Single-station decomposition of seismograms for subevent time histories

Massimo Di Bona and John Boatwright

1 Instituto Nazionale di Geofisica 00161, Rome, Italy
2 US Geological Survey, 345 Middlefield Rd, Menlo Park, CA 94025, USA

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SUMMARY
We have adapted an iterative least-squares inversion with positivity constraints to the problem of deconvolving the recordings of a small earthquake from the recordings of a large composite earthquake. The inversion uses an $F$ test to minimize the number of non-zero amplitudes or subevents in the solution set, by requiring each added subevent to reduce the variance significantly. We apply this inversion to the analysis of strong motion recordings of a set of moderate-sized (4.0 ≤ $M_L$ ≤ 6.4) aftershocks of the 1976 Friuli earthquake. The earthquake pairs are selected by comparing $S$ trigger times and polarization diagrams. The deconvolved time series are interpreted as the time history of slip events or stress release in the composite earthquake, depending on whether the recordings of the small earthquake are unfiltered or filtered to simulate the waveforms radiated by subevents. Deconvolving with the filtered recordings reduces the variance more rapidly, as a function of the number of subevents, than deconvolving with the unfiltered recordings. The deconvolved time series for three of the four earthquakes analysed indicate that the rupture processes resemble the failure of asperities where the initial subevent has the largest stress drop and the later subevents have longer durations and weaker stress drops.

Key words: complex earthquakes, Green's functions, seismogram decomposition.

INTRODUCTION
Over the past 10 years, many seismological papers have considered the problem of modelling the time history of a complex earthquake as the sum of the time histories of a set of subevents. Much of this modelling has been motivated by Hartzell's (1978) insight that the recordings of small earthquakes contain the propagation characteristics necessary for modelling large earthquakes, and therefore yield empirical Green's functions which are more appropriate than the synthetic seismograms generated by modelling the wave propagation through an inadequately known velocity structure. Often, however, these techniques have been applied using synthetic Green's functions; because of its relatively long-period character and the difficulty of recording small earthquakes at teleseismic distances, most teleseismic data are analysed using synthetic Green's functions.

A range of different techniques, including both time-domain and frequency-domain deconvolutions, have been used to analyse this problem. In particular, two of the most widely used time domain techniques, that is, the methods of Kikuchi & Kanamori (1982) and Hartzell & Heaton (1985), which were developed to analyse teleseismic data, are similar to the technique which we propose and test in this paper. We will introduce our technique by first discussing the different assumptions and characteristics of these two methods.

The method of Kikuchi & Kanamori analyses seismograms iteratively, choosing the delay for each subevent by finding the maximum of the cross-correlation of the Green's function with the residual of the fitted data, that is, the data minus the previously fitted subevents. The amplitude of each chosen subevent is determined from the value of the cross-correlation; if desired, only subevents with positive amplitudes can be chosen. Kikuchi & Kanamori's method does not refit the amplitudes of the entire set of subevents when each new subevent is added. The analyst's judgement, rather than a statistical test, is used to determine the appropriate number of subevents.

The method of Hartzell & Heaton also fits the complex waveform iteratively, that is, by adding individual subevents. The delay for each subevent is obtained by finding the maximum cross-correlation of the Green's
function with the residual of the fitted data. When each subevent is added, the inversion refits the amplitudes for the entire set of subevents using an algorithm from Lawson & Hanson (1974) which solves the least-squares problem with inequality constraints. The total number of subevents is determined as the set of all subevents with positive amplitudes.

In analysing teleseismic data, Hartzell & Heaton (1985) found it necessary to damp their inversions: that is, they minimized the total length of the parameter vector to obtain a stable solution. The damping parameter is selected by trial and error. Bezeghoud & Madariaga (personal communication 1988) indicate that the method of Hartzell & Heaton produces subevent time histories which are generally smoother than the time histories obtained using the method of Kikuchi & Kanamori. The divergence of the results from the two methods indicates only that the inversions are poorly conditioned on teleseismic data, owing to the indeterminacy of the long-period components of the body waves radiated by shallow earthquakes.

The inversion technique which we present in this paper contains elements from both of the methods described above. It resembles both methods in that subevents are added iteratively to the solution set of subevents. As each new subevent is added, the amplitudes for the entire set of subevents are recalculated, following Lawson & Hanson (1974). Instead of including all the subevents with positive amplitudes, however, the iteration continues until the error, divided by the number of degrees of freedom, reaches a minimum. The total number of subevents chosen by this procedure is generally fewer than that used by the method of Hartzell & Heaton.

We apply this time-domain inversion to the analysis of accelerograms recorded at Gemona del Friuli, Italy. Following the Friuli mainshock of 1976 May 6, the Istituto Nazionale di Geofisica installed a SMA-1 accelerometer at Gemona from June to September 1976. This instrument recorded a large number of aftershocks at relatively small epicentral distances, from 2 to 20 km. Rovelli et al. (1991) have analysed these accelerograms in detail, selecting 24 accelerograms triggered by P-waves for the spectral analysis of the S-waves. An interesting characteristic of this data set is the wide range in local magnitude of the earthquakes: from roughly $M_L \approx 4$ to $M_L > 6$ (Rovelli et al. 1991). The wide range in magnitude of the earthquakes recorded at this station makes the Gemona data set particularly suitable for studying large earthquakes by using recordings of small earthquakes as empirical Green’s functions (Hartzell 1978).

