The Possibility of Hyperon Superfluids in Neutron Star Cores

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The possible existence of hyperon superfluidity in neutron star cores is studied by using several $^1S_0$ $YY$ interactions from the OBE baryon-baryon potentials. It is found that not only $\Lambda$ but also $\Sigma^-$ and $\Xi^-$ hyperons could very likely be superfluid.

With the increase of the total baryon density $\rho$ toward the center of neutron stars, hyperons (Y) begin to appear as new constituents. The subject of hyperon-mixing has gathered much attention from an early stage of theoretical works on neutron stars. The hyperon fraction $y_Y$ increases with $\rho$ and in the core region they are important components comparable to nucleons, interestingly affecting the properties of neutron stars. Here we address the question of whether $\Lambda$, $\Sigma^-$ and $\Xi^-$ admixed could be superfluid. The occurrence of hyperon superfluidity plays a key role in the rapid cooling scenario of neutron stars, i.e., the so-called “hyperon cooling”, as one of the non-standard cooling scenarios to explain the unusually low surface temperatures observed for some neutron stars. In a preceding work, we concentrated our attention on the case of the $\Lambda$ superfluid, and we showed that $\Lambda$ superfluidity exists, though in a limited density region, with the critical temperature $T_c \sim 10^8 - 10^9$ K. In the present work we discuss the cases of $\Sigma^-$- and $\Xi^-$-superfluids in reference to the $\Lambda$ case. Unfortunately, our present knowledge of $YY$ (and also $YN$) interactions is very limited as compared to the $NN$ interaction. For this reason, we calculate the pairing energy gap by choosing several baryon-baryon (BB) potentials based on the hypothesis of SU(3) invariance and see what can be said regarding the existence or nonexistence of $Y$ superfluids. In this choice, we pay particular attention to the compatibility with hypernuclear data.

Although hyperons participate in the high-density region ($\rho \geq 2\rho_0$; where $\rho_0 = 0.17$ nucleons/fm$^3$ is the nuclear density), the fractional density $\rho_Y (\equiv y_Y \rho)$ is relatively small because $y_Y$ is not large (10% – 30% at most), and thus the Fermi energy $\epsilon_{FY} = \hbar^2(3\pi^2\rho_Y)^{2/3}/2M_Y$, with $M_Y$ the hyperon mass) is rather low. Therefore, the pairing interaction responsible for $Y$ superfluidity should be that in the $^1S_0$ pair state. Because there are different Fermi surfaces for every hyperon species, the pairing correlation can be restricted to that of the same $Y$ species. Then the energy gap equation to be solved here has a well-known form of the $^1S_0$-type.  

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The baryon-baryon (BB) interaction models of the one-boson-exchange (OBE) type are extended from those of NN on the basis of $SU(3)$ invariance. Hence the YY interaction is more or less constrained by the condition to reproduce NN andYN scattering data, since in the model, the B-B-meson coupling constants are determined through the so-called mixing angles $\theta$ and F-D ratios $\alpha$. Generally speaking, however, the BB interactions at present are not accurately determined, due to the lack of sufficient scattering data. In such a situation it is crucial to take account of the information from the hypernuclear data. Fortunately, in the case of the $\Lambda\Lambda$ interaction, this can be checked by the separation energy of the $\Lambda\Lambda$ pair in the analyses of double $\Lambda$ hypernuclei, such as $^{10}\Lambda\Lambda$Be and $^{13}\Lambda\Lambda$B.\(^{17,18}\) In the case of $\Sigma^-\Sigma^-$ and $\Xi^-\Xi^-$ interactions, such a test is lacking. Therefore, it is more suitable to use the $\Sigma^-\Sigma^-$ and $\Xi^-\Xi^-$ interactions taken from the BB interaction model whose $\Lambda\Lambda$ interaction is compatible with the double $\Lambda$ hypernuclei data.

We consider three cases for the $^1S_0$ pairing interaction $V_{YY}$ described by $r$-space potentials, which are commonly based on the OBE scheme with the introduction of octet baryons plus nonet mesons with the $SU(3)$ symmetry hypothesis. The main differences between these cases regard the meson species introduced and the treatment of the short-range interaction. We now describe these cases.

[A] ND-Soft:\(^{1,17}\) This is a soft-core version of the original Nijmegen-D (ND)\(^{19}\) hard-core potential, abbreviated here as “ND-Soft”, and is constructed so as to fit the $t$-matrix from the original ND potential. It is expressed simply as a superposition of three-range Gaussian functions. The use of the soft-core version, instead of the hard-core potential, is for the sake of convenience in the treatment of the gap equation.

