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Examination of the Summer Stratification

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The seasonal stratification in lakes is examined by a qualitative evaluation of important physical processes and with the aid of a mathematical model. From the description of the physical processes it is realized that the observed vertical temperature distribution is the synthesis of a number of complicated and inter-dependent phenomena. The mathematical model is employed for a quantitative estimate of the importance of lake morphometry and transparency. It is found that the vertical variation of horizontal area and variations in transparency strongly influence the thermal structure of a lake. Measurements of the temperature distribution in Lake Velen are compared to predictions obtained by the mathematical model. The agreement is found to be satisfactory and it is concluded that the mathematical model provides a realistic way of studying and predicting the summer stratification.

Introduction

The paper presented intends to provide a description of physical processes and lake characteristics important to the thermal structure of a lake.

Firstly a section is devoted to a qualitative description of the relevant physical processes, while the next section makes use of a mathematical model in order to obtain qualitative information. The mathematical formulation of this model is given in Appendix A. In the final section field measurements of the thermal structure of Lake Velen are compared with predictions obtained by the mathematical model.

Few words are needed for justifying a study of the vertical temperature structure of lakes. The following points are sufficient.

- The thermal structure is the frame for all biological life. Thus, knowledge of this structure is a basic requirement for all ecological and water quality models.
- Reservoirs and lakes are often used as cooling ponds by power plants. The consequences of this thermal impact must be estimated and interpreted.
- Most lakes are receiving some kind of sewage. The vertical temperature distribution has a dramatic effect on how sewage is spread within a water body.

More examples could easily be given, but the reader will probably already appreciate the practical importance of the problem.

In the light of this it is not surprising that a number of mathematical models of the problem has been advanced during the last ten years. No attempt will here be made to review these, instead the reader is referred to a recent paper by Niiler and Kraus (1977).

Physical Processes Affecting the Thermal Structure

The processes to be discussed in this section are summarized in Fig. 1. As can be seen, there are a number of complicated and interdependent processes to be considered. In order to simplify the discussion, it is useful to distinguish between processes affecting the heat content of the lake and those that merely redistribute heat within the lake.

Processes affecting the overall heat content

The principal source of heat energy for most lakes is incoming long and shortwave radiation. A small fraction of this flux will be reflected at the surface, while the bulk of it will be stored in the lake. According to the Stefan-Boltzmann law longwave radiation will be emitted from the water body. This term is thus always a sink term in the heat budget. Another surface flux of heat, which can be a sink or a source term, is due to conduction and evaporation. These are the processes at the air-water interface. Empirical formulas for their determination are available, see for example Robinson et al. (1972), Davies et al. (1975) and Weisman (1975).

Also affecting the overall heat budget is the heat flux to the sediments.