We modify the analysis of Hartzell (1978) and many others, however, by filtering the time histories of the small earthquakes which are used as Green’s functions. Boatwright (1988) demonstrates that the asperity rupture process is more appropriate than the crack rupture process as a model for a subevent within a composite rupture. Unfortunately, small earthquakes or aftershocks whose recordings might be used as Green’s functions generally exhibit crack-like rupture processes. In order to obtain an equivalent subevent waveform, we use a simple filtering method (Boatwright 1988). The filtering operation is causal and minimum phase; its effect is to amplify the low frequencies without altering the high frequencies.

The filtering to obtain subevent waveforms is particularly appropriate when it is combined with an inversion technique which minimizes the number of subevents in the solution set. As we demonstrate by comparing their respective variance reductions, Green’s functions with filtered waveforms are more appropriate than Green’s functions with unfiltered subevents: that is, the variance reduction per subevent is greater and the maximum variance reduction is obtained for a smaller number of subevents. Our deconvolutional results indicate that the inversion technique proposed in this paper can reliably resolve the time history of either the stress release or the moment release in a complex earthquake.

**MOMENT RATE FUNCTIONS AND STRESS RATE FUNCTIONS**

Generalizing the decomposition proposed by Hartzell (1978) to model multiple slip events yields the kinematic decomposition

$$u_k(x,t) = \sum_i^{\infty} \sum_j^{\infty} \Delta u_{ij} g_k(t - T_c(\xi_i, x) + T_c(\xi_j, x) - T(\xi_i) - \Delta t)$$

(1)

where $\Delta u_{ij}$ is the slip in the small earthquake, which is assumed to occur within one time step $\Delta t$ and one square rupture grid $\Delta \xi^2$. $T$ is the function that converts the time step to the wave propagation time $T_c(\xi, x)$ representing the differential traveltime for the wavetype $C = S$ or $P$ radiated by the small earthquake at $\xi$ and the grid point $i$ at $x$. Because the grid point continues for more than one time step in the composite earthquake, the summation for the radiation from the grid point $i$ runs over $n + 1$ time steps beginning at the rupture time $T(\xi_i)$. Note that this decomposition is only appropriate for frequencies less than the corner frequency of the small earthquake, implying that the time step should be chosen as $\Delta t < f^{-1}$. We can rewrite the summation over time in equation (1) as the convolution

$$u_k(x,t) = \frac{\mu}{M_{og}} \int \Omega_c(x,t') g_k(t + T_c(\xi, x) - t') dt'$$

(2)

where $M_{og} = \mu A \Delta u_{s}(\Delta \xi)^2$ is the seismic moment of the small earthquake. Then we can readily identify the summation over the grid points as the integral

$$\Omega_c(x,t) = \int \Delta \hat{u}[\xi, t - T_c(\xi, x)] d\Sigma$$

(3)

that yields the waveform function for the wavetype $C$ observed at $x$ (Boatwright 1980). For a point source, where $T_c(\xi, x) = \text{constant}$, the waveform function is exactly proportional to the moment rate function. For a planar source, this relation can be expressed as $\hat{M}(t) = \mu \Omega_c(x, v, t)$ where $v$ is the normal to the fault plane (Boatwright 1984). Thus, if we deconvolve the seismograms of a small earthquake from the seismograms of a large earthquake, the
resulting time series approximate the moment rate function of the large earthquake, as projected through the body wave radiation to an observer at \( x \). The seismic moment of the earthquake,

\[ M_0 = \mu \int \Omega_C(x, t) \, dt, \quad (4) \]

is independent of the observer location.

The dynamic decomposition proposed by Boatwright (1988) describes an earthquake rupture as a distribution of stress release rather than slip events. The crucial aspect of this decomposition is the identification of the asperity-like character of a subevent embedded in a composite rupture process. Boatwright (1988) demonstrates how the waveform radiated by an asperity subevent can be approximated by filtering the waveform radiated by a crack. The method of filtering combines a low-pass Butterworth filter with a corner frequency at \( f_c \), and the convolutional inverse of a low-pass Butterworth filter with a corner frequency at \( f_c' \), where \( f_c \) and \( f_c' \) are the corner frequencies for the large and small earthquake, respectively. The low-frequency content of the resulting waveforms, which we will write as \( g_s^n(t) \), is amplified by the factor \( (f_c/f_c') \) in a causal manner while the high-frequency content is unchanged. We will call this filter a subevent filter.

Using this filtered waveform as the empirical Green's function for the stress release of a subevent yields the dynamic decomposition

\[ u(x, t) = \sum_i \frac{\Delta \sigma_i}{\Delta \sigma_s} g_s^n(t - T_i(\xi, x) + T_C(\xi, x) - T_i(\xi)) \quad (5) \]

where \( \Delta \sigma_i \) and \( \Delta \sigma_s \) are the dynamic stress drops of the grid point \( i \) and of the small earthquake, respectively. In contrast to the slip events required in equation (1), however, each grid point is assumed to radiate only once in equation (5). Note also that this decomposition is effectively broadband, in contrast to the kinematic decomposition of equation (1) which is limited to frequencies less than the corner frequency of the smaller earthquake.

