-state probabilities for the modern realistic NN potentials, CD-Bonn, $^4$) AV18 $^5$) and Nijmegen I, II and 93. $^6$) Clearly, we need a better calculation for the analysis of the $^3\text{He}(d,p)^4\text{He}$ reaction.

A theoretical analysis has been reported by the SUT group $^7$) based on a $^3\text{He}$-n-p and d-d-p three-cluster model. However, the evaluations performed to this time using this model lead only to a tiny deviation from the PWIA calculations mentioned above. Recently, the Hosei group $^8$) analyzed $T_{20}$ and $\kappa_0$ with the $^3\text{He}$-n-p cluster model using an analogy between $^3\text{He}$ and the proton ($T = 1/2, S = 1/2$). They concluded that PWIA describes the global features of the experimental data.

In this paper we would like to introduce a three-nucleon (3N) model, which, when evaluated correctly, leads to a great similarity between deuteron vector and tensor analyzing powers and those found in the reaction $^3\text{He}(d,p)^4\text{He}$. Thus in this article we do not concentrate on the spin correlation coefficients, which might depend on the deuteron wavefunction. We briefly mention the necessary additional steps in our model to calculate observables in the last section.

Model For the $^3\text{He}(d,p)^4\text{He}$ reaction we assume a model which is based on a three-nucleon reaction process. This is shown in Fig. 1 in the subprocess $p + d \to p + d$ described by the amplitude $U$. The wavefunctions for $^3\text{He}$ and $^4\text{He}$ take on maximal values if the momenta of the subclusters are zero in their respective rest systems. For $^3\text{He}$ these are the momenta of $p$ and $d$ and for $^4\text{He}$ the momenta of the two deuterons. This implies that for moving nuclei, the sub-
cluster momenta should be equal. Therefore to form the $\alpha$ particle with highest probability in the picture of Fig. 1, one has to assume that the two deuterons, $d'$ and $\tilde{d}$, have equal momenta. Similarly for $^3\text{He}$ one has to assume that the proton and deuteron, $\tilde{p}$ and $\tilde{d}$, have equal momenta. This turns out to be kinematically inconsistent. Therefore we make a choice and assume that only the two deuterons forming the $\alpha$ particle have equal momenta.

It is easy to see that our basic assumption,

$$\vec{k}_{\tilde{d}} = \vec{k}_{d'},$$

(2)

fixes the kinematics uniquely. It follows from simple kinematical arguments that

$$\vec{k}_{\text{lab}} \tilde{p} = \frac{1}{2} \vec{k}_{\text{cm}} \tilde{p} - \frac{2}{5} \vec{k}_{\text{lab}} \tilde{d} = -\vec{k}_{\text{lab}} \tilde{d}.$$  (3)

Here the superscripts lab and cm denote the laboratory and 5-body cm systems, respectively. Further, the total momentum of the picked up proton and the incoming deuteron in the lab system is

$$\vec{K} = \frac{1}{2} \vec{k}_{\text{cm}} \tilde{p} + \frac{3}{5} \vec{k}_{\text{lab}} \tilde{d}.$$  (4)

Also, we obtain the momentum of the picked-up proton in the 3-nucleon center-of-mass system of (3CM) the interacting proton and deuteron and the 3CM energy as

$$\vec{k}_{3\text{CM}} \tilde{p} = \frac{1}{3} \vec{k}_{\text{cm}} \tilde{p} - \frac{3}{5} \vec{k}_{\text{lab}} \tilde{d}, \quad E_{3\text{CM}} = \frac{3}{4m} \left(\vec{k}_{3\text{CM}} \tilde{p}\right)^2.$$  (5)

We display in Fig. 2 the relevant kinematics for the 5-body cm and the 3CM systems. From the relation

$$\vec{k}_{3\text{CM}} = \frac{2}{5} \vec{k}_{3\text{CM}} \tilde{p} - \frac{3}{5} \vec{k}_{\text{lab}} \tilde{d},$$

(6)

it follows under our condition that the angles shown in Fig. 2 are related as $\theta_{3\text{CM}} = \theta_p^{3\text{CM}} - \theta_p^{3\text{CM}}$ (note that $\theta_p \equiv \theta_p^{3\text{CM}} = \theta_p^{\text{cm}}$). The dependence of $E_{3\text{CM}}$ on $\theta_p^{3\text{CM}}$ is illustrated in Fig. 3 for 3 deuteron energies. The scattering angle $\theta_{3\text{CM}}$ is shown as a function of $\theta_p^{3\text{CM}}$ in Fig. 4 again for the same 3 deuteron energies.

Our claim is now that $\mathcal{O}(E_d, \theta_p^{3\text{CM}}) \approx \mathcal{O}_{pd}(E_{3\text{CM}}, \theta_{3\text{CM}})$, where $\mathcal{O}_{pd}$ represents the elastic $pd$ deuteron polarization observables and $\mathcal{O}$ those for the reaction $^3\text{He}(\tilde{d}, p)^4\text{He}$. 