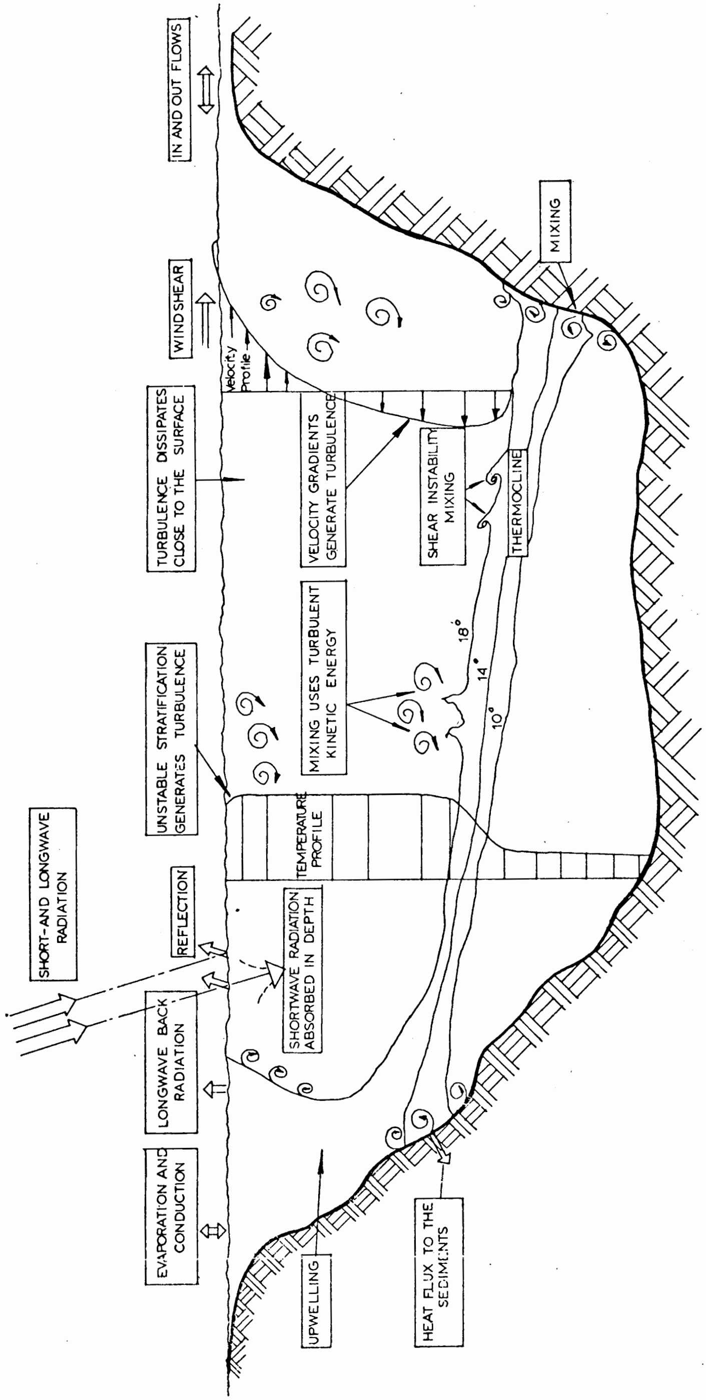


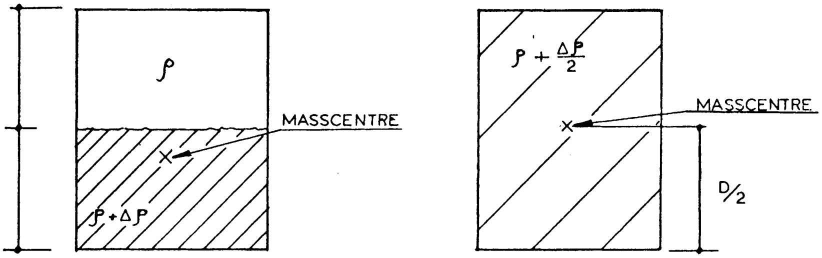
Fig.1. Illustration of physical processes affecting the thermal structure of a lake.

Measurements by Tandertz (1973) do however indicate that this flux is negligible when considering the total heat budget.

If significant inflows and outflows are present these also have to be considered.

Internal redistribution, or mixing, of heat

Much understanding of mixing processes has been gained from energy considerations. As a background to the discussion in this section the important fact that mixing of a stably stratified fluid requires energy will be illustrated, see Fig. 2. Initially the mass centre is at some level below D/2, but after the fluid has been mixed it is at D/2. The potential energy of the system has thus been increased.



a/ BEFORE MIXING

b/ AFTER MIXING

Fig.2. Internal redistribution of heat. Mixing of a stably stratified fluid requires energy.

In a lake, see Fig. 1, turbulent kinetic energy is generated by velocity gradients and unstable stratification. Most of this energy is dissipated close to the surface, but a small fraction of it will be available for mixing processes. How the actual mixing proceeds depends on the flow properties close to the thermocline. If large velocity gradients are present shear instability, or Kelvin-Helmholtz instability, may cause mixing, Thorpe et al. (1977). When no wind is present but the heat loss at the surface is substantial, for example the conditions during a cold calm night, the picture is more complicated. Deardorff et al. (1969) studied this situation in a laboratory experiment and found that a number of mixing processes, including local shear instability, is present.

