The excitation spectrum of a nucleon with spin $1/2$ is examined using the Friedberg-Lee model containing constituent quarks and a scalar meson. An appropriate method of quantization for the non-linear meson field is employed by taking account of the non-topological soliton existing at the classical level. Our model space for the nucleon resonances includes the three quark plus one meson state, in addition to the pure three quark state. The excitation spectrum in this model space reveals that the positive parity state appears as the first excited state associated with the $0^+_s$ excitation of the scalar meson. The meson excitation also generates an additional negative parity state apart from the well-known $0^+_p$ excitation of the quark.

§1. Introduction

Since the internal structure of hadrons was recognized, various kinds of quark models have been proposed. These models have been successful in describing hadron properties. However, there are some exceptions for which reasonable explanations have not been given yet. One of the most difficult problems is that the observed first excited state of the nucleon has positive parity, contradicting many predictions commonly obtained with quark models.7)

Several works suggest that extra degrees of freedom should be introduced so as to reproduce the observed spectrum of a nucleon with spin $1/2$, e.g. surface vibration in the bag model. However, plausible candidates for the extra degrees of freedom can be found in the quark models; they usually contain a gluon and/or a meson in addition to the quark. The dynamical role of these bosons for the internal structure of baryons has not been investigated thoroughly, because of the static, and in many cases perturbative, treatment applied to it.3,5)

The Skyrme model is another type of the effective model which is composed only of a meson field with a non-linear self-interaction. Although general features of the nucleon resonances are not reproduced with this model, the positive parity state appears as the first excited state, owing to the surface vibration of the topological soliton.10) This result indicates that the introduction of a quantized meson field with a non-linear self-interaction may solve the parity-ordering problem in the quark models.

In order to realize this idea, the soliton bag model originally proposed by Friedberg and Lee is one of the well-known models that can treat the meson field non-perturbatively. A non-topological soliton of the scalar meson is formed in this model due to the non-linear self-interaction of the meson and its coupling with the quarks. This soliton behaves as an effective binding potential for the quarks.

It is interesting to study the nucleon resonances in this model for the purpose
of exploring the dynamical role of the mesons. In the existing works dealing with nucleon resonances in the Friedberg-Lee model,\cite{13,14} the mean-field approximation is applied to the scalar meson field instead of properly quantizing it. Thus the excitation spectrum obtained therein exhibits the same character as usual: the first excited state is predicted to have negative parity. Furthermore, the orthogonality among the quark wave functions belonging to different excited states is lost in their treatment.

In this paper, we investigate the role of the scalar meson in the structure of nucleon resonances with spin 1/2 using the Friedberg-Lee model. Although this model does not include the pion field, which is also important in low-energy hadron phenomenology, static properties of the nucleon can be reproduced with this model reasonably well.\cite{15} We regard this simple model as suitable for the first step of our exploration. We quantize the scalar meson field by introducing fluctuations around the non-topological soliton.\cite{16,17,18} Note that the scalar meson field in the Friedberg-Lee model does not necessarily correspond to the so-called $\sigma$-meson as a chiral partner of the $\pi$-meson (or as a $\pi\pi$ s-wave resonance). In this work, following the original idea introduced in Ref. 11), we consider this scalar meson field as a purely phenomenological field to generate the binding potential for the quark. Examination of the properties of this scalar meson is left for a future work in which our model will be improved by taking into account the chiral symmetry.

This paper is organized as follows. In §2, we give the formulation of our model. For simplicity, we replace the relativistic quark by a non-relativistic quark. Calculating the spectrum for several low-lying states of the nucleon resonances, we examine whether the Friedberg-Lee model with the quantized scalar meson properly predicts the observed sequence of intrinsic parity. Our results and discussion are given in §3.

§2. Formalism

The Friedberg-Lee model with a non-relativistic quark field is based on the Hamiltonian

$$H = \int d^3r \left[ \psi^\dagger \left( -\frac{\nabla^2}{2m} + g\sigma \right) \psi + \frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \sigma)^2 + U(\sigma) \right], \quad (2.1)$$

where $\psi$ is the two-component spinor field for the constituent quark. The scalar meson field and its conjugate field are denoted by $\sigma$ and $\Pi$, respectively. The potential $U(\sigma)$ is explicitly given by $a\sigma^2(\sigma - \sigma_v)^2/2$, which involves non-linear terms up to fourth order in $\sigma$. We do not employ a more general form of $U(\sigma)$ in order to reduce the number of phenomenological parameters. The parameters $a$, $\sigma_v$, $g$ and $m$ represent the potential strength, the vacuum expectation value of $\sigma$, the meson-quark coupling constant and the constituent quark mass, respectively. The Hamiltonian (2.1) does not include direct interactions among the quarks such as the one-gluon-exchange potential.

