An Interesting New Revelation on Simultaneous Higher Order Squeezing in an Electro-Magnetic Field

Biswanath Rath

Physics Department, Ravenshaw College, Cuttack-753003, India

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We find that the superposition of coherent states $\Psi_{scs}$ that generates $|4m\rangle$ Fock states can squeeze the electromagnetic field conjugate at both fourth order and sixth order $\langle \Psi_{scs} | (\Delta E_i)^{2N} | \Psi_{scs} \rangle < \langle \alpha | (\Delta E_i)^{2N} | \alpha \rangle_{coh}$ $(i = 1$ and $2; N = 2, 3)$. A very interesting point which the present calculation reveals is that the minimum squeezing in both the cases (i.e. $N = 2, 3$) occurs at $|\alpha| = 0.67$. A further interesting point is that the smallest eigenvalue of $E_i^{2N}$ obtained by using a superposition of $|4m\rangle$ Fock states remains nearly the same as the minimum squeezing value: $\langle (\Delta E_i)^{2N} \rangle_{\text{minimum squeezing}} \approx \langle (E_i)^{2N} \rangle_{\text{smallest eigenvalue}}$ $(i = 1$ and $2; N = 2, 3)$.

The subject of squeezing in quantized electro-magnetic fields has received a great deal of attention $^{1)-5}$ because of its possible use for noise reduction in detection mechanisms. In the case of second (lower) order squeezing, $^3)$ a field in a squeezed quantum state exhibits fluctuations that are smaller than those in a coherent state in one quadrature component at the cost of larger fluctuations in the other quadrature component. If information could be impressed on and extracted from the squeezed component, this could prove valuable in the optical communication channels.

Mathematically, the quadrature field component is squeezed if

$$\langle \Phi | (\Delta E)^2 | \Phi \rangle < \langle \alpha | (\Delta E)^2 | \alpha \rangle_{coh},$$

(1)

which implies

$$\langle \Phi | (\Delta H)^2 | \Phi \rangle > \langle \alpha | (\Delta H)^2 | \alpha \rangle_{coh},$$

(2)

where $E$ and $H$ represent electric and magnetic fields, respectively. The second order fluctuation of $E$ in the given state $|\Phi\rangle$ is less than that obtained in a coherent state.$^{1)-5}$ In this context, we should check that the conjugate fields satisfy the commutation relation,

$$[E, H] = i.$$  

(3)

In the above,

$$\langle \Phi | (\Delta E)^2 | \Phi \rangle = \langle \Phi | [E - \langle \Phi | E | \Phi \rangle]^2 | \Phi \rangle$$  

(4)

and

$$\langle \Phi | (\Delta H)^2 | \Phi \rangle = \langle \Phi | [H - \langle \Phi | H | \Phi \rangle]^2 | \Phi \rangle.$$  

(5)

Physically, in second order squeezing, it is practically impossible to find out a state that can simultaneously squeeze both quadratures, because the uncertainty product

$$\langle \Phi | (\Delta H)^2 | \Phi \rangle \langle \Phi | (\Delta E)^2 | \Phi \rangle$$
has a minimum value. However, with the development of techniques for making higher-order correlation measurements in quantum optics, interest has naturally extended to the higher-order moments of the field. In this context, Hong and Mandel (HM) proposed the concept on higher-order squeezing (by visualising the concept at lower order, as given in Eqs. (1) and (2)). They defined a natural generalization of the second order by terming a state squeezed to $2^N$ order ($N = 1, 2, 3, 4, \cdots$) in $E_1$ if there exists a phase angle $\theta$ such that $\langle (\Delta E_1)^{2N} \rangle$ is smaller than that for a completely coherent state of the field. Mathematically, in terms of electro-magnetic fields, this can be written as follows: If

$$\langle \Phi | (\Delta E_1)^{2N} | \Phi \rangle < \langle \alpha | (\Delta E_1)^{2N} | \alpha \rangle \text{coh},$$

then

$$\langle \Phi | (\Delta E_2)^{2N} | \Phi \rangle > \langle \alpha | (\Delta E_2)^{2N} | \alpha \rangle \text{coh},$$

where $E_1$ and $E_2$ are the electric and magnetic or magnetic and electric fields, respectively.