The mixing processes mentioned so far may all be found in the ocean and the atmosphere and have therefore been studied in some detail. Processes specifically for lakes have received much less attention. One such process is the mixing generated by internal oscillations or seiches. When the »isotherms sweep the bottom« mixing can be expected. Another is mixing associated with upwelling which may occur during strong wind conditions since the isotherms are then known to be tilted due to pressure forces.

A few more factors, not illustrated in Fig. 1, may affect the internal redistribution of heat within the lake. To be mentioned are the thermal bar, important during the initial stratification in spring, and the Langmuir cells.

Significance of some Lake Characteristics

What are believed to be the most important of the processes just described have been included in a one-dimensional mathematical model. A complete description of the model will be presented elsewhere; in the present context only the basic equations will be provided, see Appendix A.

The mathematical model will in this section be used for investigating the significance of the morphometry and transparency of a lake. First a reference situation will be specified which has much in common with the development of the thermocline in the ocean. Features specific for a lake will then be introduced and studied.

The reference case

The development of the temperature profile will be studied for 4 weeks starting from a uniform temperature of 6°C. The wind stress and the heat loss at the surface are prescribed to be 0.04 N/m^2 and 80 W/m^2 respectively, with no variation in time. Zero heat flux and velocity are used as boundary conditions at the bottom. The specified absorption coefficient for shortwave radiation, β, is 0.3 which according to Pivovarov (1973) means that a white disc is visible down to approximately 10m. A constant area versus depth is prescribed and total depth of the lake is 30m. The calculated profiles after 2 and 4 weeks are shown in Fig. 3 as solid lines. Mixed layer temperatures are found to be 8.5°C and 12.3°C respectively.

Variation of horizontal area versus depth

The relation between lake morphometry and stratification has been studied by Mayhew (1973). He found that strong stratification is related to small values on the volume development, defined as $3D_{\text{mean}}/D$. In order to study this relationship we will assume a linear increase in horizontal area along the vertical coordinate and thus reducing Mayhew's index to 1.5, compared to 3.0 in the reference case. Except for this change all other conditions of the reference case will be retained.

In Fig. 3 it can be seen that the predicted effect on the temperature distribution is in line with Mayhew's conclusion. After 4 weeks the mixed layer temperature is 3.5°C higher than in the reference case. This effect could in fact be expected since a smaller volume of water is receiving the same amount of heat when the horizontal area is decreasing with distance from the surface.

The conclusion to be drawn from this is that the volume development has an important effect on the thermal structure of a lake.

The limited horizontal extent of a lake

Due to the limited horizontal extent of a lake pressure forces will be generated and modify the circulation pattern. Realizing that the effect of the pressure forces is to ensure overall mass conservation, tentative expressions for their determination may be formulated, see Eq. (A5), Appendix A. Formulas of this kind have been used for investigating the effect of pressure on the development of the thermocline. The horizontal dimensions were specified by an average length, chosen to be 10^5m, and an average width, chosen to be 2.10^4m.