[B] Ehime:\(^{20}\) This potential is the one from the Ehime group. It is characterized by an application of the OBE scheme throughout all the interaction ranges and by the phenomenological introduction of a neutral scalar meson to take the $2\pi$-correlation effects into account, in addition to nonet scalar mesons. It is given by a superposition of the Yukawa-type functions in $r$-space regularized by the form factors in momentum space and has velocity-dependent potentials.

[C] FG:\(^{21}\) This is the potential proposed by the Funabashi-Gifu group. Here we adopt their “Type A”. Its OBE descriptions, in which the $\sigma$- meson is included in the nonet scheme and the broad width of $\sigma$- and $\rho$-mesons are taken into account, are modified for $r \lesssim 1$ fm, with a universal cutoff in $r$-space smoothly vanishing as $r \to 0$. This potential includes the velocity dependence and retardation effects. A phenomenological repulsive core is introduced with strengths constrained by the $SU(3)$ representation of two-baryons.

Here it should be noted that the $\Lambda\Lambda-\Sigma\Sigma-\Xi N$ channel coupling effects are significant, especially for the FG potential. In order to take this into account, we add $\Delta V_{\text{sim}}(r) = 93e^{-(r/r_1)^2} - 1000e^{-(r/r_2)^2}$ MeV, with $r_1 = 1.0$ fm and $r = 0.6$ fm to $V_{\Lambda\Lambda}(^1S_0)$ in the FG potential, so that $V_{\Lambda\Lambda}^{(\text{eff})}(^1S_0) \equiv V_{\Lambda\Lambda}(^1S_0) + \Delta V_{\text{sim}}(r)$ (represented by FG’ in Fig. 1(a)) simulates the $^1S_0 \Lambda\Lambda$ phase shifts including the channel coupling effect. For the case of ND-Soft, this channel coupling effect, although small, is included similarly through the original $t$-matrix to be fitted. For the Ehime case, this effect is not involved since it is constructed in a single-channel approximation.
Fig. 1. $^1S_0$ YY interaction $V_{YY}(r)$ displayed for the three kinds of OBE potentials in the text: (a) $Y = \Lambda$, (b) $Y = \Sigma^-$ and (c) $Y = \Xi^-$ cases. The two velocity-dependent potentials (Ehime, FG) are for a $YY$-pair at the Fermi surface with the Fermi energy $\epsilon_{FY} = 15$ MeV. The FG' (the dotted curve with crosses) corresponds to $V_{\Lambda\Lambda}^{(\text{eff})}(^1S_0)$.

Finally, we wish to stress that both the ND-Soft and the Ehime $\Lambda\Lambda$ interactions reproduce the double $\Lambda$ hypernuclei data. Such a check has not yet been carried out for the FG, but we can expect such a reproduction by $V_{\Lambda\Lambda}^{(\text{eff})}(^1S_0)$, because it gives $^1S_0$ $\Lambda\Lambda$ phase shifts very close to those given by the Ehime potential and similar to those given by the ND-Soft potential.

In Figs. 1(a)–(c), the $^1S_0$ YY interaction, $V_{\Lambda\Lambda}, V_{\Sigma^-\Sigma^-}$ and $V_{\Xi^-\Xi^-}$, for these three potentials (ND-Soft, Ehime, FG) are illustrated. It is observed that $V_{YY}$ for these three potentials differ considerably. First, Fig. 1(a) shows that in spite of their equivalence in the reproduction of double $\Lambda$ hypernuclei data, $V_{\Lambda\Lambda}$ (ND-Soft) has a stronger short-range repulsion and a deeper intermediate-range attraction than $V_{\Lambda\Lambda}$ (Ehime). This characteristic is also seen for $V_{\Sigma^-\Sigma^-}$ and $V_{\Xi^-\Xi^-}$ cases (Figs. 1(b),(c)). It is of interest to see how the difference affects the resulting energy gap, since both the repulsive and attractive effects are importantly involved with the pairing problem. Second, all the $V_{YY}$ (FG) have a weaker repulsive effect (smaller core radius) and a deeper attraction, as compared with $V_{YY}$ (ND-Soft). In addition, the velocity-dependent effect of $V_{YY}$ (FG) is weaker than that of $V_{YY}$ (Ehime). These observations suggest that the YY pairing is more likely to occur for the FG potential. The use of these three potentials is expected to cover the present uncertainties in the YY interactions. Finally, a comparison of $V_{\Lambda\Lambda}$ with $V_{\Sigma^-\Sigma^-}$ and $V_{\Xi^-\Xi^-}$ for the
The three different potentials used here give similar values of \( \Sigma^- \Sigma^- \) and \( \Xi^- \Xi^- \) pairings are equally or more likely to occur as compared with the \( \Lambda \Lambda \) pairing.