The manipulations of equations (2) and (3) are readily repeated for this new decomposition, despite the fact that the convolution over time in equation (5) is implicit rather than explicit. The ground motion is obtained as the convolution

\[ u_s(x, t) = \frac{1}{\Delta \sigma_s(\Delta \xi)} \int \Lambda_C(x, t') g_s^n(t - T_C(\xi, x) + T_C(\xi, x) - T_i(\xi)) \, dt' \quad (6) \]

where the function \( \Lambda_C(x, t) \) represents the integral

\[ \Lambda_C(x, t) = \int \Delta \sigma(\xi) \delta(t - T_C(\xi, x) - T_i(\xi)) \, d\Sigma, \quad (7) \]

of the dynamic stress drop \( \Delta \sigma(\xi) \) over the fault. The waveforms \( g_s^n(t) \) radiated from each point \( \xi \) on the fault are associated with the arrival time \( T_i(\xi) + T_C(\xi, x) \) of the wavetype \( C \) at \( x \).

In analogy with the previous identification of the waveform function \( \Omega_C(x, t) \) as the moment rate function, we identify the function \( \Lambda_C(x, t) \) as the stress rate function. Thus if we deconvolve appropriately filtered seismograms of a small earthquake from the seismogram of a large earthquake, the resulting time series can be interpreted as the time history of the rate of stress release in the earthquake. Because the time delays \( T_i(\xi) \) are generally non-zero for ruptures with finite spatial extent, the stress rate functions have the same sense of projection through the body wave radiation to an observer at \( x \) as the waveform functions defined in equation (3). Where the integral of the waveform function over time yields the seismic moment, the integral of the stress rate function over time yields the total stress change of the earthquake, which can be written as the product of the average dynamic stress drop \( \Delta \sigma \) and the fault area \( \Sigma \).

**CHOOSING THE EVENT PAIRS**

The time-domain inversion procedure has been applied to ground velocity time histories which have been obtained by differing ground displacement time histories. These displacements are calculated from the digitized accelerograms by deconvolving the instrument response in the time domain using an exact deconvolution method (Rovelli et al. 1991). The quality of the ground velocity time series is generally satisfactory. In a few cases a very low-frequency trend is present which is removed using a Butterworth high-pass filter. Figs 1 and 2 show the two components of ground velocity for the large and small earthquakes which we will use for the inversions. Table 1 lists the estimates of seismic moment, corner frequency and local magnitude which Rovelli et al. (1991) obtained for these earthquakes. We will use their corner frequency estimates as the parameters of the subevent filters, as proposed by Boatwright (1988).

Three conditions must be satisfied before a small earthquake can be used as a Green's function for a larger earthquake: the seismic propagation in the earth and the seismic response below the station must be similar for the different events, and the fault mechanism must be approximately the same for the small earthquake as for all the subevents of the composite earthquake. The use of recordings from a single station assures that the site effects are similar for the small and large earthquake. The other two assumptions are not always satisfied. In fact, there may be cases in which the mechanism changes appreciably during a composite rupture process. Furthermore, when a large earthquake is recorded at a short epicentral distance, the source finiteness may yield appreciably different propagation paths for the seismic waves radiated by different subevents of the large event.

In the Gemona data set the epicentral distances range from 2 to 20 km. The two largest earthquakes, #17 and #13 (whose seismic moments are \( M_w = 5.8 \times 10^{24} \) dyne cm and \( M_w = 6.0 \times 10^{24} \) dyne cm, respectively) are located about 6 km from Gemona. The effect of source finiteness must be considered for these two events, whose waveform durations are \( \approx 4 \) s and whose source extents may be as large as 10 km.

The range of epicentral distances and azimuths in the
Figure 1. Ground velocity time series for both horizontal components of seven earthquakes in Gemona data set. The larger earthquakes are events #17, #13, #6 and #15, while the smaller ones are events #14, #8 and #4.

Table 1. Source parameters for the seven earthquakes used in this analysis. $M_o$, $f_0$, and $M_L$ are respectively the seismic moment, the corner frequency and local magnitude estimated for these earthquakes by Rovelli et al. (1991).

<table>
<thead>
<tr>
<th>Event</th>
<th>Date (GMT)</th>
<th>Time</th>
<th>$M_o$ dyne-cm</th>
<th>$f_0$ Hz</th>
<th>$M_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>#4</td>
<td>7/9/76</td>
<td>1108</td>
<td>$2.0 \times 10^{22}$</td>
<td>4.3</td>
<td>4.4</td>
</tr>
<tr>
<td>#6</td>
<td>11/9/76</td>
<td>1635</td>
<td>$1.4 \times 10^{24}$</td>
<td>0.0</td>
<td>6.2</td>
</tr>
<tr>
<td>#8</td>
<td>12/9/76</td>
<td>0120</td>
<td>$5.0 \times 10^{23}$</td>
<td>2.4</td>
<td>4.7</td>
</tr>
<tr>
<td>#13</td>
<td>15/9/76</td>
<td>0315</td>
<td>$6.0 \times 10^{24}$</td>
<td>0.7</td>
<td>6.4</td>
</tr>
<tr>
<td>#14</td>
<td>15/9/76</td>
<td>0339</td>
<td>$3.0 \times 10^{21}$</td>
<td>5.0</td>
<td>4.0</td>
</tr>
<tr>
<td>#15</td>
<td>15/9/76</td>
<td>0438</td>
<td>$1.5 \times 10^{23}$</td>
<td>2.2</td>
<td>5.2</td>
</tr>
<tr>
<td>#17</td>
<td>15/9/76</td>
<td>0021</td>
<td>$5.8 \times 10^{24}$</td>
<td>0.6</td>
<td>5.9</td>
</tr>
</tbody>
</table>
Figure 2. See caption to Fig. 1.