The meaningfulness of the concept proposed by HM is clearly reflected in their calculation for the component which is squeezed (for details see Ref. 3)). In a very lengthy paper, Buzek, Vidiella-Barranco and Knight (BVK) studied HM squeezing for non-electro-magnetic conjugate operators using a superposition of coherent states that generates $|2m\rangle$ Fock states. However, after the publication of the paper concerning higher-order squeezing by HM, Lynch contradicted some of the claims of HM regarding higher-order squeezing for $N = 2$ stating that “there exists a (Jackiw) state infinitesimally different from the coherent state in which both quadrature components are squeezed in fourth order”. Then, in the reply to Lynch’s work, HM agreed that simultaneous squeezing at fourth order is possible. Later, works of Lynch and Mavromatis almost all confirm the previous conclusion.

An interesting point regarding fourth-order squeezing ($N = 2$) is that some experimental evidence has been reported by Yurke and Stoler and some ideas have been suggested by Jex and Buzex. While the previous works have yielded much information on squeezing, but this information is not complete. Therefore, in an attempt to obtain more information on simultaneous squeezing at higher (fourth and sixth) order we discuss the following questions using the superposition of coherent states. (i) How many Fock states $|km\rangle$ can be generated by using the superposition of coherent states? (ii) For a given value of $k$, can the higher-order operators $E_2^{2N}$ ($N = 2, 3, 4, 5, \cdots$) be grouped? (iii) If a group contains more than one higher-order operator, how does their squeezing behaviour match? (iv) How does the smallest eigenvalue obtained by using the superposition of $|km\rangle$ Fock states match with the minimum squeezing value for the operators in a group?

Before we discuss the present calculation, it is worth mentioning that the phase is an important factor in the squeezing calculation, and it can be introduced in many ways: (i) only in field operators [leading to define the electric field ($E$) and magnetic field ($H$)]; (ii) only in eigenvalues (i.e. in $\alpha$); (iii) only in the state function i.e. in $\Psi_{scs}$. Even though it is possible to carry out the calculations in any one of the three cases, here we only consider the case in which the phase is in $E, H$ and in $\alpha$ of the coherent state $|\alpha\rangle$.14
(i) **Electric field and magnetic field**

Here we follow the works of Loudon, Hong and Mandel and Yariv and consider the electric field as

\[
E(r,t) = E^(-)(r,t) + E^+(r,t)
\]

\[
= \frac{1}{\sqrt{2}} [a^+ e^{i(wt-k\cdot r)} + ae^{-i(wt-k\cdot r)}]
\]

and magnetic field as

\[
H(r,t) = H^(-)(r,t) + H^+(r,t)
\]

\[
= \frac{i}{\sqrt{2}} [a^+ e^{i(wt-k\cdot r)} - ae^{-i(wt-k\cdot r)}],
\]

where

\[
[a, a^+] = 1.
\]

Hence, we have

\[
[E, H] = i.
\]

The destruction operator \(a\) and creation operator \(a^+\), when acting on a photon number state or Fock state \(|m\rangle\), give the relations

\[
a|m\rangle = \sqrt{m}|m-1\rangle,
\]

\[
a^+|m\rangle = \sqrt{(m+1)}|m+1\rangle.
\]

Similarly, the destruction operator \(a\), when acting on coherent state, gives the eigenvalue relation

\[
a|\alpha\rangle = \alpha|\alpha\rangle,
\]

where \(\alpha = |\alpha|e^{i\theta}\) and \(\theta = \arg(\alpha)\).

From the above expressions for \(E\) and \(H\), we discuss higher (fourth and sixth) order squeezing using the multi-photon state function

\[
\Psi = \sum g_m|km\rangle,
\]

employing the superposition principle, because this principle can reveal a great deal of interesting squeezing behaviour. Now, we focus our attention on Eqs. (8a), (8b) and (10) and discuss the following squeezing behaviour.

