Control of buckling behavior in origami-based auxetic structures by functionally graded thickness

Special Collection: Physics of 3D Printing

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J. Appl. Phys. 135, 105101 (2024)
https://doi.org/10.1063/5.0194238
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Cite as: J. Appl. Phys. 135, 105101 (2024); doi: 10.1063/5.0194238
Submitted: 26 December 2023 · Accepted: 10 February 2024 · Published Online: 8 March 2024

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Note: This paper is part of the special topic, Physics of 3D Printing.

ABSTRACT

Negative Poisson’s ratio in auxetic structures plays a crucial role in energy absorption and impact mitigation. Origami-based lattices within the realm of auxetic structures offer the advantage of facile fabrication and design. Nevertheless, the utilization of periodic lattices in origami-based auxetic structures constrains the available design space for achieving diverse mechanical properties. Addressing this limitation, our study introduces origami-based auxetic structures with functionally graded thickness, utilizing origami-based lattices known as Tachi–Miura polyhedra. We investigated the impact of functionally graded thickness on buckling behavior and force responses through dynamic loading experiments employing 3D-printed test pieces. The experimental results indicate that functionally graded thickness induces partial auxetic deformation in lattices, and the resulting nonsymmetric deformation prevents global buckling, thereby averting bounded forces observed in structures with uniform thickness. These findings extend the applicability of auxetic structures, spanning from energy absorption to the design of cushioning structures.

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I. INTRODUCTION

Mechanical metamaterials have garnered significant attention owing to their distinctive mechanical properties. Engineered lattice metamaterials exhibit lightweight characteristics as well as various mechanical, thermal, and acoustic properties. Among the unique mechanical attributes of lattice metamaterials, auxeticity, which is characterized by negative Poisson’s ratios, is a promising candidate for engineering applications. In contrast to conventional materials, which expand under compression, auxetic structures display lateral contraction. This inherent auxetic property enhances their resistance to indentation at points of concentrated loads, enhances resistance against shear deformation, improves fracture toughness, and boosts the efficiency of energy absorption (EA). Additionally, the out-of-bending behavior of auxetic structures results in curved surfaces with positive Gaussian curvatures. The mechanical properties and deformation modes of auxetic structures have been extensively investigated, with examples including re-entrant honeycombs, chiral structures, and arrowheads.

Furthermore, origami-based lattice structures contribute to the realization of efficient designs by harnessing auxeticity through origami folding. Notably, the auxetic properties of origami-based structures have been observed in the Miura-ori and water bomb tubes. Re-entrant shapes within Tachi–Miura polyhedra (TMP) also induce auxeticity. Additionally, investigations into multidirectional auxeticity based on Miura-ori have been conducted. The enhancement of EA through the auxeticity of TMP has been investigated and applied to applications such as head protection.

To enhance the design of auxetic lattices, functionally graded approaches have been investigated. Functionally graded origami-based structures have been investigated. This approach has also been applied to auxetic origami-based lattices. Previous studies have primarily focused on grading the mechanical properties of origami lattices based on the geometry of origami patterns. However, to the best of our knowledge, the effects of graded thickness in origami-based lattices on their mechanical properties have not been thoroughly investigated. Graded thickness in...
periodic lattices can be easily achieved without considering the connectivity of lattices between unit cells. In addition, facile fabrication of functionally graded auxetic lattices can be enabled by stacking the folded materials with varying thicknesses through plate folding or employing self-folding 4D printing techniques.

This study elucidates the control of buckling behavior in origami-based auxetic metamaterials through functionally graded thickness. The implementation of functionally graded thickness in origami-based lattices induces local buckling in low-thickness layers, preempting global buckling of lattice structures by disrupting symmetry. Experimental impact testing on 3D-printed structures has substantiated this concept. The ability to control buckling behavior broadens the spectrum of force responses, thereby expanding the potential applications of auxetic structures.

