Multi-objective rehabilitation of urban drainage systems under uncertainties

ABSTRACT
Urban drainage systems are subject to many drivers which can affect their performance and functioning. Typically, climate change, urbanisation and population growth along with aging of pipes may lead to uncontrollable discharges and surface flooding. So far, many researchers and practitioners concerned with optimal design and rehabilitation of urban drainage systems have applied deterministic approaches which treat input parameters as fixed values. However, due to the variety of uncertainties associated with input parameters, such approaches can easily lead to either over-dimensioning or under-dimensioning of drainage networks. The present paper deals with such issues and describes a methodology that has been developed to accommodate the effects of uncertainties into the design and rehabilitation of drainage systems. The paper presents a methodology that can take into account uncertainties from climate change, urbanisation, population growth and aging of pipes. The methodology is applied and tested on a case study of Dhaka, Bangladesh. The urban drainage network optimisation problem is posed as a multi-objective problem for which the objective functions are formulated to minimise damage costs and intervention costs. Two approaches were evaluated and the results show that both approaches are capable of identifying optimal Pareto fronts.

Key words | multi-objective optimisation, uncertainty, urban drainage rehabilitation

INTRODUCTION
Urban water infrastructure assets represent a vast investment which is made over many generations by both public and private sector organisations in order to support the life of urban communities. These assets are strictly non-stationary systems, but for maintenance purposes they are often regarded as being stationary such that they are maintained to provide certain levels of service (or standards) by the continuing replacement and refurbishment of their components. In the urban water management context, the term ‘asset’ refers to any physical object of an urban water system which enables the provision of related services (e.g., pipe, manhole, valve, pump, weir, reservoir, detention storage, channel, etc.). The failure of such assets can be very serious for the urban environment and population. Even though most of the actual failures tend to be localised, such as pipe burst, sewer collapse, flooding, or polluted discharges to receiving waters, the solution to such failures may involve an analysis of the whole integrated network. The sustainability of these extensive networks, which frequently, if not continuously, interact with each other (e.g., water distribution network and groundwater, groundwater and wastewater, wastewater and receiving surface water bodies), is an important issue for urban water managers. Therefore, one of the main concerns of urban water utilities is to secure and manage their investments in such a way that they meet appropriate standards and levels of service with an acceptable number and severity of failures while ensuring maximum return on their investments regardless of whether they are publicly or privately owned.
Furthermore, besides the sustainability of the operation of the existing networks, urban water utilities are also faced with extensions to the networks with the development of new urban areas, or the alignment of the existing networks when a significant part of the urban area is radically redeveloped. All of these issues highlight the need for a structured and full life-cycle management approach to the management of urban water assets, starting from planning and design, through construction and operation to dismantling and disposal. With such needs most urban water utilities nowadays are vitally interested in adopting the most appropriate asset management practice for their business, and in particular, the principles and techniques that aid responsible and transparent decision making. A formal approach to the asset management of urban water assets involves a number of activities which can be grouped into the following three groups: creation of assets (planning, prioritising and acquiring), operation of assets (including their maintenance and rehabilitation) and asset rationalisation (including their potential reuse, decommissioning and disposal). The overall concept of asset management has been described by a number of authors (see e.g., Marlow et al. 2007; Price & Vojinovic 2011).

An intervention into the physical world of water infrastructure usually involves making a choice between possible alternative forms of the intervention. Consequently a decision has to be made. Often the information needed to make a good decision is extensive, and yet decisions have to be made regularly on the basis of limited and uncertain data. Whereas decision makers endeavour to gather as complete a set of relevant data as possible they will often be frustrated by the cost and time taken to collect the data they want, the difficulties in obtaining accurate and reliable data, the impossibility of acquiring historic data that has not been recorded, and when they have appropriate data it is affected by the frequent structural changes that are made to the system and alter its performance. This is especially true of urban drainage networks. The fact that making optimal interventions for an urban drainage network is not a straightforward task can be further explained by the dynamic and integrated nature of an urban catchment, inadequate knowledge of asset condition and the complexities associated with multi-objective decision making frameworks. Furthermore, urban drainage systems are affected by a variety of future drivers that can lead to their poor functioning. In many parts of the world, the climate-related changes are likely to increase the magnitude and intensity of rainfall patterns. Structural collapses due to aging and insufficient capacity of systems, land cover and density of population may also lead to unmanageable discharges and surface flooding that is beyond the capacity of a drainage system (Arisz & Burrell 2006). Also, rapid population growth and urbanisation processes will cause an increase in impervious surfaces which will in effect reduce the infiltration capacity of the surface and increase the peak and volume of surface runoff. The present paper builds upon and connects the previous work and it describes an approach which can be applied to the management of urban drainage networks. In the context of the present work, an urban drainage network optimisation problem is defined as a multi-objective optimisation problem under uncertainty. The objective functions are formulated to minimise the rehabilitation cost and flood damage cost. The uncertain variables are assumed to be independent and follow normal distribution.