Fig. 2. Scattering angles for the 5-body (cm) and 3-body (3CM) center-of-mass systems.
Before calculating these 3N observables, we introduce one more approximation. Looking at Fig. 3 we see that $E_{3\text{CM}}$ varies with $\theta_p^{\text{cm}}$, and consequently for each $\theta_p^{\text{cm}}$ one would have to solve the 3N Faddeev equation. We avoid this for this qualitative investigation and have chosen available Faddeev results at three energies which lie in the three energy bands for $0 < \theta_p^{\text{cm}} < 40^\circ$. They are $E_{3\text{CM}} = 66.7$, 100, 133 MeV, corresponding to $E_d = 140$, 200, 270 MeV, respectively.

Results

As the NN potential, we used AV18 in the Faddeev calculations. The operator $U$ for elastic $pd$ scattering has the form (see, for instance, Ref. 9) $U = PG_0^{-1} + PT$, where $G_0$, $P$ and $T$ are the free 3N propagator, permutation operators, and a partial 3N break-up operator, which is determined by a Faddeev equation. The first term, the famous nucleon exchange term, is essentially related to the PWIA mentioned in the Introduction. In order to see the importance of solving the Faddeev equation correctly and not just replacing $U$ by $PG_0^{-1}$, we compare the corresponding predictions for $A_{yy}$ and $A_{xx}$ in Figs. 5 and 6. We see large differences, especially above about 15 degrees. Trivially, $A_y$ is identically zero when we use only the real term $PG_0^{-1}$.

Fig. 3. Effective $E_{3\text{CM}}$ energies as functions of the proton scattering angle $\theta_p^{\text{cm}}$. The solid, dashed and short-dashed lines are for $E_d = 140$, 200 and 270 MeV, respectively.

Fig. 4. Effective scattering angle $\theta_{3\text{CM}}$ as a function of the proton scattering angle $\theta_p^{\text{cm}}$ for the deuteron energies, as in Fig. 3.

Fig. 5. Tensor analyzing power $A_{yy}$ in elastic pd scattering at $E_{\text{CM}} = 133$ MeV. The solid (dashed) line is calculated from $U$ ($PG_0^{-1}$). The data point for the $^4\text{He}(d,p)^4\text{He}$ reaction (270 MeV) is from Ref. 1).

Fig. 6. The same as Fig. 5 for $A_{xx}$. 
Fig. 7. The deuteron vector analyzing power $A_y$ for the $pd$ elastic scattering at (a) $E_{3\text{CM}} = 66.7$ MeV, (b) 100 MeV and (c) 133 MeV as a function of $\theta_p^{cm}$, corresponding to the $^3\text{He} (\vec{d}, p) ^4\text{He}$ reaction for $E_d = 140$, 200, 270 MeV, respectively.

Fig. 8. The deuteron vector analyzing power $A_{yy}$ for the $pd$ elastic scattering corresponding to the $^3\text{He} (\vec{d}, p) ^4\text{He}$ reaction for $E_d = 140$ MeV (solid), 200 MeV (long-dashed), and 270 MeV (short-dashed), respectively. The data point for the $^3\text{He} (\vec{d}, p) ^4\text{He}$ reaction (270 MeV) is from Ref. 1).

Fig. 9. The same as in Fig. 8 for $A_{xz}$. The data point for $^3\text{He} (\vec{d}, p) ^4\text{He}$ reaction (270 MeV) is from Ref. 1).

Fig. 10. The same as in Fig. 8 for $A_{xz}$.

The predictions of the full Faddeev solution are given in Figs. 7–10 at $E_{3\text{CM}} = 66.7$, 100 and 133 MeV, respectively. This is compared to recent data in the case of $A_y$. We see behavior qualitatively similar to the experimental data, especially for $A_y$. For the $A_y$ data, the minima shift to smaller $\theta_p^{cm}$ values with increasing energy, as in Fig. 7. Also, for $A_{yy}$, the qualitative behavior of our model is similar to that of the experimental data, especially at the highest energy. For $A_{xz}$, the shapes are again very similar. In Figs. 8 and 9 we include one data point from Ref. 1) referring
to $E_d = 270$ MeV. This shows that our absolute values are too high.

**Summary and outlook** We assumed that the reaction $^3\text{He}(d,p)^4\text{He}$ at forward angles is mainly driven by elastic $pd$ scattering. In this model, the deuteron picks up a proton from $^3\text{He}$, scatters elastically, and then combines again with the spectator nucleons to an $\alpha$ particle. Our main assumption is that the momentum of the scattered deuteron equals the spectator momentum of the deuteron in $^3\text{He}$. This leads to a high probability of forming the final $\alpha$ particle. The resulting deuteron vector and tensor analyzing powers are in astonishingly good qualitative agreement with the data. It is important here that the elastic $pd$ amplitude is a full solution of the $3N$ Faddeev equation and not only a simple PWIA expression. This model should be generalized by including a mechanism by which also a neutron from $^3\text{He}$ can be picked up. In this case one has to use the $nd$ break-up amplitude. Since the polarization of $^3\text{He}$ is carried by more than 90% by the neutron, this second mechanism is of course necessary for a description of $C_{x,x}$ and $C_{y,y}$, and thus to determine $C_{||}$. Therefore, the proton pick-up alone is too poor for those spin correlation observables. Also we neglected the momentum distributions of the proton in $^3\text{He}$ and of the deuteron in the $\alpha$ particle. As an additional improvement, the spin of the deuteron should be properly rotated for the deuteron polarization observables. Based on the promising qualitative results achieved, it appears worthwhile to improve and enrich the model along the lines mentioned.

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