II. MATERIALS AND METHODS

A. Geometry of auxetic structures

The TMP unit cell is constituted by connecting the upper and lower origami sheets, as illustrated in Fig. 1(a). The geometry is characterized by dimensions $l$, $m$, and $d$, along with the internal angle of the parallelogram, denoted as $\alpha$. Changes in TMP geometry are delineated in relation to the folding angles $\beta$, $\gamma$, and the internal angle of the parallelogram $\alpha$.25

$$\tan(\gamma) = \tan(\alpha) \cos(\beta).$$

Here, the folding ratio related to $\beta$ is introduced as follows:

$$R = \frac{90^\circ - \beta}{90^\circ} \times 100\%. \quad (2)$$

With an increase in the folding ratio, the cross sections of TMP tubes undergo a transition from convex shapes [for example, $R = 25\%$ in Fig. 1(b)] to concave shapes [e.g., $R = 50\%$ in Fig. 1(b)]. The pairing of upper and lower origami sheets with $l : m : d = 1 : 1 : 1$ and $\alpha = 65^\circ$ forms the basis for TMP-based lattices. The variation in the cross-sectional geometry in TMP results in re-entrant shapes, contributing to auxeticity. To control the mechanical properties and buckling deformation, the thicknesses of the origami sheets are graded, as shown in Fig. 1(c). As an example of graded thickness, the thickness increases from the bottom to the top of the origami sheets in TMP. This distribution of thickness influences buckling deformation.

B. Design and 3D printing

To demonstrate the control of buckling behavior through functionally graded thickness, TMPs with uniform and graded thickness distributions were designed and manufactured. The TMP was constructed using five sets of upper and lower origami sheets, defined by the geometrical parameters $l = m = d = 12.5$ mm, $\alpha = 65^\circ$ and $R = 47.8\%$, as employed in a prior study to realize auxetic structures.30 This configuration is depicted in

FIG. 1. Geometry of Tachi–Miura polyhedron. (a) Unit cell of Tachi–Miura polyhedron composed of the upper and lower origami sheets. (b) Folding motions of Tachi–Miura polyhedron. (c) Functionally graded thickness in Tachi–Miura polyhedron. The thickness distribution increases from top to bottom.
Figs. 2(a-1)–2(a-3), where the heights of both TMPs are 130 mm. For reasonable comparison, the thickness distributions in TMP are designed such that the average thickness among the TMPs is 1.5 mm. The structure in Fig. 2(a-1) has a 1.5 mm uniform thickness, and those in Figs. 2(a-2) and 2(a-3) have thickness distributions of $t_{1.0-2.0}$ and $t_{0.5-2.5}$, respectively.

As illustrated in Figs. 2(b-1)–2(b-3), the designed TMPs were fabricated using a commercial fused filament fabrication type 3D printer (Markforged, Onyx Pro) and commercially available Nylon filament (Markforged, Nylon White). The filament had a diameter of 1.75 mm, and the pitch for stacking materials was set at 0.2 mm. The tubular structure of the TMPs was oriented vertically against the bed of the 3D printer, allowing fabrication without the need for supporting materials. Figure 3 indicates the average mass of the three test pieces used for the dynamic testing with different velocities. As the average thickness are same in the designed TMPs, the masses of the fabricated structures were approximately identical.

Importantly, the materials were utilized in a water-absorbed state to prevent fracture, as water absorption induces a transition in nylon from a glassy state to a rubbery state. Therefore, the test pieces were immersed in water at 50°C for 20 h before the experiments. Water-absorbed test pieces were extracted and stored in aluminum bags to prevent drying until the experiments began.

The stress–strain curves obtained by the three-point bending test of the fabricated rectangular plates are illustrated in Fig. 4, following JIS K7171 (plastics determination of flexural properties). The stress–strain curve indicates that the water-absorbed nylon exhibits ductile material properties with a large elongation before failure. The maximum elongations observed under a high strain rate before failure may be shorter than that achieved via quasi-static testing.40
The basic material properties of the 3D-printed filament need to be further investigated. Young’s modulus identified from the stress–strain curve was 332 MPa. The mass density measured from the rectangular plates used in the three-point bending test was 1162 kg/m³. Poisson’s ratio of nylon generally is within the range of 0.34–0.43.