WORK TO DATE

Most of the research work to date uses deterministic approaches in the design of drainage systems, by assuming that all input parameters are known with certainty. This is certainly a crude assumption and not applicable to many real-life situations. Decisions based on a deterministic approach, ignoring the effects of uncertainties may result in misleading outcomes (see e.g., Nauta 2000). One of the typical traditional approaches to alleviate this problem is to apply safety factors into the analysis. However, since this approach combines future changes into a single number it may easily lead to either over-dimensioning or under-dimensioning of a drainage network. As a result, due to over-dimensioning, more substantial funds can be unnecessarily spent on the intervention works. On the other hand, under-dimensioning of drainage systems may lead to more frequent combined sewer overflows (in case of combined systems) or flood-related problems (in case of separate stormwater systems). This situation highlights the need to advance the research in identifying, quantifying
and incorporating effects of various drivers into the design of urban drainage systems so as to make the systems more adaptable, reliable and operable in a changing environment.

Further to the above, modelling and optimisation technologies have proved to be invaluable means for dealing with various asset management problems. With instantiated models and optimisation techniques, it is possible to explore the performance of drainage networks and evaluate the effects of different intervention measures in response to various drivers. Such means enable the development of a more comprehensive approach to estimating the risks involved and addressing the uncertainties that may arise due to urbanisation and climate change by simulating a number of alternative scenarios. Several authors have made efforts in this direction (see e.g., Savic 2005; Vojinovic et al. 2006; Barreto et al. 2010). However, there are limitations in certain optimisation frameworks. The uncertainties associated with the system load and corresponding hydraulic and hydrological processes can compromise the designed systems obtained by optimisation with regard to a certain criteria (Guo et al. 2008). Therefore, it is necessary to take into account the effects of uncertainties in the optimal design of urban drainage networks.

In order to enable a sewer system to function in the long term and to make it adaptable to future requirements, it is important to develop a methodology that can identify robust solutions. Robustness can be referred to as a system’s ability to keep the same level of performance under variability of assumed and actual values (Savic 2005). If a system is less responsive to alteration in input parameters, then it is said to be robust (Uber et al. 1991). Incorporation of uncertainties into optimisation of urban drainage modelling work could yield more reliable and robust results.

The overall objective of the present work is to develop and test a multi-objective optimisation methodology that can be used for design and rehabilitation of urban drainage networks under uncertain scenarios derived from urbanisation, population growth, climate change and aging of pipes. The methodology aims to contribute towards a sound and more reliable design which can minimise the problems encountered in deterministic optimisation methodologies.

The proposed optimisation approach can be defined as a modelling methodology, combined with computational tools, to carry out optimisation problems under uncertainties (Ben-Tal & Nemirovski 2002). A number of studies have been carried out in relation to robust optimisation of water distribution systems under uncertain demand and pipe roughness. Savic (2005) developed a robust optimisation methodology (using single- and multi-objective optimisation concepts) for urban water infrastructure design/rehabilitation. In Kapelan et al. (2005), the problem of multi-objective design of water distribution systems under uncertainties was addressed. In the same work, the Latin Hypercube Sampling technique was combined with the NSGAII (Non-dominated Sorting Genetic Algorithm II) optimisation technique (Deb et al. 2002). The objectives of that work were to minimise the cost and to maximise robustness which was defined as the probability of satisfying the minimum pressure requirement at each node simultaneously. The methodology applied succeeded to generate solutions with a high level of robustness (i.e., 90% and higher).

Sun et al. (2011) developed a computationally efficient robust optimisation methodology for a water distribution system design problem under uncertainty and applied it to the New York City tunnel case study. The two objectives used in that study were to minimise the total cost and to maximise the robustness which is defined as the capacity of water distribution systems to provide sufficient water for customers in spite of demand variations. To achieve this objective, the study uses NSGAII as a multi-objective optimiser. The methodology is free of sampling techniques and the robustness is addressed through analytical formula using a parameter $\theta$ which has a direct relationship with robustness. Sun et al. (2011) concluded that the method used was computationally efficient, which has a particular importance in the case of large water distribution systems and when a sampling-based technique for robustness evaluation nested in the optimisation is impractical.

Similarly, a robust optimisation methodology in ground water remediation design was developed by Chan & Culver (2005). In Chan & Culver (2005), the target of the robust optimisation problem was to find the cost-effective remediation design by taking into account uncertainty of hydraulic conductivity values under constraints such as contaminant concentration, hydraulic heads and the well extraction rates. The same study considered optimal locations and extraction rates of pumping wells as decision variables...
and the methodology proposed was applied in two heterogeneous cases of different heterogeneity levels. The results have shown that the design obtained using robust optimisation methodology has higher costs when compared to the design obtained by the deterministic optimisation technique.