Figure 3. Polarization diagrams for the ground velocity of the seven selected earthquakes. The analysis of these diagrams allows us to select the earthquake pairs suitable for the deconvolutions. The seismic events forming a pair must exhibit similar polarization diagrams.
Gemona data set imply that not every small earthquake is an appropriate choice as an empirical Green's function for a larger earthquake. In order to choose suitable earthquake pairs, we consider two criteria. The first criterion is the comparison of the S trigger times of the two earthquakes. The second criterion is the comparison of the polarization diagrams of the horizontal ground velocity. Both the fault mechanism and the propagation effects determine the polarization diagrams; when two earthquakes form an appropriate pair, they exhibit similar polarization diagrams. Figs 3 and 4 show the polarization diagrams for the earthquakes analysed in the time-domain inversion. The event pairs which we have chosen for the deconvolutions are earthquakes #17 ($M_L=5.9$) and #14 ($M_L=4.0$), earthquakes #15 ($M_L=5.2$) and #14, and earthquakes #6 ($M_L=6.2$) and #8 ($M_L=4.7$). Of these three pairs, the polarization diagrams for earthquakes #17 and #14 appear the most similar. The ratio of the seismic moments is also the greatest for this event pair. Finally, we also analyse earthquake #13 ($M_L=6.4$) using two different small earthquakes, earthquakes #4 ($M_L=4.4$) and #14, for empirical Green's functions and compare the results of the different inversions to judge the robustness of the technique.

**AN ITERATIVE TIME-DOMAIN DECONVOLUTION**

The decomposition of a large earthquake into the set of subevents proposed in equations (1) and (5) can be expressed as the general summation

$$f_k(t) = \sum_{j=1}^{M} r_j g_k(t - \tau_j),$$

(9)

where the amplitudes $r_j$ are constrained to be positive and the $j$th subevent occurs at the delay time $\tau_j$, chosen from the set of delay times

$$\tau_j = (j - 1)\Delta \tau, \quad j = 1, 2, \ldots, M,$$

(10)

for all possible subevents, say $\Sigma^*$, measured with respect to the initial arrival time for the large earthquake. $\Delta \tau$ is the time step for the delay. The solution set $\Sigma$ of the subevents which compose the rupture process is determined by fitting the data.

The estimation of the model parameters appearing in equation (9) is an overdetermined linear inverse problem

$$d_i = G_{ij} r_j,$$

(11)

which can be solved by the method of least squares. The data vector $d_i$ of length $N=2L$ contains the data for the large earthquake with $L$ data points for each of the two horizontal components. The data vector can be written as

$$d_i = f_k(t_i)$$

where

$$i = l + (k - 1)L,$$

$$t_i = (l - 1)\Delta t, \quad l = 1, 2, \ldots, L,$$

and $k = 1, 2$ for the NS and EW component, respectively. The columns of the data kernel $G$ correspond to the two horizontal components of the Green's function, where each column $j = 1, 2, \ldots, M$ is time shifted by $\tau_j$, so that

$$G_{ij} = g_k(t_i - \tau_j).$$

Note that these time shifts should not overlap the Green's functions and the observed seismograms for the different horizontal components.

As indicated previously, the parameters $r_j$ should be constrained to be positive, regardless of whether the recordings of the small earthquake are filtered before the inversion. An algorithm to solve $L_2$ problems with inequality constraints was coded from the program provided by Menke (1984), following Lawson & Hanson (1974). The properties of the solution are described by the Kuhn-Tucker theorem; in our case the error gradient $\nabla E$ associated with each parameter $r_j$ must be non-negative. The error $E$ is defined as

$$E = \sum_{i=1}^{N} \left( d_i - \sum_{j=1}^{M} G_{ij} r_j \right)^2.$$  

(12)

The inversion technique is an iterative process which starts by initially pinning all the parameters to zero so that there are no subevents in the set $\Sigma$. At each step of an outer loop, the non-negativity of the error gradient is checked; if all gradient components are non-negative, the current set $\Sigma$...
represents the desired solution. Otherwise, the parameter $r_k$ associated with the most negative gradient is unpinned and added to the set $\Sigma$. The least-squares problem for determining the amplitudes of the subevents in $\Sigma$ is now solved. If the newly obtained model parameters are all positive, the new error gradient can be checked to find the next subevent to be added to the solution set. The parameters $r_k$ are prevented from becoming negative by an inner loop; if the least-squares solution yields one or more negative parameters, the change from the previous non-negative solution is decreased until all the parameters are non-negative. One or possibly more of the parameters are then pinned to zero, and the reduced problem is resolved. Since the number of subevents must decrease in this inner loop, the number of iterations in the solution is generally greater than the number of subevents. A flow chart describing the inversion process is shown in Fig. 5.

**FLOW CHART FOR THE INVERSION**

- **generate Green's functions for each delay time in $\Sigma^*$**
- **calculate data residual from subevents in $\Sigma$**
- **find the largest gradient $(-\nabla E)_\lambda = \max(-\nabla E)_\lambda$ for subevents $\lambda \in (\Sigma^* - \Sigma)$**
- **$(-\nabla E)_\lambda > 0$?**
  - **add subevent $\lambda$ at delay time $r_k$ to $\Sigma^*$**
  - **solve the resulting least squares problem for amplitudes $s_\lambda$ of subevents $\lambda \in \Sigma$**
  - **interpolate between new and old $s_\lambda$ to fix subevents $l \in \Sigma$ at $s_\lambda = 0$**
  - **then subtract subevents $l$ from $\Sigma$**
- **calculate variance of the new solution and F-test for significance**
- **$m_i \geq 0$ for subevents $j \in \Sigma^*$?**
  - **no**
  - **calculate variance of the new solution and F-test for significance**
- **SOLUTION**