C. Experimental setup for dynamic loading

To demonstrate the control of buckling behavior through functionally graded thickness, the buckling deformation of TMP under dynamic loading was observed using the experimental setup depicted in Fig. 5. The 3D-printed test pieces were affixed to a rigid wall using cords, with a load cell positioned behind the rigid wall. The load cell is protected by bars to prevent high-load input to the load cell when the energy absorption of the test pieces is insufficient. The impactor, propelled by compressed air, was introduced from the right side, as shown in Fig. 5. The impact mass was fixed as 103.15 kg and impact energy was controlled using projection velocities. To study the effects of impact velocity, three types of projection velocity were selected: 3.4, 6.3, and 8.5 m/s (Corresponding strain rates are 26.2, 48.5, and 65.4.). First, the impact energy at a 3.4 m/s velocity was selected as the resulting deformation was smaller than half the height of the specimen, and this system was used to compare the small deformations of three samples. The impact energy with a 6.3 m/s velocity was selected to observe further large deformations including the densification of the test pieces. Finally, the 8.5 m/s velocity was selected to deform the entire structure. In addition to the effect of velocities, the repetitive use of fabricated test pieces was investigated by repeating the 3.4 m/s injection five times for the same test pieces, where a low velocity was selected to safely repeat dynamic loading. Given that the strain rate of compression in this experiment falls within the range of 10³, the test is categorized as a dynamic-low velocity experiment. Due to mechanical resonances associated with dynamic-low velocity impacts, the noise by resonances in measured data is removed by a low-pass filter with a cutoff frequency of 1000 Hz. The deformation of the test pieces was recorded using a high-speed camera (NAC Image Technology, MEMRECAM fx-K4).

III. RESULTS AND DISCUSSION

A. Effects of functionally graded thickness

The results of dynamic testing with impact velocities of 3.4, 6.3, and 8.5 m/s are analyzed to investigate the influence of functionally graded thickness on buckling behavior and force responses. Figure 6 shows the experimental results of dynamic testing under 3.4 m/s impacts. In TMPs with uniform thickness, force responses promptly increase after impact due to auxetic deformation, continuing up to approximately a 10 mm displacement. Subsequently, the force response plateaus, indicative of global buckling occurring around a 30 mm displacement. Here, the global buckling deformation has a high-order mode characterized by an S-like shape. In the traditional theory of Euler buckling, high-order modes are produced by applying external constraints at inflection points on columns with a slender aspect ratio. In this experiment, the test pieces have a slender aspect ratio due to dynamic changes in the cross-sectional area caused by shrinking following auxetic deformation after impact. Furthermore, the shrinking by auxetic deformation also densifies the center of the origami lattice, constraining the deflection at the inflection point of the S-like shapes of the structures. In addition, the local buckling of microstructures can bring the eigenvalues of higher-order modes close to the first mode,
FIG. 6. Force response and buckling deformation induced by 3.4 m/s impact. Uniform thickness generates global auxetic deformation after impact. In contrast, the functionally graded thickness partially generates auxetic deformation from contact layers, leading to gradual increases in force responses.

FIG. 7. Force response and buckling deformation induced by a 6.3 m/s impact. Under a uniform thickness, high-energy impact requires large displacements to absorb all of the energy, causing densification of the structure after a 40 mm displacement. In addition, the functionally graded thickness increases force responses to compensate for the lack of energy absorption at small displacements when the graded thickness ranges from 0.5 to 2.5 mm, causing cracking at a displacement of approximately 70 mm of the test pieces.
which is demonstrated as the programmable higher-order mode in Euler buckling using hierarchical structures.\textsuperscript{44} In contrast, TMPs with functionally graded thickness exhibit partial auxetic deformation originating from the small-thickness layer closest to the impact point, resulting in gradual increases in force responses. The non-symmetric deformation in TMPs with functionally graded thickness prevents global buckling, leading to unbounded force responses. Notably, both TMPs, with uniform and graded thickness, demonstrate the ability to absorb the entire impact energy and recover after reaching the maximum displacement, as illustrated in Fig. 6.

Subsequently, force responses and buckling deformation resulting from a 6.3 m/s impact are presented in Fig. 7. As the 6.3 m/s impact inputs higher energy to the test pieces than the 3.4 m/s impact, larger displacements are required to absorb the entire impact energy. Consequently, such displacements induce densification of the test pieces after approximately a 40 mm displacement, resulting in high-force responses. Specifically, the functionally graded thickness causes the high-force responses to compensate for the lack of energy absorption caused by small forces at small displacements. Particularly in test pieces with functionally graded thicknesses from 0.5 to 2.5 mm, the high-force responses lead to the formation of cracks at approximately a 70 mm displacement.

Force responses and buckling deformation resulting from the 8.5 m/s impact are shown in Fig. 8. The force responses and buckling deformations before the densification process (corresponding to approximately an 80 mm displacement) are approximately identical to the experimental results with a 6.3 m/s impact velocity. For instance, cracking occurs at approximately a 70 mm displacement of the test pieces with a thickness graded from 0.5 to 2.5 mm. In contrast, the fabricated test pieces cannot absorb the entire energy of impact at a 8.5 m/s velocity, resulting in a significant increase in the force responses via densification of the origami lattices at displacements from 80 to 100 mm. During the densification stage, the force responses were similar among the three test pieces because the designed mechanical properties were altered by densification. The remaining impact energy was dissipated by collision with the bars for the protection of the load cells. Therefore, the recovered process was not evaluated in the 8.5 m/s experiments.