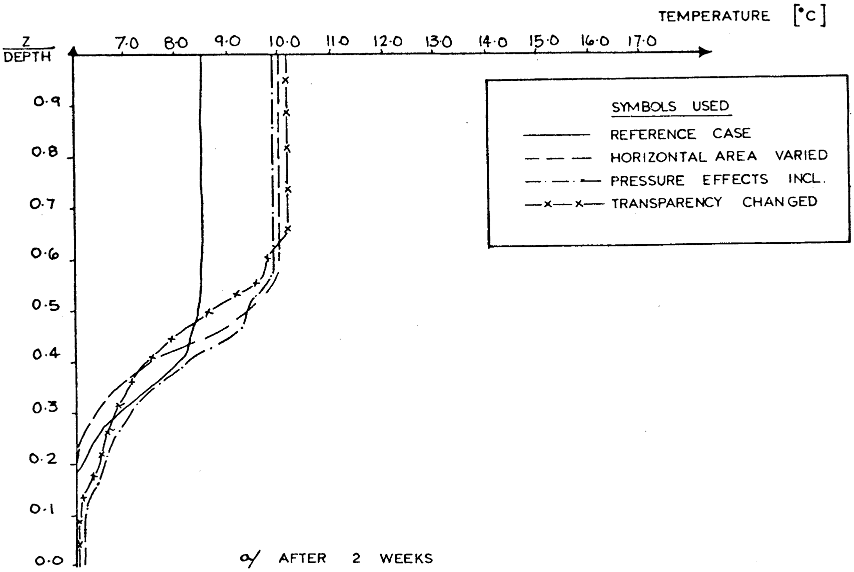
As above, a linear increase in horizontal area was prescribed. The effect on the temperature profiles can be studied in Fig. 3. The change in the temperature distribution is within the uncertainty of the pressure formula and will not be considered as significant.

Any conclusion to be drawn from this is dependent on the confidence in the formulae for the pressure gradients. The only firm statement that can be made is that the predictions presented indicate that the effect of the limited horizontal extent of a lake is small.

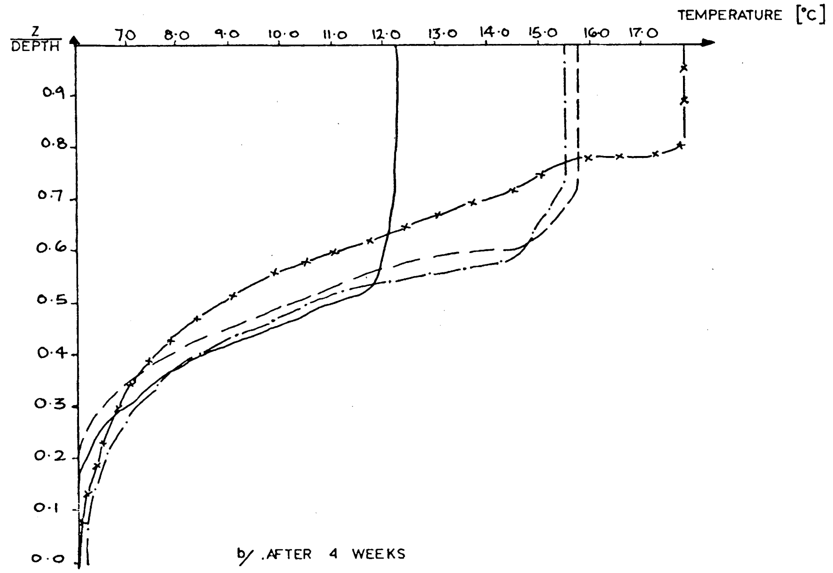
It should be noted that mixing due to upwelling, discussed in the previous section, is dependent on the horizontal extent of a lake. This process has not been taken into account since it is outside the capability of a one-dimensional model.

The effect of variations in transparency

The attenuation of shortwave radiation is normally assumed to follow an exponential decay law. The rate of decay is specified by the absorption coefficient, β. Doubling β, i.e. making the water less transparent, produces a significant effect as shown in Fig. 3. Pressure gradients and volume development were included as earlier. This effect of transparency is expected since more energy is needed to



a/ AFTER 2 WEEKS



b/ .AFTER 4 WEEKS

Fig.3. Effect of morphometry and transparency on the thermal structure of a lake. Vertical temperature distributions after 2 weeks (a) and after 4 weeks (b).

produce a mixed layer when the heat is absorbed close to the surface. The magnitude of the change of the mixed layer temperature does however emphasize the need for advanced treatments of radiation processes in natural water bodies.