Here we quantize the quark and the scalar meson fields. The conventional canon-
where the annihilation and creation operators $c_n$ and $c_n^\dagger$ satisfy the usual anti-commutation relation. In the case of a scalar meson field, which is assumed to form a non-topological soliton in the classical treatment, we quantize the fluctuations around this non-topological soliton using the canonical method:

$$\sigma(r, t) = \sigma_0(r) + \chi(r, t),$$

$$= \sigma_0(r) + \sum_k \frac{1}{\sqrt{2\omega_k}}[b_k e^{-i\omega_k t} s_k(r) + b_k^\dagger e^{i\omega_k t} s_k^*(r)].$$

Here, the annihilation and creation operators $b_k$ and $b_k^\dagger$ satisfy the usual commutation relation. Note that the meson field is not merely the semi-classical background for the constituent quark.

The complete orthogonal sets $\psi_n(r)$ and $s_k(r)$ and the eigenenergies $\epsilon_n$ and $\omega_k$ in the expansions (2.2) and (2.3) are specified by solving the eigenvalue equations

$$\left(-\frac{\nabla^2}{2m} + g\sigma_0(r)\right) \psi_n(r) = \epsilon_n \psi_n(r),$$

$$\left(-\nabla^2 + U''(\sigma_0(r)) - \omega_k^2\right) s_k(r) = 0,$$

where $n$ and $k$ denote the quantum numbers of the quark and the scalar meson, respectively. In deriving Eq. (2.5), we assume that a perturbative treatment is possible for the field $\chi$. Hence we limit the number of the scalar mesons to one and zero.

The non-topological soliton solution $\sigma_0(r)$ appearing in Eqs. (2.3)–(2.5) is obtained by solving

$$\nabla^2 \sigma_0(r) - U'(\sigma_0(r)) = 3g\psi_0^\dagger(r)\psi_0(r),$$

where $\psi_0(r)$ is the lowest-energy solution ($n = 0$) of Eq. (2.4). When we specify the right-hand side of Eq. (2.6), all three quarks are assumed to be in the ground state. To determine $\sigma_0(r)$ and $\psi_0(r)$, we solve the coupled equations Eq. (2.4) for $n = 0$ and Eq. (2.6). The non-linear terms included in $U'(\sigma)$ and the coupling between the quark and the meson lead to the existence of the soliton solution $\sigma_0(r)$. Then we obtain $\psi_n(r)$ for other $n$ and $s_k(r)$ by solving Eqs. (2.4) and (2.5).

The soliton solution also behaves as the effective binding potential for the quarks, which has the asymptotic value $g\sigma_v$ for infinitely large $|r|$. Therefore, in addition to the bound state solution, there exist continuum states. In the following calculations, we do not take into account several quantum corrections, such as the loop correction. We consider only the three-quark bound states for the nucleon and neglect all contributions of the continuum states.

Here we construct the single baryon states in the Fock space defined by the operators in Eqs. (2.2) and (2.3). They are classified into two types: the pure
three quark state and the three quark plus one scalar meson state. We use the ket $|N,LS;J⟩$ to represent the pure three quark state, where $N$ is a collective quantum number including the spatial quantum number ($n$) and the spatial symmetry, $L$ is the total orbital angular momentum, $S$ is the intrinsic spin, and $J$ is the total spin of the state. For example, $N = [(0s)(0p)^2]_M$ means that two quarks are excited to the $0p$ orbit, while the other one is in the $0s$ orbit. This state has mixed symmetry with respect to particle exchange, denoted by the subscript $M$. We use a symmetric combination of the spatial and the $SU(6)$ spin-flavor quantum numbers for the three quark state, because the color part is always antisymmetric by itself. We write the three quark plus one meson state as

$$|N,LS;J_{3q},l;J⟩ = [b^+_k|N,LS;J_{3q}]_J,$$

(2.7)

where $J_{3q}$ is the total spin of the three quark state, and $b^+_k$ operates on the meson vacuum, that is, the pure three quark state. Inclusion of this state distinguishes our study from the conventional ones because they usually employ only the pure three quark state to describe baryon resonances.