**UNCERTAINTY**

Two of the greatest problems faced by decision makers are the incompleteness in the relevant data set and uncertainty in the data that is available. Incompleteness of the data can be overcome by acquiring more data, but where this applies to historic data the only possible way left open is to use a simulation model to replicate what happened historically, provided the relevant input data is given. The treatment of uncertainty in the data is another major issue affecting a decision. In fact, there is uncertainty not only in the data but in the tools used to analyse and make use of the data. Decision making itself is fraught with uncertainty.

Uncertainty can be represented in terms of the two common concepts of probability and possibility; see Maskey et al. (2004). Traditionally, uncertainty has been represented by probability, although since the 1970s possibility theory has been used increasingly as an alternative. The theory of possibility is similar to, yet conceptually different from, the theory of probability. Probability depends on the frequency of occurrence of an event, while possibility focuses on the meaning of an event. It does not imply that a high degree of possibility leads to a high degree of probability. If, however, an event is not possible, it is also improbable. Therefore, possibility can be viewed as an upper bound for probability. This weak connection between probability and possibility is defined by Zadeh (1978) as the possibility/probability consistency principle.

In probability theory, all uncertainties are assumed to be random and therefore they can be represented by probability distributions of events. Uncertainty in possibility theory however, is quantified in terms of imprecision and vagueness, and is represented by a possibility distribution. Possibility is a fuzzy measure, which is a function taking a value between 0 and 1. This indicates the degree of evidence or belief that a certain element \( x \) belongs to a set (Zadeh 1978; Dubois & Prade 1988).

Uncertainty can be characterised as either quantitative or qualitative. Quantitative uncertainty is expressed using numerical values. Typically, these numerical values have a meaning from a probabilistic point of view. For example, ‘the probability for the water level at the road junction to exceed a value of 24.3 m during a 10 year rainfall event is 4%’. In the quantitative approach, the laws that describe the physical phenomena are assumed to be known. However, the value of some of the parameters, for example, surface runoff coefficients or pipe roughness, may not be known precisely. The uncertain parameters and variables are treated as random variables, that is, variables which have known probabilities to exceed (or not) certain values. The random variables are characterised by a probability density function and its moments (mean, variance, etc.).

Qualitative uncertainty can be expressed using qualitative terms. It is used when the sources of uncertainty cannot be estimated precisely using numerical values. For example, ‘the water level is very likely to be high’. In this statement, the probability for the water level to be high is not represented in numerical terms, but using what is called a linguistic variable. The most widely used method to deal with qualitative uncertainty is fuzzy set theory (a case of possibility theory), in which the linguistic variables are represented by a membership function. The membership function is the representation of the degree of belief in the statement.

Van Gelder (2000) divides the nature of uncertainty as: inherent uncertainty and epistemological uncertainty. Inherent uncertainty represents natural variability in time and/or in space and therefore it is referred to as the randomness in samples. For example, predicting maximum rain intensity that will occur next year is a good example for inherent uncertainty, it has inherent uncertainties in time (temporal variation of rainfall intensity) and inherent uncertainties in space (fluctuation in local topographies). It is
broadly understood that such uncertainty is irreducible due to its intrinsic nature. On the other hand, the epistemological uncertainty is a manifestation of imperfection or lack of knowledge of all the causes and effects in physical systems and/or lack of sufficient data. Model uncertainty (due to lack of understanding of the physics) and statistical uncertainty (due to insufficient data) are the two main types of epistemic uncertainties. Epistemic uncertainties can be reduced when knowledge increases and sufficient data is available. The more data available, the smaller the epistemic uncertainty is. Data collection, theoretical research and expert judgment may reduce the epistemological uncertainty by increasing the information level on the area under consideration (Van Vuren 2005). Example of epistemic uncertainty is rainfall-runoff processes or the sewer-related processes.

A number of authors have used different approaches to address uncertainties in urban drainage modelling (see e.g., Clemens 2001; Vojinovic et al. 2003; Aronica et al. 2005; Hansen et al. 2005; Vojinovic 2007; Thorndahl 2008). Even though, there are so many sources of uncertainties (both inherent uncertainty and epistemological) in the design/rehabilitation of urban drainage systems, the present work deals with uncertainties related to climate change, urbanisation, population growth and aging of pipes. Since these sources of uncertainties have an effect on the magnitude and intensity of rainfall events, distribution and areal coverage of paved surfaces they also have direct impact on flow characteristics inside the drainage system. To reflect the variability in design parameters resulting from such sources of uncertainties (i.e., effects of climate change on rainfall characteristics, urbanisation/population growth and aging of pipes), a range of possible values are used in the present work. The effects of climate change, urbanisation/population growth and aging of pipes were accounted by varying rainfall series, percentage of impervious areas in the subcatchment data and roughness values of pipes. Rainfall uncertainty was considered as intrinsic and uncertainties related to per cent impervious area and pipe roughnesses were considered epistemological. The uncertain variables are assumed to have normal probability distribution of assumed mean value and standard deviations estimated as a certain percentage of the mean values.