Although the strain rate in these dynamic loadings was on the order of $10^3$, higher strain rates could achieve different results. For instance, the impact resulting from a strain rate of $10^3$–$10^5$ causes plastic wave propagation, while strain rates exceeding $10^5$ cause shock wave propagation.\textsuperscript{42} Video files in the supplementary material provide detailed motions of the TMPs.

TMPs with uniform thickness exhibit highly efficient energy absorption, maximizing the area under the force-displacement curves by bounding maximum forces to prevent structural fracture. This force response is well-suited for applications in energy absorption structures. Conversely, the functionally graded thickness does not bind the force by avoiding global buckling as a result of partial

**FIG. 8.** Force response and buckling deformation induced by 8.5 m/s impact. The fabricated test pieces can not absorb the entire energy of impact with 8.5 m/s velocity. The remaining impact energy is dissipated by collision with the bars for the protection of load cells. Therefore, the recovered process was not evaluated in the 8.5 m/s experiments.
auxetic deformation. The non-uniform mechanical responses achieved by controlling buckling are the main advantage of auxetic structures with a functional thickness because non-uniform responses are often required in engineering applications. For instance, structures, such as protective gear, packaging materials, and automotive systems for crash safety, that ensure protection require a gradual transition between hardness and softness. In addition, non-uniform responses are also essential in improving the comfortability of products such as seats, helmets, and wearable devices by balancing softness with comfort and hardness for support. Therefore, the functionally graded thickness in TMPs broadens the range of force responses in auxetic structures by controlling buckling behavior, thereby expanding the potential applications of auxetic structures.

B. Repetition of dynamic testing

To assess the potential for repetitive use, a dynamic test with a 3.4 m/s impact was conducted five times for each TMP with uniform and functionally graded thicknesses, as illustrated in Fig. 6. The corresponding force responses are presented in Fig. 9. In all instances, the impact energy was entirely absorbed during the first to fifth impacts. However, a larger displacement was necessary to absorb the entire impact energy after the second impact, attributed to softened force responses. Specifically, the force responses of the TMP with uniform thickness softened after the second impact due to sustained plastic deformation resulting from global buckling. In contrast, the force responses of TMP with functionally graded thickness stabilized after the third impact. Importantly, none of the test pieces experienced fracture during repetitive use under the condition.
3.4 m/s impact. Therefore, repetitive auxetic deformations with functionally graded thickness can be achieved using 3D-printed test pieces, indicating the potential for repetitive applications in energy absorption and cushioning structures through 3D printing technology under low-velocity impacts. To realize the repetitive use of these structures under high-velocity impacts, materials with high strength must be used to avoid cracking.

IV. CONCLUSION

This study delves into the manipulation of buckling behavior through functionally graded thickness in origami-inspired auxetic structures. In auxetic structures with uniform thickness, forces increase due to auxetic deformation following an impact, with the force bounded after bifurcation buckling, resulting in a global-S-like shape.

In contrast, functionally graded thickness leads to the gradual generation of localized buckling, characterized by partial auxeticity, preserving auxetic behavior throughout the deformation process. Therefore, the introduction of varying thicknesses in auxetic structures allows for a spectrum of buckling behaviors and force responses. The plateaued forces resulting from global buckling are advantageous for energy absorption, as the area under force responses is maximized within the constraints of maximum force, while gradual force increases were ideal for cushioning applications.

Furthermore, 3D-printed functionally graded origami lattices using nylon materials could sustain force responses after repeated dynamic loading, suggesting potential repetitive uses in energy absorption or cushioning structures. Therefore, the incorporation of functionally graded thickness in auxetic structures presents opportunities for diverse practical applications, offering versatile load-response characteristics.

SUPPLEMENTARY MATERIAL

See the supplementary material for the videos related to Figs. 6–8.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to Dr. Hidekazu Nishigaki, Dr. Masato Tanaka, and Dr. Shoko Arita at Toyota Central R & D Labs, Inc. for their valuable advice.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

S. Tomita: Conceptualization (equal); Investigation (equal); Methodology (lead); Visualization (lead); Writing – original draft (lead). K. Shimanuki: Investigation (equal). K. Umemoto: Conceptualization (equal); Supervision (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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J. Appl. Phys. 135, 105101 (2024); doi: 10.1063/5.0194238

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