Calculations of the energy gap \( \Delta Y \) are performed when \( \rho_Y \) and the effective-mass parameter \( m_Y^* (= M_Y^*/M_Y) \), where \( M_Y^* \) is the effective mass) are given. A realistic value of \( m_Y^* \) is available only for \( \Lambda \), and not for \( \Sigma^- \) and \( \Xi^- \). This is because the \( \Lambda N \) and \( \Lambda \Lambda \) interactions responsible for the single-particle energy (hence \( m_A^{\Lambda}(q) \approx U_0 + \hbar^2 q^2/2M_A^{\Lambda} \) with momentum \( q \), are reliably constrained by the relatively abundant information from hypernuclei data. Here we are content to use a common \( m_Y^* \), assuming that \( m_Y^* \) is not so different among the \( Y \) species.

For convenience, the results are presented in terms of the critical temperature \( T_{cY} \) related to \( \Delta Y \) by \( T_{cY} \approx 0.66 \Delta Y \) (in MeV) \( \times 10^{10} \) K. Figures 2(a), (b) and (c) correspond to \( T_{c\Lambda} \), \( T_{c\Sigma^-} \) and \( T_{c\Xi^-} \), respectively. When \( T_{cY} > T_i \approx 10^8 \) K, the internal temperature of normal neutron stars, \( Y \) superfluidity is realized. We note the following points:

(i) Since the values of \( T_{c\Lambda} \) are well above \( 10^8 \) K, \( \Lambda \) superfluidity can be realized in a hyperon-mixed core of neutron stars, as shown in a previous paper \( \Lambda \) the ND-Soft and the Ehime potentials. The results for the FG potential newly adopted confirm the existence of a \( \Lambda \) superfluid.

(ii) The three different potentials used here give similar values of \( T_{c\Lambda} \), both in magnitude and in \( \rho_Y \) dependence, in contrast with the \( T_{c\Sigma^-} \) and \( T_{c\Xi^-} \) cases, because the uncertainties in the \( YY \) interactions are by far smaller in the \( \Lambda \Lambda \) interactions, due to the constraint provided by double \( \Lambda \) hypernuclei data. A rapid decrease on the high \( \rho_Y \) side, resulting mainly from the growth of repulsive effects with increasing \( \rho_Y \) due to the repulsive core and the velocity dependence, is reflected faithfully by the differences among the three potentials.

(iii) By combining the hyperon-core model, namely the \( Y_Y\rho_Y \) relationship, with the \( T_{c\Lambda}\rho_Y \) relationship, we can determine the \( \Lambda \) superfluid density region. For example, the \( \rho_Y\rho_Y \) relationship, which differs greatly among the works of different authors, is given as \( y_A \approx \{2.0, 5.0, 8.6\} \% \) \{12, 18, 24\} \% corresponding to \( \rho = \{2.5, 3.0, 4.0\} \rho_0 \) for the hyperon-core model by Pandharipande \( \Lambda \) Shaffner-Mishustin \( \Lambda \). These imply \( \rho_N = \{0.05, 0.15, 0.34\} \rho_0 \) \{0.30, 0.54, 0.96\} \rho_0 \). Then, for the \( T_{c\Lambda}\rho_Y \) relationship from the ND-Soft, we have the \( \Lambda \)-superfluid up to \( \rho \approx \{3.3 \rho_0 \} \approx 2.5 \rho_0 \) and the existence region \( \rho \approx \rho_i - 3.3 \rho_0 \) \( \rho_i - 2.5 \rho_0 \) with \( \rho_{t\Lambda} \approx 2.3 \rho_0 \) \{2.0 \rho_0 \} defined here by the condition \( y_A(\rho_i) = 0.1 \%. \) For the \( T_{c\Lambda}\rho_Y \) from the FG, we have the region \( \rho \approx \{2.3 \rho_0 - 5.0 \rho_0 \} \approx 2 \rho_0 \) for Pandharipande (Shaffner-Mishustin) case.

