APPENDIX

Estimation of Deviation Factor

For a first approximation, the angle of deviation $\delta$ between the blade angle $\beta$ and the outlet gas angle $\alpha_3$ in an axial-flow turbine meter, Fig. 3, is assumed to be of a similar nature to that in a low-speed axial-flow compressor. The angle of deviation $\delta$ can be expressed as [9]

$$
\delta = \frac{m(S/C)^{1/2}}{V}
$$

where $m$ is a dimensionless number depending upon the blade profile and blade angle.

With reference to Fig. 3, and for small gas deflection angle $\theta$

$$
\theta = \tan \delta = \frac{V_{\alpha}/V_{\alpha_3}}{V_{\alpha}/(Q/A)}
$$

(31)

Substituting (31) into (30)

$$
\delta = \frac{m(S/C)^{1/2}V_{\alpha}}{(Q/A)}
$$

(32)

But

$$
\Delta \theta = \frac{\tan \beta - \tan \alpha_3}{(Q/A)(\tan \beta - \tan \alpha_3)}
$$

(33)

For small deviation angle $\delta$

$$
\delta = \tan \delta = \tan (\beta - \alpha_3)
$$

(34)

But

$$
\tan (\beta - \alpha_3) = (\tan \beta - \tan \alpha_3)/(1 + \tan \beta \tan \alpha_3)
$$

(35)

Equation (34) then becomes

$$
\delta = (\tan \beta - \tan \alpha_3)/(1 + \tan^2 \beta)
$$

(36)

Substituting (35) into (33), and also using (32)

$$
\Delta \theta = \frac{(Q/A)(1 + \tan^2 \beta)\delta}{m(1 + \tan^2 \beta)(S/C)^{1/2}V_{\alpha}}
$$

Therefore, $\Delta \theta$ can be expressed by equation (4) in the analysis

$$
\Delta \theta = \eta V_{\alpha}
$$

where

$$
\eta = \frac{m(1 + \tan^2 \beta)}{(C/S)^{1/2}}
$$

$\eta$ is a dimensionless number called deviation factor and depends upon the blade angle $\beta$, the solidity $(C/S)$ of the rotor blade and profile of the rotor.

DISCUSSION

W. G. Fox

The turbine meter is one of a class of fluid meters whose performance cannot at present be predicted with satisfactory accuracy based on construction features and dimensions alone. The performance characteristics of such devices must therefore be determined by means of flow calibration. However, flow calibration facilities are limited, particularly with gases, in terms of the variety of fluids and operating conditions available. Hence, the practical necessity of extrapolating the data from calibration with one fluid at one set of operating conditions to prediction of performance with another fluid at different operating conditions invariably arises.

The authors of this paper have recognized this practical problem and have formulated a method for making the extrapolation. The complexity of the fluid mechanics involved is evident from even a cursory examination of their paper, which, as the authors state, is a simplified analysis. The result of their efforts is the best available extrapolation technique toward the solution of a very real and present problem and thus represents a significant contribution to the art of turbine flow metering.

Without detracting from the value of this work, it seems important to emphasize the perspective with which it should be viewed. That is, the proposed method is an educated engineering approximation for extrapolating data. As the authors state, “this analysis will be quite useful in the proper selection of meters as well as the prevention of meter misapplication.” They further state that the performance of the meter when operating with other gases under other operating conditions “can be predicted reasonably.” It seems certain that they did not mean to imply this method could be used in lieu of actual flow calibration at the new conditions with equal confidence in either result. The question then is how accurately this extrapolation could, with confidence, predict performance.

The confidence one places in this extrapolation method is derived from two sources. The first source is a comparison of the predicted results with actual calibrations under the new conditions, while the second is consideration of the validity of assumptions and techniques used in the authors' analysis. On the first point, the authors have not presented any such data. This is not without good reason—the difficulty of obtaining accurate calibration of a turbine meter on gases other than air already has been noted.

With respect to the second point, there are some techniques and assumptions made by the authors that need to be examined.

The first has to do with their interpretation of the term $M_r$, which they call the total resisting torque. This term is used in their fundamental equation [equation (2)], and serves as the basis for the remainder of the analysis. This term should be defined more correctly as the total external retarding torque, where external refers to those torques that are applied from outside of the control volume of interest. While the authors have not defined their control volume, it is apparent that the planes perpendicular to the axis at leading and trailing edges of the blades form two of their control volume boundaries.

This definition is a crucial point, for if they define the remaining boundaries as the pipe wall and the cylinder at the hub radius, then the torques between the fluid and the blade are internal and the only external torques are what the authors have called the nonfluid retarding torques. On the other hand, if they define their control volume as the fluid contained between the blades, then the external retarding torques are equal to just the torques due to the fluid-blade interaction, not the sum of these torques plus that due to nonfluid forces. As a matter of interest, the nonfluid forces must be equal and opposite to those due to the fluid-blade interaction.

This is the same comment as that made by Leroy H. Smith, Jr., in his discussion of the authors' previous paper [8]. In the authors' closure to this discussion, they stated that Mr. Smith's interpretation of the driving torque was not what the authors' had defined. This is true, but Mr. Smith and this discusser are noting that, when using the momentum approach, the authors are not at liberty to define the retarding (or driving) torques as they would be defined in a free body diagram of the rotor but must consider only those torques which are external to the control volume.