METHODOLOGY

Problem formulation

As introduced earlier, an urban drainage network optimisation problem is posed here as a multi-objective optimisation problem and sources of uncertainty are: effects of climate change (i.e., it is reflected by variation of rainfall data), urbanisation and population growth (i.e., it is reflected by variation of impervious areas percentage) and aging of pipes (i.e., it is reflected by variation of roughness values of pipes). The numerical modelling work involved the use of one-dimensional (1D) and two-dimensional (2D) models for simulation of flows through pipes and across the urban floodplain. The objective functions are formulated in such a way to minimise both the rehabilitation cost and the flood damage cost. The decision variable used is pipe diameter and the optimisation process searches for an optimal set of pipe diameters that need to be allocated to those pipes with inadequate flow capacities. Other decision variables such as the length of pipes, slope of the pipes, the layout or the implementation of real time control can still be used with this optimisation framework depending on the particularities of each case. The optimisation procedure was done to improve the performance of the system and to satisfy the above two objectives (i.e., to minimise the rehabilitation cost and the flood damage cost). The multi-objective optimisation problem ends with the creation of a Pareto front with non-dominated solutions.

Hydraulic modelling

A coupled 1D/2D hydraulic model was used in the present work. The hydrological rainfall-runoff process and routing of flows in drainage pipes are simulated using the 1D sewer network model EPA SWMM5 (Rossman et al. 2005). SWMM5 solves the conservation of mass and momentum equations (the Saint Venant equations) that govern the unsteady flow of water through a drainage network of channels and pipes by converting the equations into an explicit set of finite difference equations and using a method of successive approximations under relaxation to solve them. When the capacity of the pipe network is exceeded, excess flow spills into the two-dimensional model domain from
the manholes and is routed using the non-inertia 2D overland flow model.

The SWMM source code was modified so that the surcharge in the sewer network is represented in terms of hydraulic head rather than overflow volumes. The interacting discharges are determined by the weir or orifice equations by taking into account the hydraulic head at manholes and the aboveground surface for every time step of the sewer network model and treated as point sinks or sources, correspondingly, in the 2D model within the same time interval. A 2D non-inertia model developed and described in Seyoum et al. (2012) was used. The peak water depths computed by the 2D model were used for calculation of flood damage.

The non-inertia 2D model represents the urban topography by the ground elevations at the centres and boundaries of cells on a rectangular Cartesian grid and determines the water levels at the cell centres and the discharges (velocities) at the cell boundaries, Figure 1. The alternating direction implicit finite difference procedure is used to solve the governing equations. The schematisation of the topography coupled with the way the governing equations are solved allows for a good representation of small-scale topographical elements including defined flow paths such as road networks and channels (see also Seyoum et al. 2012; Vojinovic et al. 2015).

The bidirectional interacting discharge is calculated according to the water level difference between sewer network nodes and overland surface. The upstream and downstream levels for determining discharge are defined as

$$h_U = \max\{h_{mh}, h_{2D}\} \quad \text{and} \quad h_D = \min\{h_{mh}, h_{2D}\},$$

respectively where $h_{mh}$ is the hydraulic head [m] at manhole and $h_{2D}$ is the water surface elevation [m] on the 2D grid.

The crest elevation $Z_{crest}$ is assumed to be equal to the cell elevation where the manhole is located.

The interacting discharges are calculated using equations of free weir, submerged weir and orifice depending on the upstream and downstream water levels as shown in Figures 2 and 3. The free weir equation is adopted when the crest elevation $Z_{crest}$ is between the values of the upstream water level $h_U$ and the downstream water level $h_D$, as shown in Equation (1). The discharge is calculated by using the weir equation below:

$$Q = \frac{\text{sign}[h_{mh} - h_{2D}]C_w W \sqrt{2g (h_{2D} - Z_{crest})^{3/2}}}{\text{C}_{wW} \sqrt{2g h_{U}/C_{0}h_{crest}}}$$