(iv) As compared with \( T_{c\Lambda} \) in Fig. 2(a), \( T_{c\Sigma^-} \) for \( m_{\Sigma^-}^* = 0.8 \) in Fig. 2(b) is by far larger for all three potentials; by a factor of about 5 for the ND-Soft and Ehime potentials and by one order of magnitude for the FG potential, and the rapid decrease of \( T_{c\Sigma^-} \) is pushed to the higher fractional density side. This feature results from stronger attractive effects of \( V_{\Sigma^-\Sigma^-} \) compared to those of \( V_{\Lambda\Lambda} \) as seen in Fig. 1. Even if \( m_{\Sigma^-}^* \) turns out to be extremely small, e.g. \( m_{\Sigma^-}^* = 0.6 \), the results are
Fig. 2. The critical temperature $T_{cY}$ of the $^1S_0 Y$ superfluid versus the $Y$ fractional density $\rho_Y$ plotted for an effective-mass parameter $m_Y^* = 0.8$ and for the three potentials in Fig. 1: (a) $Y = \Lambda$, (b) $Y = \Sigma^-$ and (c) $Y = \Xi^-$ cases. The internal temperature of normal neutron stars is around $10^8$ K. For reference, $T_{cY}$ corresponding to $m_Y^* = 0.6$ and $m_Y^* = 1.0$ are plotted for $\Sigma^-$ and $\Xi^-$ cases. Results of $T_{c\Lambda}$ for FG case are obtained from the FG' potential in Fig. 1(a).

comparable to (larger by an order of magnitude than) $T_{c\Lambda}$ for the ND-Soft and the Ehime cases (the FG cases). Although at present a realistic value of $m_{\Sigma^-}^*$ from $\Sigma^- N$ and $\Sigma^- \Sigma^-$ interactions tested by hypernuclei is not available, it is worth mentioning that $m_{\Sigma^-}^* \gtrsim 1$ and also $m_{\Xi^-}^* \gtrsim 1$ have been obtained from $G$-matrix calculations with the ND potential.\(^{22}\) For $m_{\Sigma^-}^* = 1$, $T_{c\Sigma^-}$ obtained from the ND-Soft is almost equal to $T_{c\Sigma^-}$ obtained from the Ehime with $m_{\Sigma^-}^* = 0.8$. Based on these we can say that, in addition to $\Lambda$ superfluid, $\Sigma^-$ superfluid is realizable in neutron stars.

(v) As for the $\Xi^-$ case, we have results very similar to those of $\Sigma^-$ for the Ehime and the FG cases, and the discussion in (iv) also applies here. For the ND-Soft case, $T_{c\Xi^-}$ is somewhat smaller than $T_{c\Lambda}$, but it is well above $10^8$ K. When $m_{\Xi^-}^* \gtrsim 1$ for the ND potential is taken into account, $T_{c\Xi^-}$ becomes by far larger than $T_{c\Lambda}$, as shown in Fig. 2(c) by the solid curve with $m_{\Xi^-}^* = 1$. Thus we expect that $\Xi^-$ superfluid also exists in neutron star cores.

(vi) The existence density regions for the $\Sigma^-$ and $\Xi^-$ superfluids can be obtained in manner similar to that discussed in (iii), depending on the hyperon-core model. Roughly speaking, we have a wider region for the $\Sigma^-$ superfluid than for the $\Lambda$ superfluid, due to that $T_{c\Sigma^-} \gtrsim T_{c\Lambda}$, $\rho_{t\Sigma^-} \sim \rho_{t\Lambda}$ and $y_{\Sigma^-}(\rho) \sim y_{\Lambda}(\rho)$. On the other hand, for the $\Xi^-$ superfluid we have a narrow region around the neutron star center, since usually $\rho_{t\Xi^-} \gg \rho_{t\Lambda} \sim \rho_{t\Sigma^-}$ due to the larger mass.

In conclusion, we can expect that not only $\Lambda$ but also $\Sigma^-$ and $\Xi^-$ are superfluid in a hyperon-mixed core of neutron stars. To this time, only the $\Lambda$ superfluid has
been discussed.\textsuperscript{16, 23} The present work represents the first investigation of $\Sigma^-$ and $\Xi^-$ cases. Of course, in order to have realistic superfluid density regions, further studies are necessary. In particular, $\nu Y$, $m_Y^*$ and $\Delta_Y$ should be calculated systematically from the same $BB$ interaction model and by making effort to constrain the $BB$ interactions in accordance with hypernuclear data. In future studies, it is of special interest to employ the $YY$ potentials obtained by a quark model approach recently developed. Finally, the results obtained here support qualitatively the idea of “hyperon cooling” scenario combined with $Y$ superfluidity, since in this scenario, the realization of superfluidity for all the $Y$ species is a key ingredient.

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\begin{enumerate}
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