By using the expansions (2.2) and (2.3), the Hamiltonian (2.1) becomes

$$H = H_0 + H_I,$$

(2.8)

$$H_0 = E_\sigma + \sum_n \epsilon_n c^+_n c_n + \sum_k \omega_k b^+_k b_k,$$

(2.9)

$$H_I = \int d^3r \left( g \psi^\dagger \psi \chi - g \langle 0 | \psi^\dagger \psi | 0 \rangle \chi + O(\chi^3,\chi^4) \right),$$

(2.10)

where $E_\sigma$ is the classical energy of the non-topological soliton and $|0⟩$ indicates that all three quarks occupy the ground state with no meson. The difference between the present model and the usual Friedberg-Lee model is the appearance of the third term in Eq. (2.9) and $H_I$, which results from the fact that we quantize the scalar meson field. In our model, the scalar meson contributes to the excitation energies if the baryon state includes one meson. There exists explicit coupling of the quark with the meson in $H_I$. Because of the perturbative treatment of $\chi$, higher-order corrections due to $O(\chi^3,\chi^4)$ terms are not considered here.

To obtain the energy eigenvalues of the Hamiltonian (2.8), we employ the diagonalization method. As for the basis states, we use the single baryon states given above.

§3. Results and discussion

3.1. Parameters

We set the constituent quark mass at 300 MeV, as is commonly used in various non-relativistic models. We determine the values of the three other parameters, $a$, $\sigma_v$ and $g$, by referring to the observed spectrum for the spin 1/2 nucleon resonances: the first and second excited states are the positive and negative parity states, and their excitation energies are about 500 and 600 MeV, respectively. Many results
of the constituent quark models suggest that the mass of this positive parity state cannot be explained by the orbital or nodal excitations of the quark, while the negative parity state is described by the orbital excitation of the quark.\textsuperscript{22) This observation motivates us to find parameter sets which satisfy the following condition: the first excited state is due to the one meson 0s-excitation, instead of the quark excitation, with excitation energy 500 MeV, and the second one is due to the orbital 0p-excitation of the quark with excitation energy 600 MeV.

The results of our search for values of $a$ and $\sigma_v$ are shown in Fig. 1 as functions of $g$. We find monotonic behavior of these parameters. For values of $g$ smaller than 20, the quark single particle state has no bound states above the 0p orbit. In the following calculation, we consider only those parameters for which there are at least four bound states, 0s, 0p, 0d and 1s orbits for the quark single-particle state. For values of $g$ larger than 50, although curves do not appear in Fig. 1, we can easily obtain the values of $a$ and $\sigma_v$ by using extrapolation.

Now let us calculate other quantities obtained with the single-particle wave functions and examine their dependence on the parameters given in Fig. 1. First, we calculate the root mean square radius of the 0s-quake wave function. We find that this quantity is nearly constant at about 0.56 fm for each value of $g$, because the potential felt by the 0s quark does not change significantly.

However, we can see the difference between each parameter set if we observe the 0s-meson wave function. In Fig. 2, we see that the 0s-meson wave function is gradually pushed out for larger values of $g$. The wave functions of other excited states exhibit the same behavior as the 0s-meson wave function. Thus we can specify the best-fit parameters if there are some observables which are sensitive to the change in the root mean square radius of the meson wave function. But unfortunately we have no information about such observables.
3.2. Spurious states

Before going into the diagonalization, we consider the spurious states inevitably appearing in our formalism. These states do not describe the internal excitation but the excitation of the center of mass (c.m.) motion. To remove them from our basis states, the shell model technique is useful for our practical calculation: we project out the spurious states by calculating the matrix elements of an artificial potential \( V(R) \) that depends only on the c.m. coordinates \( R \) of the system.\(^{19} \)

We take the harmonic oscillator form for \( V(R) \) (\( \propto R^2 \)) to simplify the calculation.