**Figure 5.** A flow chart for the deconvolution, containing the algorithm for a least-squares inversion with positivity constraints. Two loops are used. The outer one iteratively determines the delay times and amplitudes $r_k$ for the set of subevents $\Sigma$ which fit the data. At each iteration the parameter with the most negative gradient $\nabla E$ is added to the set $\Sigma$. Then the least-squares problem is solved to compute the amplitudes of the current subevents in $\Sigma$. The inner loop subjects the parameters $r_k$ to the positivity constraints. The solution is reached when all the components of the error gradient are non-negative or when the variance is not improved sufficiently. At the beginning, all the parameters are pinned to zero so that there are no parameters in $\Sigma$. The set $\Sigma^*$ includes all possible delay times.

We have modified Menke's code by introducing an F test. Where a model with a larger number of parameters might minimize the error $E$ more than a model with less parameters, the improvement might not be statistically significant. The F test is used at the end of the outer loop of the inversion algorithm to evaluate the variance reduction associated with each new subevent: if the variance is significantly improved the process continues, otherwise the previously determined values for the model parameters are taken as the solution. We will consider a number of threshold values for the F test.

**OPTIMIZING THE DECONVOLUTIONS**

As an illustrative example, we consider the results of the deconvolution on the event pair composed by the large ($M_L = 6$) earthquake # 17 and the small ($M_L = 4$) earthquake # 14. The deconvolution was run on the time histories of the ground velocity. The sampling rate for the data is $\Delta t$, and it is reasonable to set $\Delta t$ as a multiple of $\Delta t$. For $\Delta t \geq \Delta t$, however, the empirical Green's functions must be smoothed by an operator of equivalent width $\Delta t$, otherwise the artificial zeros at the time points between the possible subevent times can introduce spurious high frequencies into the synthetic waveforms (Hartzell & Heaton 1985). A similar phenomenon occurs when $\Delta t$ is less than the minimum time resolution of the data and the number of subevents is restricted.

The optimum value of $\Delta t$ should be determined from the high-frequency content of the data. For these deconvolutions, the Gemona data were decimated to a $\Delta t$ equal to approximately 0.025 s. Rovelli et al. (1991) have determined $\kappa = 0.042 \pm 0.006$ s as the decay constant describing the path and site attenuation (Anderson & Hough 1984) of the Gemona data. The original choice of $\Delta t = \Delta t$ produced relatively rough stress rate functions and some high-frequency oscillations in the synthetics. The choice of $\Delta t = 2\Delta t = 0.05$ s yields the most appropriate stress rate functions and synthetic ground velocities. This result implies that the sampling of the subevents should be generally constrained as $\Delta t \approx \kappa$, because the attenuation removes the higher frequency information.

The duration for the time window $t_{\text{max}}$ that includes the subevents determines the number $M$ of the possible delay times through equation (10). Two different choices may be considered for this duration. One of these fixes $t_{\text{max}} = 5.0$ s, equal to the time window available for the smaller event. Although the use of the short Green's function in the longer time window can lead to possible misidentifications of late reverberations as source excitation, inspection of the seismograms of the candidate Green's function earthquakes suggests that almost all of the energy radiated by these earthquakes arrives in the initial 1.5 s of the S-wave.

If the inversion is performed without any F test, the total number of subevents in the solution set is determined by the inversion algorithm by fitting the data. This unrestricted inversion is equivalent to the original algorithm of Lawson & Hanson (1974). The F test appropriately restricts the number of subevents in the solution set. We estimate the variance of a particular model fit as the error of the fit $E$ divided by the number of degrees of freedom $v$. In this case, $v = N - Q$, where $Q = M$ is the number of subevents in the
solution set $\Sigma$. Then the $F$ ratio for any two models (say models 1 and 2) is

$$F_{12} = \frac{(N - Q_2)E_1}{(N - Q_1)E_2}. \quad (13)$$

When the probability associated with this $F$ ratio and these associated degrees of freedom $P(F_{12}, v_1, v_2)$ falls below the significance level $P_{\text{sig}}$, the iterative process is interrupted.

Figure 6 shows the behaviour of the variance in the deconvolutions with the filtered Green's functions (curve $\alpha$) and with the unfiltered Green's functions (curve $\beta$) for the pair of earthquakes #17 and #14. Fig. 7 shows the $F$ test probability as a function of iteration number for the same inversions as Fig. 6. While the $F$ test probability for the filtered Green's functions is less variable than the probability for the unfiltered Green's functions, it is not monotonic for either inversion.

Figure 7. $F$ test probability versus the number of iterations for the inversion with the earthquake pair formed by the events #17 and #14. The curves (a) and (b) show the variance reductions with and without filtering the smaller event to obtain an asperity-like waveform, and for $\tau_{\text{max}} = 5$ s. The curve (c) shows the variance reduction obtained by smoothing the smaller event with a triangular function of half-width 0.42 s. The overall reduction of variance for the filtered Green's functions is greater than that for the unfiltered Green's functions and for the smoothed Green's functions.