If both the upstream and downstream water levels are above the crest elevation of the manhole, then either the submerged weir or the orifice equation is used to calculate the interacting discharge. If the upstream water depth above the manhole crest is less than the area of the manhole divided by the weir crest width ($h_U - Z_{crest}) < A_{mh}/W$ then the submerged weir equation (Equation (2)) is used, or otherwise the manhole is considered fully submerged and the orifice equation (Equation (3)) is used

$$Q = \frac{\text{sign}[h_{mh} - h_{2D}]C_w W \sqrt{2g(h_U - h_{crest})(h_U - h_D)^{1/2}}}{\text{C}_{wW} \sqrt{2g (h_U - h_D)^{1/2}}}$$

$$Q = \frac{\text{sign}[h_{mh} - h_{2D}]C_w A_{mh} \sqrt{2g (h_U - h_D)^{1/2}}}{\text{C}_{wW} \sqrt{2g (h_U - h_D)^{1/2}}}$$
where, $Q$ is the interacting discharge [m$^3$/s], whose positive value meant surcharge flow from sewer toward overland and negative value meant drainage flow from surface into sewer; $C_o$ is the orifice discharge coefficient; $C_w$ is the weir discharge coefficient and $W$ is the weir crest width.

**Damage costs**

The assessment of damage in the present work has a diverse combination of water depth and land uses which have been broadly categorised as residential, commercial, government, educational & religious, business, non-governmental and industrial. Damage cost calculations were carried out for each grid based on the land use type and the maximum flood depth. The depth damage curves developed in the previous work of Ahmed (2008) and Matungulu (2010) were used in formulating damage cost equations for each land use. In this work, 10 land use types including open space and five water depth ranges were defined. Based on this a total of 50 damage functions were formulated and used in the calculation of potential damage costs per grid cell of the 2D model. The resolution of the 2D model was 10 m and the equation used for computation of damage at each grid is formulated as follows:

$$\text{Damage Cost}_{[i, j]} = (\alpha + \hat{\alpha}+) \cdot \text{MaxWdpth}_{[i, j]}$$  \hspace{1cm} (4)

where, $\alpha$ is a slope of the curve based on the value of the land use, the higher the land use the higher the value; $\beta$ is an intercept based on land use and the water depth, if the water depth is zero, this value is zero; $\text{MaxWdpth}_{[i, j]}$ is the maximum flood depth at cells $[i, j]$.

**Intervention cost**

The intervention cost ($C_T$) is calculated based on the cost per unit length of the new pipe multiplied by its length

$$C_T = L_i \sum_{i=1}^{n} C(D_i)$$  \hspace{1cm} (5)

where $n$ is the number of pipes in the network; $C(D_i)$ is the cost per unit length of the pipe (i) with diameter $D_i$ and length $L_i$.

The database of pipe unit costs is comprised of the pipe material cost and the cost of labour needed for excavation and reinstatement.

**Robustness**

The robustness of each selected solution is calculated based on the following assumptions. The robustness level of the original pipe network with worst case scenario (maximum value of all the uncertain variables) is defined as zero and a solution with zero damage cost and with worst case scenario is defined as having 100% robustness level. The robustness levels of the solutions obtained by the optimisation process lie between the above two robustness levels and are calculated by interpolation of the damage costs and robustness levels.

**Optimisation framework**

A Latin Hypercube Sampling (LHS) scheme that connects to the hydraulic model and the optimiser was written in C++ code and it was used in the present work for the purpose of sampling statistical distributions and uncertainty analyses. Each uncertain variable was assumed to have normal probability distribution. In terms of the rainfall data and percentage of impervious areas, the mean is assumed to be 20% higher than the current values and to have standard deviation of 10 and 30% of mean values for each time series rainfall data and for each subcatchment respectively. In terms of the roughness values (which reflect the condition of pipes), the mean values were assumed to be 40% higher than the minimum value within the range. The standard deviations were assumed to be 10 and 30% of the mean value. Also, the relationships between uncertain variables were assumed to be independent or uncorrelated (i.e., the effects of one uncertain variable could not affect the other).

Two intermediate C++ routines that connect the optimiser (i.e., NSGAII algorithm), the hydraulic model (i.e., 1D/2D model) and the LHS technique were written. The first routine was used to run the hydraulic model and to compute the two objective functions (damage cost and rehabilitation cost) which were then passed to the optimiser to search for best candidates from a range of alternatives. The second routine connects the sampling technique, hydraulic model and the
optimiser in such a way that it updates the input data for the hydraulic model by interpreting the randomly generated population and by providing the generated samples of uncertain variables from the LHS technique. An illustration of the robust optimisation framework is given in Figure 4.