We take special notice of the spurious states appearing in negative parity states, since these states correspond to the negative parity resonances observed unambiguously in experiments. First, we consider the pure three-quark states with the \( 0p \) excitation of one quark. Applying the above described technique, we can extract two of the three states belonging to these negative parity states,

\[
\left| [(0s)^2(0p)]_M, 1s; \frac{1}{2} \rightangle,
\]

with \( s = \frac{1}{2} \) and \( \frac{3}{2} \). Next, we consider the three quark plus one meson states when one of their constituents is excited to the \( 0p \) orbit. We obtain three states after removing the spurious state:

\[
\left| [(0s)^2(0p)]_M, 1s; \frac{1}{2}, 0; \frac{1}{2} \rightangle,
\]

with \( s = \frac{1}{2} \) and \( \frac{3}{2} \), and \( \lambda_1 \left| [(0s)^3]_S, 0 \right\rangle \frac{1}{2}; \frac{1}{2}, 1; \frac{1}{2} \rangle + \lambda_2 \left| [(0s)^2(0p)]_S, 1s; \frac{1}{2}; 1; \frac{1}{2} \rightangle \), where \( \lambda_1 \approx \lambda_2 \), which is found by numerical calculation. We note that the contributions from the spurious energy, which is inevitably included in this system, are not removed in this prescription. Although a few methods have been proposed for this purpose (for example, see Ref. 20)), no reliable method has been established to resolve this problem. Therefore, we do not touch on this problem now and leave it as a future subject.

There is no effect of the spurious motion on the first positive parity excited state. Also, because other positive parity states appear in the higher energy region, we do not touch on the spurious states here.

3.3. Excitation spectra

We now consider the excitation spectra obtained by diagonalizing the Hamiltonian using several parameter sets obtained above. The number of basis states is 14 for the positive parity states and 8 for the negative parity states. We list several lower basis states in Table I and display the excitation spectrum in Fig. 3. The ground state energy is set to zero in this figure. Our effective model is developed so as to reproduce the observed structure of the nucleon, but this model is not capable of determining the absolute value of the ground state energy.

The following features are common to all spectra obtained for each value of \( g \). The ground state is mainly composed of the three quark state in which all three quarks occupy the \( 0s \) orbit. This is consistent with many results of other quark models.

We obtain the positive parity state as the first excited state. Its main component is the three \( 0s \) quark plus one \( 0s \) meson. This result is obtained, because we can determine the appropriate parameter sets satisfying the condition given in §3.1. We also find that the meson-quark interaction \( H_I \) has only a weak influence on the structure of each state due to the small overlap between the quark and the meson.
Table I. Several examples of the basis states. The spatial symmetry ($S$ or $M$) is indicated by the irreducible representation of $S_3$.

<table>
<thead>
<tr>
<th>positive parity</th>
<th>negative parity</th>
</tr>
</thead>
<tbody>
<tr>
<td>[(0s^3)_s, 0, \frac{1}{2}, \frac{1}{2}]</td>
<td>[(0s^3)_s, 1, \frac{3}{2}, \frac{1}{2}]</td>
</tr>
<tr>
<td>[(0s^3)_s, 0, \frac{1}{2}, \frac{1}{2}, 0; \frac{1}{2}]</td>
<td>[(0s^3)_s, 0, \frac{1}{2}, 0; \frac{1}{2}]</td>
</tr>
<tr>
<td>[(0s^3)_s, 0, \frac{1}{2}, \frac{1}{2}, 0; \frac{1}{2}]</td>
<td>[(0s^3)_s, 0, \frac{1}{2}, 0; \frac{1}{2}]</td>
</tr>
</tbody>
</table>

Fig. 3. The excitation spectra of the spin 1/2 nucleon resonance for the four parameter sets. The positive and negative parity states are indicated by the solid and the dotted lines, respectively. The energy of the ground state is set to zero. The negative parity states above the first excited state are doubly degenerate.

wave functions: The quark wave function is concentrated in the central region of the nucleon by $g\sigma_0(r)$, while the meson wave function is distributed around the surface by $U''(\sigma(r))$.

Brown et al. introduced the breathing mode in the MIT bag model to explain the excitation energy of the Roper resonance. The distribution of the quantized bag surface is very similar to that of the scalar meson in our model. Also it is worth noting that both the bag and the scalar field are related to the confinement phenomenon. Thus our model can also be interpreted as a microscopic expression of the breathing mode.

