Theory of the $C$-Meson*

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In this talk I would like to discuss some aspects of the $C$-meson theory which seem to be related to the topics of this workshop.

This theory is one of the achievements made just after the war under severe circumstances. I think that it should be remarked upon for two reasons. The first is that it is a typical example of the application of Sakata's philosophy of the methodology of science, and the second is that it has played an important historical role as a motive to Tomonaga's renormalization theory.

Let us start from Sakata's philosophy. He believed that there must be the law of development of science. So if we can find this law and apply it we shall be able to arrive at the goal much more efficiently than without it. With such a methodology any clever and diligent person can make a useful contribution to science even if he is not a genius. This was the belief and the hope of Sakata.

As such a methodology he adopted the idea of Taketani who was his intimate friend. Taketani stated that our cognition of Nature is a dialectic logical process, proceeding spirally repeating the three stages of phenomenological, substantialistic and essentialistic, corresponding to the three stages of an sich, für sich and an und für sich of Hegel.

In the phenomenological stage separate facts are described. In the substantialistic stage substantial structure embodying the occurrence of phenomena is learnt, and in the essentialistic stage it is made clear what phenomenon an entity with a given structure causes under a given condition.

I feel that Sakata understood especially the importance of the substantialistic stage. This is perhaps related to the experiences of his young days in which various difficult problems of the inner-nucleus electrons were solved trivially by the discovery of the neutron or the introduction of the neutrino, not by the modification of physical law, as suggested by Bohr.

At the end period of the war Sakata's group — I was one of the members — was staying at Fujimi to escape from the war fire. In the autumn of 1945 we invited Taketani and discussed how to attack the divergence difficulties of the quantum field theory according to his methodology.

We discussed various theories proposed by that time. Among them the cut-off method of March and other, the subtraction theories of Bhabha and others or the $\lambda$-limiting process of Dirac attempt to remove the divergence by subjecting the theory to certain manipulations and so were looked upon as belonging to the phenomenological stage. On the other hand, the modified electrodynamics proposed

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by Mie, Born and Bopp are attempting to get a finite theory by clarifying the substantial structure of the basic interactions and so were regarded as belonging to the substantialistic stage.

Born’s theory is famous for its non-linearity. But as Bopp has pointed out its key point is not the non-linearity but that its Lagrangian involves derivatives higher than the second. In fact he succeeded in constructing a theory which is linear and still gives finite self-energy.

Thus we paid special attention to Bopp’s theory and tried to make its more detailed analysis. Soon it became clear that this theory is equivalent to a mixed theory of the electromagnetic field and a neutral vector meson field with negative energy. It is clear that such a type of theory can lead to a finite theory. For example, the static potential of a point charge is \( e^2/r \) and \(-g^2 e^{-kr}/r\) for the electromagnetic field and the neutral vector meson field with negative energy respectively. Therefore with the condition \( e = g \) the total static potential is

\[
e^2 \left( \frac{1}{r} - \frac{e^{-kr}}{r} \right) ,
\]

which is finite at \( r = 0 \).

Thus Sakata regarded that the essence of Bopp’s theory lies in the fact that it is a mixed theory, and inspired by this, he arrived at the viewpoint that part of the origin of the difficulties of quantum field theory lies in the formal logical treatment of the object and so it would be a promising path to the solution of the present difficulties to investigate the internal correlations between all fields interacting with the same particle and all particles acting as sources of the same field.

Thus he proposed mixed field theory as a guiding principle to solve the self-energy problem. This is to mix various fields and by suitably adjusting the coupling scheme try to eliminate the divergence.

He tried to modify Bopp’s theory from this viewpoint and proposed to replace the neutral vector field by a neutral scalar field to avoid the negative energy difficulty and named it cohesive meson or C-meson, since it acts as a kind of cohesive force to compensate the infinite Coulomb repulsion.

Following this line relativistic calculation was done for the electromagnetic self-energy of the electron or the proton, and it was shown that if the coupling constant \( f \) satisfies the condition

\[
f^2 = 2e^2
\]

the result is finite at least in the second order of the perturbation calculation.

Tomonaga, who heard this news, tried to apply the C-meson theory to the problem of radiative correction to the elastic scattering of an electron by a fixed potential. As pointed out by Bethe and Oppenheimer in this process the effect of the real and the virtual

![Fig. 1.](https://academic.oup.com/ptps/article-abstract/doi/10.1143/PTPS.105.193/1837734/05-March-2019)
photon cooperate in a delicate way and so it is a very sensitive test of the adequateness of any modification of quantum electrodynamics. Tomonaga showed that the radiative correction is given by

$$\delta \sigma_0 = 0\left\{ \frac{3e^2}{2\pi} \frac{m^2(p-q)^2}{(p^2+m^2)(p^2+pq+2m^2)} \int_{s_0}^\infty \frac{dk}{k} - \frac{4e^2}{3\pi} \int_{s_0}^\infty \frac{dl}{l} \right\}$$  \hspace{1cm} (3)$$

with

$$\sigma_0 = \text{const} \left| eV(p, q) \right| \frac{p^2+(p \cdot q)+2m^2}{p+m^2}$$  \hspace{1cm} (4)$$

and pointed out that the first divergence is just canceled by the C-meson hypothesis. He went further and showed that $\delta \sigma_0$ can be written is

$$\delta \sigma_0 = \frac{\partial \sigma_0}{\partial m} \delta m + \frac{\partial \sigma_0}{\partial e} \delta e,$$  \hspace{1cm} (5)$$

where $\delta m$ and $\delta e$ are given by

$$\delta m = m \frac{3e^2}{2\pi} \int_{s_0}^\infty \frac{dk}{k}, \quad \delta e = -e \frac{2e^2}{3\pi} \int_{s_0}^\infty \frac{dl}{l}$$  \hspace{1cm} (6)$$

with the first and the second term in Eq. (3) corresponding to the first and the second term of Eq. (6) respectively.

These results show that in this process the divergence can be ascribed to the modification of $m$ and $e$ by $\delta m$ and $\delta e$, and that the C-meson hypothesis is successful in eliminating the mass type divergence $\delta m$. Once this was made clear, perhaps it was not so difficult for Tomonaga to arrive at the idea of renormalization, abstracting the content of the C-meson theory. Thus the C-meson theory led Tomonaga to the detailed analysis of the nature of divergences in quantum electrodynamics and was a direct motive to his renormalization theory. In this respect he was especially lucky in that he had completed by that time a completely relativistically invariant formulation of the quantum field theory, which enabled him to make these analysis less ambiguously.

The mixed field theory was applied to other problems, too. For example, Umezawa et al. applied it to the problem of the self-energy of the photon. They showed that the divergence can be eliminated by suitably mixing the charged fermion and the boson fields. It is a realistic background of the regulator method proposed by Pauli and Villars, and also is a prototype of supersymmetry which is now discussed widely. From these results it may be said that the C-meson theory — or more generally the mixed field theory — has played a respectable role in the study of quantum electrodynamics in Japan after the War.

References

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