Figure 8. Stress rate functions obtained for the event #17, using #14 as Green's function, after that filtering in order to simulate the radiation from a subevent. These results have been obtained performing an $F$ test with significance threshold $P_{\text{sig}} = 0.6$, $P_{\text{sig}} = 0.5$ and without an $F$ test (A, B and C, respectively), for $\tau_{\text{max}} = 5$ s. In (C) the number of subevents are larger than the one in (B), but there are no significant differences between the results obtained in the two cases. In (A) the stress rate function is extremely sparse, because of the smaller number of subevents.
In contrast, the variances plotted in Fig. 6 exhibit relatively smooth behaviours. For the filtered Green's functions, the minimum variance occurs at the 55th iteration when \( Q = 55 \), while for the unfiltered Green's functions, the minimum variance occurs at the 68th iteration when \( Q = 66 \). Each of these minima are associated with the first passage of \( P(F, \nu_2, \nu_{2+}) = 0.5 \); setting \( \Psi_\text{sig} = 0.5 \) then yields the minimum variance \( E/\nu \).

Figure 8 shows the stress rate functions obtained using the filtered Green's functions and setting \( \Psi_\text{sig} = 0.6, 0.5 \) and 0.0; the number of subevents obtained in the inversions are 13, 55 and 76, respectively. Note that setting \( \Psi_\text{sig} = 0 \) makes the inversion formally equivalent to the method of Lawson & Hanson (1974). Despite the larger number of subevents, the stress rate function is not significantly changed from the function obtained by setting \( \Psi_\text{sig} = 0.5 \). In contrast, however, the stress rate function obtained by setting \( \Psi_\text{sig} = 0.6 \) is extremely sparse. The corresponding synthetics for \( \Psi_\text{sig} = 0.6 \) and 0.5 are plotted in Fig. 9, below the observed ground velocity for the large earthquake.

**COMPARING GREEN'S FUNCTIONS FOR SUBEVENTS**

The iterative deconvolution can also be performed without filtering the velocity time series of the small earthquake, that is, using a crack-like waveform. The variance for this case (curve \( \beta \)) is plotted together in Fig. 6 with the variance (curve \( \alpha \)) obtained using the subevent filter, while the \( F \) test probability for this case is plotted in Fig. 7(b). Two differences are immediately apparent from this comparison: first, the variance reduction for the deconvolution with the

![Figure 9](https://academic.oup.com/gji/article-abstract/105/1/103/666874)
filtered Green's functions is more rapid than that with the unfiltered Green's functions, and second, the variance reduction for the deconvolution with the filtered Green's functions is always greater than the variance reduction with the unfiltered Green's functions.

The rapid decrease of the variance suggests that the filtered Green's functions are more appropriate models for the subevents of large earthquakes. However, this decrease is expected for any filtering technique which enhances the low-frequency content of the Green's functions. As a comparison, we plot in Fig. 6 the variance reduction obtained by smoothing the Green's functions with a triangle function of half-width 0.42 s (curve γ), which reaches a minimum variance even more rapidly. However, the overall reduction of variance obtained using the filtered Green's functions is greater than that obtained using the smoothed Green's functions. Although we cannot demonstrate that the subevent filter proposed by Boatwright (1988) and used in this research yields the optimal filter for those deconvolutions, Fig. 6 indicates that the filtered Green's functions are an improvement over both the unfiltered Green's functions and the smoothed Green's functions.

The smaller number of filtered than unfiltered Green's functions required to obtain the minimum variance also corresponds with the obvious difference between equations (1) and (5): equation (1) includes a summation over the number of time steps during which each grid point slips.

The moment rate function obtained by deconvolving the unfiltered Green's functions from the observed waveforms is plotted in Fig. 10(b). The plotted moment rate function corresponds to the case where the variance is minimized, that is, for \( P_{\text{eq}} = 0.5 \). It is interesting to compare this function with an equivalent moment rate function obtained by filtering the stress rate function shown in Fig. 8(b) using the subevent filter (Fig. 10a). This double filtering is similar to the pre-whitening strategy discussed by Kanasewich (1976, p. 126): the subevent filter broadens the spectrum of the empirical Green's function.

The moment rate function in Fig. 10(a) strongly resembles the moment rate function obtained using the unfiltered Green's functions (Fig. 10b), except that it does not go to zero until the end of the waveform. This aspect of the doubly filtered moment rate function appears to be physically appropriate, as one might intuitively expect some part of the rupture area to be slipping throughout a complex rupture process. The total moment of the doubly filtered moment rate function is about 45 per cent more than that of the moment rate function determined using the unfiltered Green's functions.

Figure 10(c) plots the result of the spectral deconvolution performed with smoothing the amplitude spectrum for the EW component of the two earthquakes #17 and #14. Although there is a low-frequency trend present, the spectral deconvolution clearly shows the first large subevent; the later smaller subevents are obscured by the deconvolutional noise. This comparison demonstrates the utility of the positivity constraints.

**RESULTS FOR OTHER EARTHQUAKE PAIRS**

In addition to the earthquake pair analysed in the last two sections, four other earthquake pairs have been analysed: #15 and #14, #6 and #8, #13 and #4, and #13 and #14. In each pair, the first earthquake listed is the largest. The significance level for the \( F \) test was set to \( P_{\text{eq}} = 0.5 \) for all of these deconvolutions.

Figure 11 shows the stress and moment rate functions obtained by deconvolving earthquake #14 from earthquake #15, while Fig. 12 shows the fit between the observed and the synthetic waveforms. The stress and moment rate functions are relatively simple and suggest an asperity failure as a model for the rupture process of earthquake #15. The stress rate function is dominated by an initial subevent with a large stress drop, which is followed by two weaker subevents. The stress drops of the later subevents
Decomposition of seismograms

The last 3.5 s of the inversion window contains a few weak subevents which probably represent deconvolutional noise rather than actual subevents of the rupture process. These subevents are manifest more clearly in the stress rate function than in the moment rate function, indicating that they are relatively deficient in low-frequency content. They may represent scattered energy which is not perfectly modelled by the Green’s function recordings.