However, we stress here that we can reproduce the parity of the first excited state without introducing any artificial degrees of freedom. The surface of the MIT bag model has no dynamical origin, and the quantization was carried out in a somewhat ad hoc way. On the other hand, the scalar meson field is one of the dynamical degrees of freedom in our model from the start, and it is treated on an equal footing with the quarks.

The first excited state is due to the $0s$ excitation of the scalar meson in the nucleon, or equivalently the monopole mode of the fluctuation of the soliton surface.
This structure is consistent with the idea of Morsch et al. They consider the Roper resonance as the monopole excitation of the nucleon (the compression mode) using the analysis of the scalar-isoscalar excitation of the nucleon due to α-p scattering.

Now we turn to the negative parity states. Above the first excited state, there are two negative parity states ($S = 1/2$ and $3/2$) degenerate in energy. Their main components are the three quark state with the $0p$ excitation of one quark. Their internal structure is consistent with many results of other quark models, and they are often allocated to the observed resonances $N(1535)$ and $N(1650)$. This mass difference is usually attributed to the tensor interaction of the one-gluon-exchange or one-meson-exchange potentials, which are not included in our present model.

In addition to these two states, another negative parity state appears above them by about 250 MeV in our model, which is mainly composed of the three quark plus one meson state. The appearance of this state is characteristic of our model. The correspondence of this state with experimental observation is not known, because there are no observed states in the neighborhood of well-known negative parity states. However, we believe that this state is neither superfluous nor a serious problem in our model. Before identifying our excited states with the observed resonances, we must apply the present model to the pion-nucleon reaction. It is quite possible that a pion probe cannot see this third negative parity state due to its weak coupling with the pion-nucleon channel.

Finally, we consider the $g$ dependence of the spectrum. The first and second excited states exhibit a weak $g$ dependence, because the excitation energies of their main components, the $0s$ and $0p$ quarks and the $0s$ meson, are constrained in the parameter search. In contrast to these states, other excited states exhibit prominent $g$ dependence. The positive parity excited states shift downward as $g$ increases, while the negative parity states remain at nearly constant energies. This behavior is not due to the quark-meson interaction $H_I$, but it is closely related with the $g$ dependence of the quark single-particle energy level. The positive parity excited states considered here include the quark in the $1s$ or $0d$ orbit. These single-particle energy levels for each parameter set. The energy difference between $0s$ and $0p$ is constrained to be 600 MeV.

![Fig. 4. The quark single-particle energy levels for each parameter set. The energy difference between $0s$ and $0p$ is constrained to be 600 MeV.](https://academic.oup.com/ptp/article-abstract/105/3/449/1834323)
states shift downward as $g$ increases as shown in Fig. 4. The negative parity states include the quark in the $0p$ orbit, whose excitation energy is constrained by the condition given in §3.1.

In the case of a large value of $g$, we find positive parity states with relatively small excitation energies, which may not correspond to the actual nucleon resonance, even if we project out the spurious states. From this observation, it seems that a large value of $g$ may not be appropriate. Furthermore, the work of Ref. 15) suggests that small values $g \approx 10$ should be used.

§4. Summary

We have calculated the excitation spectrum of nucleon resonances with spin $1/2$ using the Friedberg-Lee model. We have quantized the scalar meson field around the classical solution for the non-topological soliton instead of using the mean-field approximation. The positive parity state appears as the first excited state in our calculation. This is consistent with the observed nucleon resonance spectrum. The first excited state is mainly composed of three $0s$ quarks plus one $0s$ meson, and we believe that this first excited state may correspond to the Roper resonance. This interpretation is significantly different from the conventional one in quark models. For example, in the constituent quark models with the harmonic oscillator confinement potential, the Roper resonance has been described as a $2\hbar\omega$ excitation of the constituent quark. Above this state, according to our results, there appear two degenerate negative parity states. Their structures are consistent with those in the usual quark models. There is another negative parity state above these two states. This is characteristic of the present model.

We have shown that the dynamical treatment of the scalar meson in the constituent quark model may solve the parity ordering problems in the nucleon spectrum. Thus we believe that the Friedberg-Lee model with quantized scalar meson field is worth studying further as an effective model for baryon resonances.

To improve our present model, we must include the pion field, which is important in the low energy phenomenology. This can be achieved along the line of Ref. 23). We will also apply our model to the analysis of pion-nucleon reactions. This application is important to confirm the idea that the nucleon resonances are described as composites of quarks and mesons.

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