The Thermal Structure of Lake Velen

Description of the lake

Lake Velen is located at approximately 50°N, 14°E, which is between Lake Vänern and Lake Vättern in Sweden. The main features of the lake configuration are shown in Fig. 4. The length and width are about 6000 and 1000 m respectively and the maximum depth is 17m. The variation of horizontal area versus depth, found to be important in the previous section, is also shown in Fig. 4. As can be seen, the bulk of the water volume is concentrated rather close to the surface. Inflows and outflows are insignificant.

Lake Velen is strongly stratified each summer with a temperature difference between surface and bottom water of the order 12-15°C.

For a detailed description on the lake, see Falkenmark (1971).

Application of the mathematical model

In addition to the morphometric data, given above, information about initial, boundary and transparency conditions are needed.

Heat flux at the surface and shortwave radiation data was provided by Rodhe (1977). These data were given as 24 hours averages which is not suitable for shortwave radiation. Instead it was assumed that the total daily flux is gained during 12 hours, distributed as a sine function.

The absorption coefficient for shortwave radiation, β, is not well known for Lake Velen. Occasional measurements of the depth where a white disc becomes invisible have however been made. These measurements gave a depth of the order of 3 m which according to Pivovarov (1973) gives $\beta \approx 1.0$. This was the value adopted.

The time series of wind velocities was provided by Matsson (1977). These data were given on an hourly basis which is believed to be a necessary resolution since a gale of a few hours duration may have a significant effect on the thermal structure. A drag coefficient of $1.3 . 10^{-3}$ was used to relate the wind velocity to the shear stress at the surface.

The predicted period is five months long starting on 1 May 1971, since the lake was then homotherm at 5.4°C.

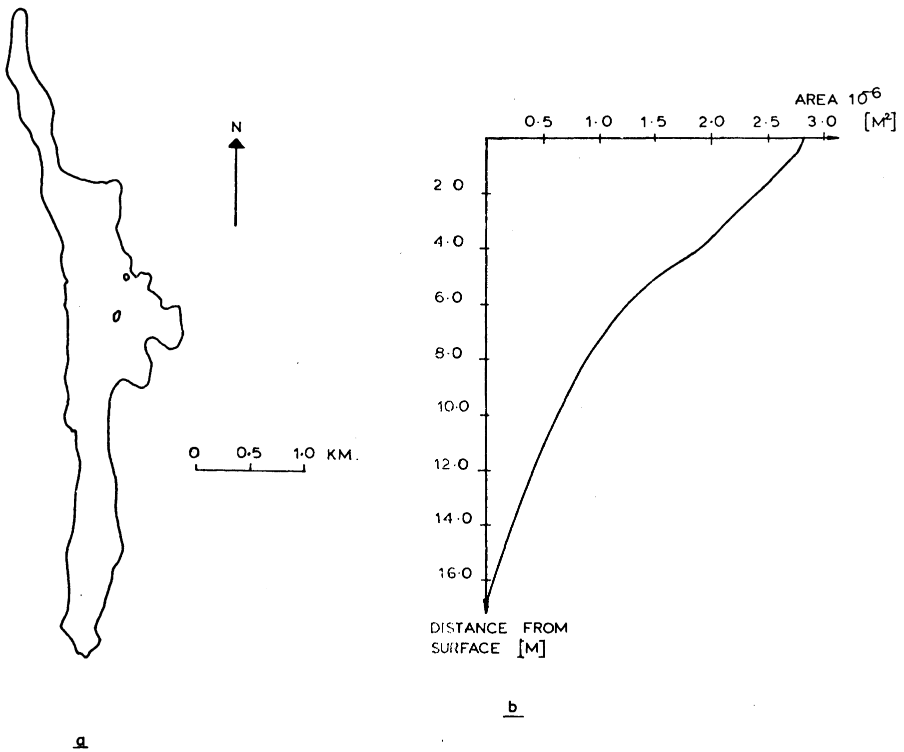
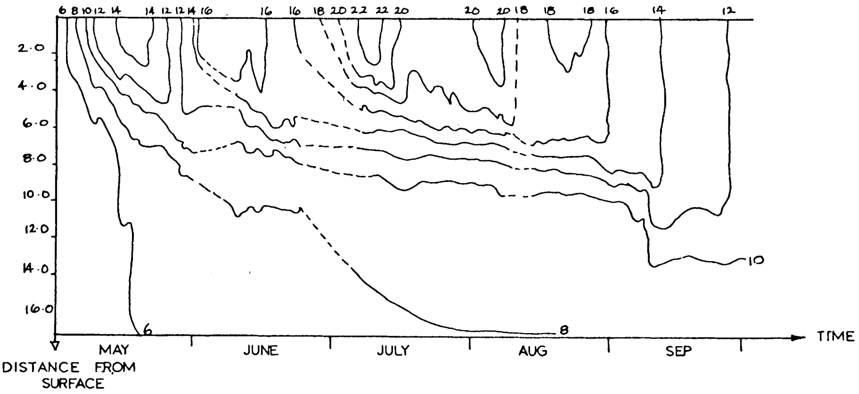