In the present work, two approaches were adopted to handle the robust optimisation procedure and multiple runs were performed in each approach with different number of LH samples \((N_s = 5, N_s = 10 \text{ and } N_s = 20)\) and different standard deviations \((STDV = 10\% \text{ of mean and } 30\% \text{ of mean})\) to test the performance of the multi-objective optimisation methodology with increased uncertainty values. Moreover, a number of trial runs were carried out to find an optimal Pareto front. Also, by taking into consideration the high computational costs involved, the population number of 60 and generation number of 40 were used in both approaches.

**First approach**

The first approach of the optimisation methodology (i.e., Approach 1) incorporates uncertainties in the evaluation of objective function through multiple realisations of uncertainties in each generation. Each chromosome in a population was evaluated for each number of LH samples from each sub-interval. This was done by running the hydraulic model for a number of samples \((N_s)\) for each chromosome and calculating the values of objective functions of each chromosome and for each LH sample. Based on the values of the two objective functions the optimiser will keep the best population that dominates the search space (i.e., Pareto front) and produce new populations for next generations. After processing of generated populations for the next run and calling the LH technique for preparation of input data (uncertainty quantification), the code updates the hydraulic model with new pipe diameters and new uncertainty sets and the same procedure continues for a number of generations. This approach is referred to as a multiple realisation method and it is illustrated in Figure 5.

**Second approach**

The second approach (i.e., Approach 2) of the multi-objective optimisation methodology was adopted from Savic (2005). This approach is based upon modification of the NSGAII

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**Figure 4 | Illustration of the robust optimisation framework.**
algorithm and it incorporates uncertainties into the optimisation process through single realisation of the uncertainty sets in each generation. The following procedure was applied:

1. Assigning the minimum chromosome age \((MA)\) as zero, each chromosome in a population was evaluated with one realisation of uncertainty sets and objective values were calculated. After that, using the fast non-dominated sorting algorithm the non-dominated Pareto front was identified and recorded for further re-evaluation of objective functions.

2. By increasing the age of previously generated chromosomes by one, objective values were calculated as average of present and past values over the chromosomes’ age.

3. Children population was created from parent population using genetic operators and zero \(MA\) was assigned for each new chromosome and objective functions were evaluated.

4. By combining the parent and children population the non-dominated Pareto front was identified and kept recorded.

5. The age was increased by one and the objective functions of each chromosome in the previously recorded non-dominated Pareto front (i.e., the first step) were recalculated. This front was then combined with the fronts identified in the fourth step and new non-dominated Pareto fronts were identified using the fast non-dominated sorting algorithm. For the chromosomes in the current best Pareto front to dominate the chromosomes in the previously identified best Pareto front, their chromosome age should be equal to or greater than some specified \(MA\). Then the third and fourth steps were repeated until the pre-defined number of generations was reached and the best value of \(MA\), which is a criterion to select chromosomes that can survive for long generations, was determined on a case by case basis. An optimum value of minimum chromosome age 30 was used in the present work. The search procedure used in this approach is given in Figure 6.

**CASE STUDY AREA**

The case study area used in the present work is Segunbagicha area in Dhaka, Bangladesh, Figure 7. The city has been experiencing frequent flood-related problems for
many years. The Segunbagicha catchment has a drainage area of 8.3 square kilometres and includes the most important business and government office areas of Dhaka City. Floods caused by intense local rainfall occur in the built-up areas of the city several times a year.

The drainage network consists of 88 sewer links with a total length of 13,635 m. Of these 75 links are circular pipes with a total length of 11,308 m and 13 links are box culverts with a total length of 2,327 m. The circular sewer pipe diameters range from 450 to 5,500 mm and the box culvert sizes are between 2.5 by 2 m and 5.5 by 4.3 m. The slopes of the pipes range from 0 to 10%. The underground drainage system consists of circular pipes, box culverts, basins and pumps. Stormwater from sub-catchments is drained by sewer pipes to two basins from which sewage is pumped to Tongi Khal river system. Figure 8 shows the schematisation of network used for the simulation in SWMM.

The resolution of the available digital terrain model (DTM) was 10 m and it was used for setting up the 2D model, Figure 9. An event of 100 year return period 1 hour rainfall is used for simulation of the drainage network in the case study area.

**RESULTS**

**Results obtained from Approach 1**

The results obtained from Approach 1 with $N_s = 5$, and $N_s = 10$ samples and standard deviations of 10 and 30% of
the mean values of the distribution of the uncertainty sets are given in Figure 10.

The solutions obtained from Approach 1 ranged from $0.54$ to $17.30$ million in damage costs and $19.71$ to $54.80$ million in intervention costs. From the Pareto optimal fronts five solutions were selected from each case for comparison purposes. Solution 1 corresponds to a very high damage cost and minimum intervention cost, whereas solution 2 corresponds to a very high intervention cost and minimum damage cost. Solution 2 would be the preferred one from the flood damage perspective and solution 1 would be the preferred one from the intervention cost perspective. Solutions 3 and 4 are compromised solutions and they are located in the central part of the Pareto front. Solution 5 is the solution that reduces the existing damage cost with minimum rehabilitation. For the case where \( N_s = 5 \) and \( STDV = 30\% \) of mean (see also Figure 10(c)) the damage cost, intervention cost and robustness level of the five selected points in the Pareto front are presented in Table 1.