(a) It is found from the work of BVK that the superposition of \(|\alpha\rangle\) and \(|-\alpha\rangle\) can generate a \(|2m\rangle\) Fock states:

\[
|\phi_{1,scs}\rangle = A^{1/2}[|\alpha\rangle + |-\alpha\rangle],
\]

with

\[
A = \frac{1}{2[1 + \exp(-|\alpha|^2)]}.
\]

In terms of Fock states, the expression in Eq. (11a) can be written as

\[
|\phi_{1,scs}\rangle = \frac{1}{\sqrt{\cosh|\alpha|^2}} \sum_{m=0}^{\infty} \frac{\alpha^{2m}}{\sqrt{(2m)!}}|2m\rangle.
\]
It is seen that by using the function $\phi_{1,scs}$, only one component, either the electric field or magnetic field, will be squeezed to $2N$ order, because $|2m\rangle$ Fock states can interact with all the combination of $a^+a$ yielding a phase factor of $2(wt - \theta - k \cdot r)$ or its multiple. Of course, the choice of the phase plays an important role. For example, if $2(wt - \theta - k \cdot r) = \pi/2$ then $H$ will be squeezed but not $E$. Similarly, if the choice of the phase is $2(wt - \theta - k \cdot r) = \pi$ then, $E$ will be squeezed but not $H$. However, BVK superposition of coherent states is not the only superposition that can yield $|2m\rangle$ Fock states. In fact, if we consider the superposition of $|i\alpha\rangle$ and $|-i\alpha\rangle$ states, then we can generate $|2m\rangle$ Fock states also:

$$|\phi_{2,scs}\rangle = A^{1/2}[|i\alpha\rangle + |-i\alpha\rangle],$$

with

$$A = \frac{1}{2[1 + \exp(-|\alpha|^2)]}.$$ 

In terms of Fock states, the expression in Eq. (12a) can be written as

$$|\phi_{2,scs}\rangle = \frac{1}{\sqrt{(\cosh|\alpha|^2)}} \sum_{m=0}^{\infty} (-1)^m \frac{\alpha^{2m}}{\sqrt{(2m)!}}|2m\rangle.\quad (12b)$$

It is interesting to note that the function in Eq. (12a) can squeeze only the $E$ component when the phase is $2(wt - \theta - k \cdot r) = \pi/2$, but not the $H$ component of the field. Similarly, if the choice of phase is $2(wt - \theta - k \cdot r) = \pi$, then the magnetic field will be squeezed, but not the electric field. In other words the function in Eq. (12a) or (11a) can squeeze only a single component in higher-order $E^{2N}$ or $H^{2N}$. That is, either

$$\langle \Phi_{1,scs}|(\Delta E)^{2N}\Phi_{1,scs}\rangle = \langle \phi_{2,scs}|(\Delta H)^{2N}\phi_{2,scs}\rangle$$

or

$$\langle \Phi_{1,scs}|(\Delta H)^{2N}\Phi_{1,scs}\rangle = \langle \phi_{2,scs}|(\Delta E)^{2N}\phi_{2,scs}\rangle$$

will be squeezed only. Suppose we mix Eq. (11a) and Eq. (12a). Then one will find that the superposition generates $|4m\rangle$ Fock states:

$$|\Psi_{scs}\rangle = B^{1/2}[|\alpha\rangle + |-\alpha\rangle + |i\alpha\rangle + |i\alpha\rangle]$$

with

$$B = \frac{1}{8\exp(-|\alpha|^2)[\cosh|\alpha|^2 + \cos|\alpha|^2]}.\quad (13b)$$

In terms of the number state basis, Eq. (13a) can be written as

$$|\Psi_{scs}\rangle = \frac{\sqrt{2}}{\sqrt{(\cosh|\alpha|^2 + \cos|\alpha|^2)}} \sum_{m=0}^{\infty} \frac{\alpha^{4m}}{\sqrt{(4m)!}}|4m\rangle.$$ 