Fig.4. The morphometry of Lake Velen. Lake configuration (a) and horizontal area versus depth (b).

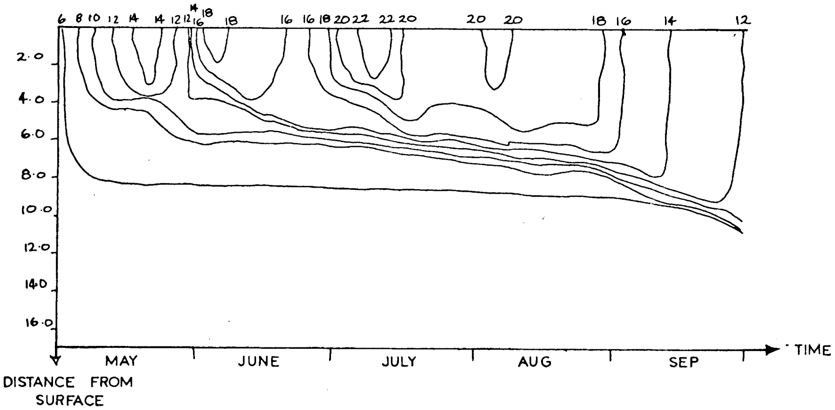
Results and discussion

The measured isotherm depths versus time for the predicted period are shown in Fig. 5a. Dashed lines in this figure indicate periods when the automatic recorders did not work. Predicted isotherms are shown in Fig. 5b and as can be seen the overall agreement with measurements is good. The main discrepancy is encountered for the bottom water which is found to be warmer than predicted. The reason for this is probably due to internal oscillations which produce most of the mixing below the thermocline, as described in the section on physical processes. This is a process which is outside the capability of a one-dimensional model. Of course it would be possible to prescribe a suitable minimum value on the eddy diffusivity to account for this mixing process but this would only be a form of curve fitting and was therefore not done.

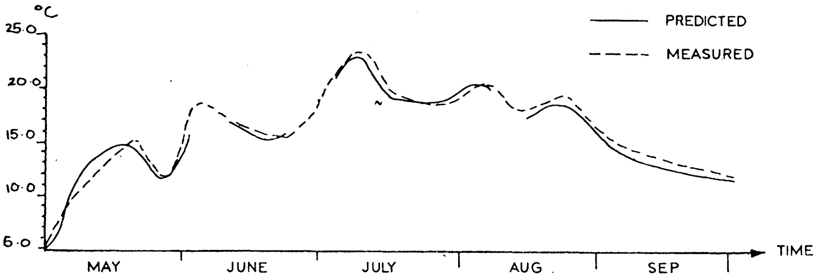
Measured and predicted temperatures 1 m below the surface are shown in Fig. 5c. The agreement is impressive. The small differences found are well within the error range caused by the uncertainty in the boundary conditions.



a/ MEASURED



b/ PREDICTED



c/ MEASURED AND PREDICTED TEMPERATURE 1M. BELOW THE SURFACE

Fig.5. Measured and predicted seasonal stratification in Lake Velen.