To demonstrate the results obtained from different standard deviations of distributions of uncertain variables (\( STDV = 10\% \) of mean and 30\% of mean) and different number of samples of distribution of each uncertain variable (\( N_s = 5, N_s = 10 \) and \( N_s = 20 \)) comparisons were made and given in Figure 11 and Figure 12, respectively.

Standard deviations of distribution of uncertainty sets (10 and 30\% of mean values) produced different Pareto fronts of robustly optimised solutions. Pareto optimal fronts with standard deviation of 30\% of mean value appeared to be higher than the Pareto optimal fronts with standard deviation of 10\% from the mean values (see Figure 15). This indicates that when the deviation of the uncertain values from the mean value increases the damage cost and the intervention cost increase. This can be due to the fact that a large standard deviation increases the possibility of large values of uncertain variables to be taken into account. And from sensitivity analysis results it can be seen that an increase in the values of uncertain variables results in an increase in the damage cost. As a result, the likelihood of selecting cost-effective pipe diameters which can convey additional flows due to uncertainties to a certain threshold level increases. And this leads to an increase in rehabilitation cost.

As can be observed from Figure 12, robustly optimised solutions obtained from \( N_s = 20 \) samples have a large distribution of solutions over the Pareto front and hence the coverage and extension of the Pareto optimal front will increase. It is obvious that when the number of LH samples increases the possibility of covering a large spectrum of distribution of uncertainty sets will also increase. This is due to the fact that in the case of Approach 1 each LH sample of distribution of uncertainty sets has a contribution in the objective value evaluations. Also, as the number of samples increases, the possibility of objective function evaluations with a large portion of the uncertainty sets may also increase.

Results obtained from Approach 2

In Approach 2, in addition to fitness values of chromosomes, the required minimum age of chromosomes (i.e., the number of generations in which the chromosome stayed alive) was used as the criterion in selecting the non-dominated solutions. The same genetic algorithm parameter
sets were used as in Approach 1 and an additional parameter referred to as ‘minimum chromosome ages (MA)’ was introduced. Multiple runs were performed to find the optimal values of MA for the case study data set. Thus, the optimal value of minimum chromosome age for this case study was 30. Furthermore, for the optimisation process the values of $N_s = 5$ and $N_s = 10$ samples of uncertain variables with standard deviations of 10 and 30% of mean values of the uncertainty sets were used. The non-dominated solutions obtained from Approach 2 are shown in Figure 13 below.

The solutions obtained from Approach 2 ranged from $0.68 to $18.29 million in damage costs and $19.46 to $51.94 million in intervention costs, Figure 13. Similar to the case of Approach 1, five solutions were selected from the Pareto optimal front based on the criteria used in Approach 1. For example in the case of $N_s = 5$, $MA = 30$ & $STDV = 30\%$ of mean, the damage cost, intervention cost and robustness level of these selected points are represented in Table 2 below.

The comparison of Pareto optimal fronts for different standard deviations ($STDV = 10\%$ of mean and $30\%$ of mean) and different number of LH samples ($N_s = 5$ and $N_s = 10$) are shown in Figure 14 and Figure 15.
From Figure 14, we can observe that variations of solutions between $STDV$ of 10 and 30% of the mean values are seen only in a certain part of the Pareto fronts (central part). However, in both limbs (i.e., lower and upper limbs) of the Pareto optimal fronts, solutions from both cases overlay together. This could be explained by the effects of changes in standard deviations in Approach 2 which are addressed through one realisation rather than multiple realisations like in the case of Approach 1. Also, since only one LH sample contributes to objective value evaluations in each generation there is no significant variation between the Pareto optimal fronts obtained from 5 and 10 numbers of LH samples.