(b) From Eqs. (8a) and (8b) we note that (i) if the superposition of coherent states generates $|4m\rangle$ Fock states, then it will squeeze the fourth order in $E$ or $H$, because the maximum value of $a$ or $a^+$ is 4. Further, simultaneous squeezing can be visualised, because, by definition, $E$ and $H$ both contain terms of the same sign,
that can interact with $|4m\rangle$ Fock states. The next question is why the sixth order will be squeezed by $|4m\rangle$ Fock states. The answer to this is that $a^4$ is maximum in $E_4$, but not in $E_6$. Hence $E_6$ will contain terms that can interact with minimum $|4m\rangle$ states, yielding an equal value in its conjugate also, i.e. in $H_6$. Hence, $E_4^4$ and $E_6^6$ are the only two higher-order powers of $E_i$ that can only be squeezed using $|4m\rangle$ Fock states:

$$\langle \Psi_{scs} | (\Delta E)^{2N} | \Psi_{scs} \rangle = \langle \Psi_{scs} | (\Delta H)^{2N} | \Psi_{scs} \rangle < \langle \alpha | (\Delta E \text{ or } H)^{2N} | \alpha \rangle_{coh}. \quad (14a)$$

$$(N = 2, 3)$$

Other higher orders can be squeezed with $|4m\rangle; |8m\rangle \cdots$ Fock states. For example, in $E_2^{2N}$, $N = 4$ and $5$ can be grouped together, because $E_4^8$ and $E_6^{10}$ can experience squeezing behaviour using the multi-photon states $|km\rangle$ for $k = 4$ and $k = 8$. Hence, these two operators can be put in one group. In the opinion of the present author, the behaviour for $k = 4$ should be practically the same as that with $k = 8$. Similarly, one can group higher-order operators.

In this context, the author would like to state that any superposition of coherent states that can generate a Fock states basis $|km\rangle$, where $k = 4, 8 \cdots$ can be proposed for observing simultaneous squeezing, but the minimum can only be achieved with $k = 4$, because this is the nearest to the ground state. Unfortunately, the author has not been able to construct a squeezed state with a $|8m\rangle$ Fock state basis by superposition of coherent states. For this reason, he has not considered specific calculations beyond $N = 2$ and $3$.

(c) In order to make a comparative study of the squeezing behaviour at fourth order and sixth order, we present a graphical behaviour of simultaneous squeezing.

(d) From the above physical argument, we believe that $|4m\rangle$ Fock states, with use of the superposition principle, can yield simultaneous squeezing at fourth order as well as at sixth order. An interesting question which may come to the reader’s mind is that, supposing one proposes a superposition of $|4m\rangle$ Fock states and calculates the smallest eigenvalue, then, how is the smallest eigenvalue compared with the minimum squeezing value? In other words, the comparison that holds between the smallest eigenvalue and the minimum squeezing value at fourth order will also be valid at sixth order i.e.,

$$(E_4^{2N} \text{ or } H_4^{2N})_{\text{smallest eigenvalue}} \simeq (E_4^{2N} \text{ or } H_4^{2N})_{\text{minimum squeezing value}} \quad (14b)$$

In this case also, we present the explicit calculation for $N = 2$ and $3$. However, if one desires to make some squeezing calculation without making use of the superposition principle, this can be done for any value of $k$, by suitably adopting the procedure for the generation of $g_m$ either with a perturbative approach or a non-perturbative approach, or any other suitable approach.

(ii) Simultaneous squeezing at fourth order

Let us now consider simultaneous squeezing at fourth order in detail. It is not difficult to see that $E_4 \neq H_4$. However it is seen that $E_4$ and $H_4$ contain the same term having the same sign, i.e. $E_4 = H_4 = Y$, where

$$Y = (1/4)[a^4 e^{-4i(wt - k \cdot r)} + (a^+)^4 e^{4i(wt - k \cdot r)} + 12a^+ a + 6(a^+)^2 a^2 + 3]. \quad (15a)$$
At this point we would like to state that terms of the type
\[ y = \frac{1}{4}[4a^+a^3e^{-2i(wt-kr)} + 4(a^+)^3ae^{2i(wt-kr)} + 6[a^2e^{-2i(wt-kr)} + (a^+)^2e^{2i(wt-kr)}]] \]  
(15b)
can increase or decrease the value of \( Y \), leading to \( E^4 \neq H^4 \).

From Eq. (15a) we find that only \(|4m\rangle\) Fock states lead to same value for both \( E^4 \) and \( H^4 \). At the same time, it is not difficult to show that Eq. (15b) will not contribute to \(|4m\rangle\) Fock states. Such \(|4m\rangle\) Fock states can be generated by the superposition of coherent states as in Eq. (13a). It is seen that when \(|4m\rangle\) states are taken into account, the matrix elements of both components are the same. Hence, the state \(|\Psi_{scs}\rangle\) yields the same value for both components.