Conclusions

It seems that the vertical variation of horizontal area has a more pronounced effect on the thermal structure than the pressure gradients. Further work is however needed before a firm statement about pressure effects can be given. The effect of transparency is, on the other hand, much clearer. Changing the absorption coefficient from 0.3 to 0.6, which is within the error range when simple field methods are used, produces a substantial change in the temperature profile. It is therefore concluded that reliable information about the absorption coefficient is essential.

From the comparison between measured and predicted stratification in Lake Velen, it may be stated that the mathematical model is capable of predicting the overall-temperature structure in small lakes in a satisfactory way. It has also been found that mixing processes which are outside the reach of a one-dimensional model affect the temperature below the thermocline. The variation of the temperature 1 m below the surface is found to be predicted accurately by the model. It is therefore concluded that mathematical models provide a realistic way of examining the summer stratification in lakes.

Notification

Area	horizontal area
c	specific heat
c_1, c_2, c_3, c_μ	constants in turbulence model
D	depth
Dmean	mean depth
f	Coriolis parameter
g	gravitational acceleration
H	heat content
i	time-step
k	turbulent kinetic energy
k_p	constant in pressure-formula
p	pressure
Ri_f	flux Richardsson number
S_H	source term in heat energy equation
t	time
T	temperature
T_0	reference temperature
u	velocity in x-direction

v	velocity in y-direction
x	horizontal coordinate
y	horizontal coordinate
z	vertical coordinate
α	volume expansion coefficient
β	absorption coefficient for shortwave radiation
μ	laminar viscosity
μ_T	turbulent viscosity
μ_{eff}	effective viscosity, $\mu+\mu_T$
ρ	density
$\Delta\rho$	density difference
σ	Schmidt or Prandtl number

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Appendix A

Basic equations of the mathematical model

The treatment of the mathematical formulation will be very brief in the present context as a complete derivation will be presented elsewhere. It has merely been included as a background to the predictions in the two previous sections.

The basic equation, when studying the thermal structure, is of course the heat energy equation. For the present onedimensional treatment of the problem, it may be written as:

$$\frac{\partial H}{\partial t} = \frac{\partial}{\partial z}\left(\frac{\mu_{\mathrm{eff}}}{\rho\sigma_{\mathrm{eff}}}\frac{\partial H}{\partial z}\right) + S_H \qquad \text{(A1)}$$

Change in heat content — Laminar and turbulent diffusion — Source term due to shortwave radiation

Where H is heat content ($= T\rho c$), μ_{eff} effective viscosity, σ_{eff} effective Prandtl number, ρ density and S_H source term. The cartesian coordinate system used is defined in Fig. 6. In this equation it has been assumed that an effective transport coefficient, $\mu_{\mathrm{eff}}/\sigma_{\mathrm{eff}}$, may be formulated by simply adding the laminar and turbulent contributions. Thus:

$$\frac{\mu_{\mathrm{eff}}}{\sigma_{\mathrm{eff}}} = \frac{\mu}{\sigma_L} + \frac{\mu_T}{\sigma_T} \quad ; \qquad \text{(A2)}$$

where μ, σ_L and μ_T, σ_T are laminar and turbulent viscosities and Prandtl numbers respectively. The absorption of shortwave radiation, as denoted by S_H, is

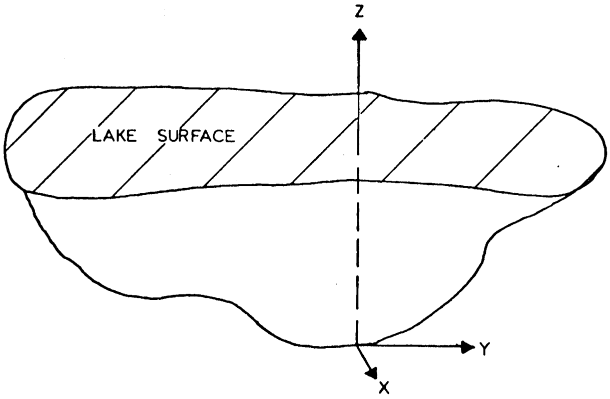

Fig.6. Definition of the coordinate system.

assumed to follow an exponential decay law; for details see Dake and Harleman (1969).