**Comparison of results obtained from different approaches**

Both approaches of multi-objective optimisation identified solutions as Pareto optimal fronts. Solutions obtained from Approach 2 cover a larger spectrum of the solution domain than the solutions obtained from Approach 1. Also, the discrepancy between both approaches decreases...
when the number of LH samples increases. This could be due to different mechanisms which are used to incorporate uncertainties into the optimisation process in each approach and different ways of evaluating objective functions. In the case of Approach 1, objective function evaluations were carried out as an average of objective values for each LH sample of uncertainty sets. When the number of LH samples increases the coverage range of the uncertainty set also increases and Pareto optimal fronts having large distribution of the solution domain could be obtained. Moreover, comparison of values given in Table 1 and Table 2 show that solutions obtained from Approach 1 are more robust than those obtained from Approach 2. Even solutions having almost the same intervention cost in both cases gave more robust solutions in the case of Approach 1. This could be due to the fact that having exposures for a large portion of distributions of uncertainty sets in the optimisation process increases the possibility of selecting robustly optimised pipe diameters that can perform well over a range of uncertainty set realisations. This approach also has disadvantages with respect to the computational time required to evaluate objective functions for each chromosome. For example, in the case of 20 LH samples, 20 objective value evaluations were carried out for one chromosome in a population. Therefore, for 60 populations there were 1,200 objective value evaluations were carried out for one chromosome in a population.

This implies that for the same number of populations two generations of Approach 1 will result in equal objective function evaluations to that of 40 generations of Approach 2. More generally, for any numbers of $N_s$ having the same...
number of populations and generations there is the same number of objective function evaluations in the case of Approach 2. However, in the case of Approach 1 the number of objective function evaluations is dependent on the number of LH samples used. So, the discrepancy of number of objective function evaluations for Approach 1 and Approach 2 becomes larger when the number of LH samples increases.

In order to illustrate the difference that can be obtained between different approaches, the results obtained from the deterministic approach were also generated and plotted against the results obtained from the optimisation approaches under uncertainty (i.e., Approaches 1 and 2). For simulations using a deterministic approach, the percentage of impervious surfaces, rainfall and roughness values were kept fixed and their mean values were used. In terms of the NSGAII parameter set, the same parameter sets used for Approaches 1 and 2 were also used for the deterministic approach. The Pareto optimal fronts obtained from the deterministic optimisation and optimisation under uncertainty ($N_s = 10 \& STDV = 50\%$ of mean) are given in Figure 16.

CONCLUSIONS

Uncertainties associated with population growth, urbanisation, climate change effects and aging of pipes are continuously bringing an increasing challenge to urban water managers. Further to that, there is a need to develop the tools and techniques that can enable development and implementation of more effective and robust solutions. In the present work, an optimisation framework has been developed and implemented in the C++ code to carry out the rehabilitation of urban drainage networks under...
uncertainties. The tool developed is flexible and it can integrate several objectives and levels of performance or costs, allowing their optimisation in a multi-criteria setting. It allows the integration of hydraulic modelling software (1D/2D models) with cost estimation tools and a multi-objective optimisation algorithm for minimising any numerical objective and particularly it has been used to minimise damage costs and intervention costs. Within the framework developed, two approaches of multi-objective optimisation considering uncertainty were evaluated. The first approach (i.e., Approach 1) accounts for uncertainties in the objective function evaluation process and the second approach (i.e., Approach 2) accounts for uncertainties in the optimisation process. In both approaches, the hydraulic model, the sampling technique and the optimiser were connected together to enable execution of the optimisation procedure. Both approaches identified robustly optimised solutions in terms of the damage cost and intervention cost. The two

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Damage cost (million dollar)</th>
<th>Intervention cost (million dollar)</th>
<th>Level of robustness (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solution 1</td>
<td>17.91</td>
<td>20.69</td>
<td>16.16</td>
</tr>
<tr>
<td>Solution 2</td>
<td>0.61</td>
<td>49.58</td>
<td>79.18</td>
</tr>
<tr>
<td>Solution 3</td>
<td>5.73</td>
<td>24.15</td>
<td>18.61</td>
</tr>
<tr>
<td>Solution 4</td>
<td>4.94</td>
<td>25.57</td>
<td>34.36</td>
</tr>
<tr>
<td>Solution 5</td>
<td>9.09</td>
<td>21.17</td>
<td>18.15</td>
</tr>
</tbody>
</table>

Figure 14 | Pareto optimal fronts obtained from different STDV.

Figure 15 | Pareto optimal fronts for different number of LH samples.
optimisation approaches have produced almost the same results. However, slightly more robust solutions were obtained from Approach 1 when the large number of LH samples was applied.

The comparison made between deterministic and optimisation approaches (Approaches 1 and 2) under uncertainties shows that the difference can be substantial and it may lead to under-design of the system. As anticipated, the optimisation methodology considering uncertainty identified design solutions with larger intervention and damage costs. The larger intervention costs are due to additional capacity of the drainage system required to convey additional flows due to uncertainties. To enhance the present approach it is recommended that future research is conducted to include an objective function that reflects the likelihood of the failures and their severity, since this is an important issue regarding the operation of any drainage system. From the work to date it can be concluded that the effects of uncertainties should be incorporated in the optimal design/rehabilitation of drainage systems in order to achieve more robust solutions.

REFERENCES


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