Now we use the function \(|\Psi_{scs}\rangle\) in Eq. (13a) to evaluate \( E^4 \) and \( H^4 \). The expression in terms of \(|\alpha|\) is
\[ \langle \Psi_{scs}|(\Delta E)^4|\Psi_{scs}\rangle = \langle \Psi_{scs}|(\Delta H)^4|\Psi_{scs}\rangle = \frac{3}{4} + 3|\alpha|^4 \left[ \frac{\cosh |\alpha|^2 - \cos |\alpha|^2}{2 \cosh |\alpha|^2 + \cos |\alpha|^2} \right] \]
\[ + \frac{|\alpha|^4}{4}(e^{4i(wt-\theta-k \cdot r)} + e^{-4i(wt-\theta-k \cdot r)} + 3|\alpha|^2 \left[ \frac{\sinh |\alpha|^2 - \sin |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right]. \]  
(16a)

It is seen that when \( 4(wt - \theta - k \cdot r) = \pi \), Eq. (16a) can exhibit squeezing, and the squeezing value is
\[ Y = \langle \Psi_{scs}|(\Delta E)^4|\Psi_{scs}\rangle = \langle \Psi_{scs}|(\Delta H)^4|\Psi_{scs}\rangle \]
\[ = \frac{3}{4} + 3|\alpha|^4 \left[ \frac{\cosh |\alpha|^2 - \cos |\alpha|^2}{2 \cosh |\alpha|^2 + \cos |\alpha|^2} \right] \]
\[ - \frac{|\alpha|^4}{2} + 3|\alpha|^2 \left[ \frac{\sinh |\alpha|^2 - \sin |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right]. \]  
(16b)

The minimum value of squeezing can be found in two different cases: In the first case, we find the value of \(|\alpha|\) (we call this value “alpha”) from the relation \( \frac{dY}{d|\alpha|} = 0 \) for which Eq. (16b) is minimal. In the second case, we plot the function Eq. (16b) for different values of \(|\alpha| = alpha \) and find the minimum value of \( Y \). Here we follow the procedure for the second case, because we can note the values of \( Y \) for different \( alpha \).

In Fig. 1 we show the variation of \( E^4 \) or \( H^4 \) along the \( Y \) axis and \(|\alpha| = alpha \) along the \( X \) axis. We find that the minimum value of \( Y \) (at \(|\alpha| = 0.67\)) is 0.69992.

Further, we notice that simultaneous squeezing exists in the range \( 0 < |\alpha| < 0.7 \). However, if \( 4(wt - \theta - k \cdot r) = \pi/2 \), then no squeezing is seen.

(iii) Simultaneous squeezing at sixth order

As in the case of fourth-order squeezing, at sixth order \( E^6 \neq H^6 \). However, if we consider terms having the same sign, then we have \( E^6 = H^6 = Z \), where \( Z \) is given by the following:
\[ Z = (1/8)[(15a^4 + 6a^+a^5)e^{-4i(wt-kr)} + (15(a^+)^4 + 6(a^+)^5a)e^{4i(wt-kr)} + 90a^+a + 90(a^+)^2a^2 + 20(a^+)^3a^3 + 15]. \]  
(17a)
It is also seen that $Z$ can be squeezed by the same function $|\Psi\rangle$ as in Eq. (16b). Using the appropriate choice of the phase $4(\omega t - \theta - k \cdot r) = \pi$, it is not difficult to show that Eq. (17a) can exhibit squeezing. We find the squeezing value of $Z$ as

$$Z = \langle \Psi_{scs} | (\Delta E)^6 | \Psi_{scs} \rangle = \langle \Psi_{scs} | (\Delta H)^6 | \Psi_{scs} \rangle$$

$$= \frac{1}{8} + \frac{90|\alpha|^2}{8} \left[ \frac{\sinh |\alpha|^2 - \sin |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right] - \frac{30|\alpha|^4}{8} + \frac{90|\alpha|^4}{8} \left[ \frac{\cosh |\alpha|^2 - \cos |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right]$$

$$+ \frac{20|\alpha|^6}{8} \left[ \frac{\sinh |\alpha|^2 + \sin |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right] - \frac{12|\alpha|^6}{8} \left[ \frac{\cosh |\alpha|^2 - \sin |\alpha|^2}{\cosh |\alpha|^2 + \cos |\alpha|^2} \right],$$

To study squeezing, we plot Eq. (17b) along the $Y$ axis for different values of $|\alpha| = \text{alpha}$ along the $X$ axis in Fig. 2.