The effective transport coefficient in the heat energy equation is a variable with a local value closely related to the turbulence structure. It is well known that turbulence is generated by velocity gradients and we therefore have to include the momentum equations in a proper analysis.

Within the present assumptions they may be written as:

in x-direction: $$\frac{\partial \rho u}{\partial t} \equiv \frac{\partial p}{\partial x} + \frac{{\sim}\partial}{\partial z}\left(\frac{\mu_{eff}}{\rho}\frac{\partial \rho u}{\partial z}\right) + f\rho v \; ; \qquad (A3)$$

in y-direction: $$\frac{\partial \rho v}{\partial t} \equiv \frac{\partial p}{\partial y} + \frac{{\sim}\partial}{\partial z}\left(\frac{\mu_{eff}}{\rho}\frac{\partial \rho v}{\partial z}\right) - f\rho u \; ; \qquad (A4)$$

Where u and v denote velocity in x and y-direction respectively, p pressure and f the Coriolis parameter. The pressure gradients may be calculated from the mass conservation requirement that the mean velocity is zero in both the x and y-direction when averaged over the total depth. If $\bar{u}$ is the mean velocity in the x-direction we may write:

$$\frac{\partial p^{i+1}}{\partial x} = \frac{\partial p^{i}}{\partial x} + k_p \bar{u} \; ; \qquad (A5)$$

where i is timestep and k_p a constant. This formula is only illustrative since k_p is a function of timestep, dimension of the lake, etc. Details will be presented elsewhere. From Eq. (A5) one may anyway grasp the basic idea; a positive $\bar{u}$ increases the pressure gradient which through Eq. (A3) will decrease $\bar{u}$.

Introducing the momentum equations did not result in a way to prescribe the effective exchange coefficient for heat. The velocity profiles are however, an essential piece of information in the turbulence model employed for determining μ_T. It is beyond the scope of this paper to describe the turbulence model to any length, instead, the reader is referred to Launder and Spalding (1972). For a recent review of the turbulence modelling technique, see Reynolds (1976). In the present study it is assumed that the eddy viscosity can be defined by the relation:

$$\mu_T \equiv c_\mu \, \rho \frac{k^2}{\varepsilon} \; ; \qquad (A6)$$

where k is turbulent kinetic energy, ε its dissipation rate and c_μ an empirical constant. Exact transport equations for k and ε can be derived from the Navier-Stokes equations and thereafter »modelled« to the following form:

Turbulent kinetic energy:

$$\frac{\partial k}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\mu_{\mathrm{eff}}}{\sigma_k}\frac{\partial k}{\partial z}\right) + \frac{\mu_T}{\rho}\left\{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right\}(1 - Ri_f) - \varepsilon \; ; \qquad (A7)$$

Dissipation of turbulent kinetic energy:

$$\frac{\partial \varepsilon}{\partial t} \equiv \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\mu_{\mathrm{eff}}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial z}\right) + \frac{\mu_T}{\rho}\frac{\varepsilon}{k}\left\{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right\}(c_1 - c_3 Ri_f) - c\frac{\varepsilon^2}{2k} \qquad (A8)$$

where:

$$Ri_f = \frac{2\,\alpha\, g\,(T - T_0)\,\frac{\partial T}{\partial z}}{\sigma_T\left\{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right\}} \qquad (A9)$$

is the flux Richardsson number.
c_1, c_2, c_3, c_μ, k and σ_ε are numerical constants.

This mathematical formulation, ((A1)-(A9)), has been verified against laboratory experiments and found to be capable of describing the physical processes present in the thermocline problem.