As indicated in Table 1, the corner frequency of earthquake #15 is relatively close to that of the Green’s function earthquake; consequently, the moment rate functions obtained by filtering the stress rate function and by deconvolving the unfiltered Green’s functions are very similar. Note that the moment release of the two later subevents is somewhat greater than the moment release of the first subevent, in contrast to the stress release. This difference reflects the non-localized aspect of the moment release in a composite earthquake, where the stress release on a given fault area extracts slip from previously faulted areas as well.

Deconvolving earthquake #8 from earthquake #6 yields the stress rate functions plotted in Fig. 13(c). The stress rate exhibits a clear periodicity which is also apparent in the observed EW ground velocity in Fig. 14. This anomalous periodicity suggests that a higher mode surface wave which is approximately polarized onto the EW component has been excited by the larger earthquake but not by the Green’s function earthquake. In order to consider the effect of such an inconsistency on the inversion, we have performed the deconvolution on each component separately. Figs 13(a) and (b) show the stress rate functions obtained from the components NS and EW, respectively. All the synthetic waveforms are presented in Fig. 14 together with the observed ground velocity.

When the two components are deconvolved together, the results are dominated by the EW component because the amplitude of the EW component is much larger than that of the NS component. Note that this periodic motion appears in the synthetic NS waveform plotted in Fig. 14(c) although it is not present in the observed NS ground velocity. When the deconvolution is run on the NS component alone, the periodicity in the resulting stress rate function disappears, as shown in Fig. 13(a). Instead of the periodic subevents observed in Figs 13(b) and (c), the stress rate function deconvolved from the NS component contains a relatively long duration (~1 s) subevent which has a larger moment release than the initial subevent.

Thus the recordings of earthquake #8 are not quite suitable as Green’s functions for earthquake #6, possibly because of a difference in depth between the two earthquakes. This problem is endemic to the use of small earthquakes as Green’s functions. Large earthquakes radiate energy from a range of azimuths and depths, so that different propagation effects may have to be incorporated for different subevents. The effects, however, on the derived stress rate functions does not seem to be pathological: the waveforms for the later subevents are distorted but the total stress rate appears to be approximately conserved.

The deconvolutions of earthquake #13 by using earthquakes #4 and #14 as Green’s functions exhibit an even more pronounced periodicity. The stress rate functions are plotted in Fig. 15. Relatively high stress rate values are obtained for an overall duration of 4.5 s, but they arrive as five distinct subevents with a characteristic delay time of 0.85 s. The results from the earthquake pair discussed previously indicate that this periodicity probably results from a waveform mismatch between the Green’s functions and the large earthquake. The observed velocity waveforms for earthquake #13 are plotted in Fig. 16(a); both horizontal components are dominated by energy at period equal to 0.85 s.

In addition to this misfit at low frequency, the phase characteristics of the earthquakes #4 and #14 also indicate that they are imperfect Green’s functions for earthquake
Figure 12. Synthetic waveforms obtained from the inversion with the pair including the earthquakes #15 and #14. In (A) the observed ground velocity is shown. In (B) and (C) the filtered and the unfiltered Green's functions have been used, respectively.

#13. The synthetic waveforms plotted in Figs 16(b) and (c) show that earthquake #4 fits the beginning of the waveform, while earthquake #14 fits the section of the waveform between 2 and 3 s. This variable fit suggests the possibility of using two different Green’s functions simultaneously in the deconvolution. The different Green’s function earthquakes might describe the propagation characteristics from different source regions or the polarizations obtained from different focal mechanisms. The algorithm can be modified to include more than one possible Green’s function in the inversion scheme, by increasing the number of columns of the kernel $G_{ij}$.

Figure 13. Inversion results obtained by using only one component each time, NS and EW respectively in (A) and (B), for the pair of earthquakes #6 and #8. The stress rate function produced with both components is shown in (C). An $F$ test with $P_{sig}=0.5$ has been used. The comparison between the three functions shows that the EW component has more weight than the NS component.
Decomposition of seismograms

The inversion technique which we have derived and tested in this paper determines a minimal set of positive amplitudes and time delays for the subevents in a composite rupture process. The inversion is performed entirely in the time domain, iteratively fitting the waveforms of the composite earthquake by adding together empirical Green's functions; the solution set is increased subevent by subevent until the variance is maximally reduced. In this sense, the inversion process emulates the intuitive decomposition which most observational seismologists use to interpret the recordings of a large earthquake: that is, iteratively identifying a set of subevents by recognizing simpler waveforms embedded in the complex seismograms.

The inversion yields reasonable results when the recordings of the large and small earthquake exhibit similar characteristics, implying that the earthquakes share similar fault mechanisms and propagation effects. In fact, the inversion appears relatively robust, in that the deconvolved waveforms are distorted but not unstable, even when there are clear inconsistencies in the phase and frequency content of the seismograms. It is likely that this robustness is partly derived from the simultaneous deconvolution of the two horizontal components of the S-waves. As shown in Fig. 10(c), the spectral deconvolution cannot clearly resolve the later and weaker subevents which compose the last part of the rupture process, because of the relatively high noise level. We note that Frankel et al. (1986) and Mori & Frankel (1989) mostly use the method of spectral division.