**It is seen that in this case also the minimum value of squeezing occurs at $|\alpha| = 0.67$.** Hence our conclusion is that if superposition of coherent states leads to the generation of $|4m\rangle$ Fock states, then simultaneous squeezing is possible at higher order. Further, the most interesting observation for $N = 2$ and 3 is that the minimum squeezing occurs at $|\alpha| = 0.67$. Mathematically, one can find $|\alpha|_{\text{minimum}}$ from $\frac{dZ}{d|\alpha|} = 0$. However, in the opinion of the author, it would be very difficult to explain physically why the simultaneous squeezing realizes the minimum value at $|\alpha| = 0.67$.

(iv) Smallest eigenvalue: superposition of $|4m\rangle$ Fock states

From Eq. (13a), it is seen that only Fock states of the type $|4m\rangle$ are generated and such a state becomes the squeezed state for $N = 2, 3$. Now, let us define a wave function using the superposition of $|4m\rangle$ Fock states and study the smallest eigenvalue using the matrix diagonalisation method. Here we use $\Psi_1$ in terms of Fock states $|4m\rangle$ to find the smallest eigenvalue by solving the eigenvalue relation

$$L|\Psi_1\rangle = \lambda|\Psi_1\rangle$$
assuming

$$\Psi_1 = \sum_{m=0,1,2\ldots} A_{4m} |4m\rangle$$  \hspace{1cm} (18b)

and using the normalisation condition\textsuperscript{15)}

$$\langle \Psi_1 | \Psi_1 \rangle = 1.$$  \hspace{1cm} (19)

Here we take $L = H^{2N}$ or $E^{2N}$ ($N = 2, 3$). In the standard matrix diagonalisation procedure, we obtain a three-term recurrence relation involving $A_{4m}$ as

$$A_{4m+4} T_m + A_{4m} Q_m + A_{4m-4} G_m = 0,$$  \hspace{1cm} (20)

where

$$T_m = \langle 4m| L_{N} |4m + 4\rangle,$$  \hspace{1cm} (21a)

$$Q_m = \langle 4m| L_{D} - \lambda |4m\rangle,$$  \hspace{1cm} (21b)

$$G_m = \langle 4m| L_{N} |4m - 4\rangle.$$  \hspace{1cm} (21c)

Here $|m\rangle$ is the $m$th state wave function of the harmonic oscillator. In Table I we display the convergence of the smallest value of $\lambda$. It is seen that this convergence is too fast.

<table>
<thead>
<tr>
<th>$N$ (matrix size)</th>
<th>Value of $(E$ or $H)^2N$ ($N = 2, 3$)</th>
<th>Value of $(E$ or $H)^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.700 008</td>
<td>1.523 950</td>
</tr>
<tr>
<td>15</td>
<td>0.698 364</td>
<td>1.477 182</td>
</tr>
<tr>
<td>25</td>
<td>0.698 364</td>
<td>1.476 523</td>
</tr>
<tr>
<td>45</td>
<td>0.698 364</td>
<td>1.476 522</td>
</tr>
<tr>
<td>75</td>
<td>0.698 364</td>
<td>1.476 522</td>
</tr>
</tbody>
</table>
In conclusion, we have found the interesting fact which arises in simultaneous squeezing for $N = 2, 3$ that both $Y$ and $Z$ attain their minimum value when $|\alpha| = 0.67$. It would be very difficult to explain physically why the squeezing for $N = 2$ and $3$ attains their minimum value at $|\alpha| = 0.67$. But this point should encourage interest in experiments.\textsuperscript{11), 12} However it is interesting to note that using the superposition of $|4m\rangle$ Fock states the smallest eigenvalue is nearly the same as the minimum squeezing value (see Table II).

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**References**