**Figure 14.** Synthetic waveforms obtained from the inversion with the earthquakes # 6 and # 8. The observed ground velocity is shown in (A). The synthetic waveforms produced by using only one component each time are shown in (B). In (C) both components have been used. A higher mode surface wave present particularly on the EW component is reproduced also in the NS synthetic waveform, while it is not present in the data for the NS component. This fact indicates that earthquake # 8 is not completely suitable as a Green's function.

**Figure 15.** Results obtained from the inversion with the earthquake # 13 using the events # 4 and # 14 as Green's functions. An F test has been performed with $P_{\text{test}} = 0.5$. The stress rate obtained with the Green's function # 4 and # 14, respectively, is shown in (A) and (B). The persistence of relatively high stress rate values within a duration longer than 4 s and their periodicity is suspect. Probably, the smaller earthquakes # 4 and # 14 are not suitable as Green's functions for the event # 13.

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**DISCUSSION**

The inversion technique which we have derived and tested in this paper determines a minimal set of positive amplitudes and time delays for the subevents in a composite rupture process. The inversion is performed entirely in the time domain, iteratively fitting the waveforms of the composite earthquake by adding together empirical Green's functions; the solution set is increased subevent by subevent until the variance is maximally reduced. In this sense, the inversion process emulates the intuitive decomposition which most observational seismologists use to interpret the recordings of a large earthquake: that is, iteratively identifying a set of subevents by recognizing simpler waveforms embedded in the complex seismograms.

The inversion yields reasonable results when the recordings of the large and small earthquake exhibit similar characteristics, implying that the earthquakes share similar fault mechanisms and propagation effects. In fact, the inversion appears relatively robust, in that the deconvolved waveforms are distorted but not unstable, even when there are clear inconsistencies in the phase and frequency content of the seismograms. It is likely that this robustness is partly derived from the simultaneous deconvolution of the two horizontal components of the S-waves. As shown in Fig. 10(c), the spectral deconvolution cannot clearly resolve the later and weaker subevents which compose the last part of the rupture process, because of the relatively high noise level. We note that Frankel et al. (1986) and Mori & Frankel (1989) mostly use the method of spectral division.
for P-waves with far less noise than the S-waves which we analyse in this paper.

The minimization of the set of subevents that is achieved using the F test is another significant advantage of this inversion technique. Hartzell & Heaton (1985) apply the algorithm from Lawson & Hanson (1974) with damping to obtain a stable solution. This damping minimizes the total amplitude of the subevents; but yields little constraint on the number of subevents with relatively low amplitudes. Because we are interested in the significance of subevents late in the waveforms, minimizing the number of subevents is a more appropriate constraint than minimizing the total amplitude of the subevents.

The moment rate function obtained for earthquake #15 indicates the importance of this minimization. The source process for this earthquake appears confined to the first 1.5 s; the four low-amplitude subevents which occur in the next 4 s are probably generated by a mismatch of the scattered energy present in the seismograms. If the algorithm of Lawson & Hanson (1974) had been used, it is probable that there would be more low-amplitude subevents in this time window that might serve to obscure our resolution of the stopping and healing of the rupture, as evident in the last 2 s of the windows plotted in Fig. 8.

The stress rate and moment rate functions obtained by deconvolving earthquakes #17, #15 and #6 show that this inversion can reliably identify subevents which occur late in the rupture process. These results can be used to investigate the general stopping behaviour for the rupture of these earthquakes. The late subevents in earthquakes #17 and #6 appear to represent secondary ruptures which release significantly less stress than the initial subevents. Although it is not possible to locate these subevents spatially, it is reasonable to assume that they are clustered around the initial subevent. Instead of the abrupt stopping behaviour often assumed for theoretical rupture models, these results suggest that the stopping of real earthquakes resembles an intermittent diffusion process, where the initially concentrated stress release spawns successively weaker and weaker ruptures which act to smooth the spatial distribution of the stress drop.

This interpretation of the stopping behaviour of real earthquakes suggests that frequency characteristics of the subevents might systematically change as the rupture stops in the gradual fashion. The subevent filtering which we have used in this paper assumes that the frequency characteristics of the subevents are effectively constant throughout the waveform. Because the inversion is performed in the time domain, however, it is possible to use different filter characteristics for subevents with different delay times. Wennerberg (1990) suggests that it might be feasible to invert for the appropriate filter characteristics for the subevent at the same time as the algorithm searches for the delay time. A more thorough investigation of the stopping
behaviour in earthquakes will probably require such a detailed approach.

CONCLUSIONS

It is necessary to have recordings of earthquakes with very different sizes to fully exploit the use of empirical Green’s functions to investigate source processes. The data set recorded at Gemona del Friuli in 1976 is perfectly suited to this analysis technique, because the earthquakes span a wide range of magnitudes and they occurred at relatively small hypocentral distances from the station.

The inversion technique proposed and tested in this paper can resolve either stress release or moment release as a function of time during a complex rupture process. The resolution of the stress release is somewhat better than the resolution of the moment release, but this result depends on the assumption that the frequency characteristics of the subevents are constant throughout the rupture process.

This inversion technique is relatively robust, permitting seismologists to obtain detailed information about the subevents which comprise a large composite earthquake, in regard both to their stress release and their relative delays. Even the small amplitude subevents which make up the tail of the source process can be resolved, so that it is possible to investigate in much greater detail how rupture stops in real earthquakes.

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