To be more explicit, for full fluid guidance:

$$
T_k - T_D = \rho Q V_{\theta_k} = T_L
$$

(37)

and not

$$
T_k = T_D + T_R = \rho Q V_{\theta_2}
$$

(38)

8 Ibid., pp. 24-25, 123-124.
where \( T_L, T_D, \) and \( T_E \) are the tangential components of lift, drag, and external torques, respectively.

The authors have treated \( T_E \) as the nonfluid retarding torque \( M_N \) and \( T_D \) as the fluid retarding torque \( M_f \). Thus, equation (38) as employed in the paper becomes

\[
M_f + M_N = M_s = \rho pQ \bar{V}_0 \tag{38}
\]

However, the momentum analysis requires the use of equation (37) which can be expressed as

\[
T_L - M_f = \rho pQ \bar{V}_2 - M_s \tag{39}
\]

In their previous paper, they had assumed the nonfluid forces to be negligibly small, and therefore they dropped this term from their analysis. In such a case, the component of lift must equal exactly the component of drag, under equilibrium conditions. However, in the present paper they have not dropped out this term; thus they have no choice but to consider both components. In fact, if the nonfluid forces are not zero, the components of lift must exceed those due to drag by an amount such that the torque due to this difference is exactly equal and opposite that due to nonfluid forces. A more complete exposition of the suggested analysis, together with a discussion of the distinctions between the momentum and airfoil approaches, is contained in the paper by M. Rubin, et al.

One of the key points in the authors’ extrapolation method is the determination of the “density-effect” term \((1 + \eta)M_s\). The value of this term, determined at the flow rate at which the rotor starts, is assumed constant. This seems to be an unrealistic assumption. In order for this term to be constant, either both the deviation factor and the nonfluid retarding torque \( M_f \) must be constant, or their product must be constant.

The deviation factor is a measure of the finite guidance capacity of the rotor blading. It seems difficult to assume this factor is constant over the range of operating speeds from start (at which the rotor slip is nearly 100 percent) to maximum operating speed (at which the slip is perhaps 10 percent or less).

It is even more difficult to assume the nonfluid retarding torques (due mainly to bearing loads and register gear train) are independent of speed, especially since, in the example given, the operating speed goes from essentially zero to about 5400 rpm.

The possibility that the product \((1 + \eta)M_s\) is constant is also a difficult assumption to make, since \( \eta \) has a value considerably less than unity, while the nonfluid retarding torques (for ball bearings at least) are related to the ball retainer drag, the rolling friction caused by the applied load, and the viscous drag caused by the lubricant, all of which are functions of the rotational speed.

With the questions that have been raised concerning (a) the application of the basic equation and (b) the validity of one of the key assumptions, it is difficult for this discusser to have confidence that this method will predict performance at the new conditions to some high degree of accuracy.

Montgomery R. Shafer

The authors are to be complimented upon the presentation of a paper analyzing the separate effects of density and the Reynolds number upon the performance of gas turbine meters under those conditions in which the compressibility effect is of secondary importance. The particular procedure presented describes a method of establishing a family of performance curves applicable to a single gas at selected pressures. A more general type of curve, applicable to any fluid may have advantages in some applications. One form of such a chart may be derived as follows:

If compressibility is insignificant, the following general relations can be derived by dimensional analysis for a turbine meter of fixed shape:

\[
\frac{\omega}{\omega_1} = \phi(M_D \rho/\mu) \tag{40}
\]

\[
\frac{\omega}{\omega_1} = \phi(M_D \rho/\mu) \tag{41}
\]

\[
\frac{\omega}{\omega_1} = \phi(M_D \rho/\mu D) \tag{42}
\]

where \( D \) is a characteristic length designating the size of the meter.

From these relations it is seen that the performance of a turbine meter of a given shape could be shown, for example, as a plot of \( \omega/\omega_1 \) versus \( m/\mu D \) for a family of curves, with each curve representing a constant value of the product \( M_D \rho/\mu D^2 \). This chart would then be applicable to all operating conditions of the meter, provided the compressibility effect is insignificant.

If the consideration is further restricted to a particular meter of fixed size, and assuming the nonfluid retarding torque is constant under all operating conditions, the relation can be reduced to

\[
\frac{\omega}{\omega_1} = \phi(m/\mu, \rho/m) \tag{43}
\]

A chart of this relation may be obtained conveniently using the techniques described in the paper. The calibration is performed with a convenient gas of known density and viscosity and the starting flow rate measured. These experimental data are then used to obtain \((\Delta\omega)/\omega_1\) versus \( m/\mu \), which has been plotted as the solid curve in Fig. 13. This corresponds to curve 2 of Fig. 7 and represents the hypothetical condition in which \( M_D \rho/\mu D^2 \) is substantially equal to zero.

Other curves representing selected constant values of the ratio \( m/\mu \) may be located and plotted quickly by computing the corresponding value of \((\Delta\omega)/\omega_1\) by the relation

\[
(\Delta\omega)/\omega_1 = \left( \frac{m^* \rho}{\rho^* m^2} \right) \tag{44}
\]

where \( m^* \) is the starting mass flow rate of the turbine meter using a gas of density \( \rho^* \). It should be noted \((\Delta\omega)/\omega_1\) is a constant for each selected value of the ratio \( m/\mu \). Consequently, the corresponding curve is displaced by a constant amount from the \((\Delta\omega)/\omega_1\) curve throughout the entire range of \( m/\mu \).

This resultant chart is applicable to any fluid at any temperature and pressure, provided the operating range of \( m/\mu \) is within that range covered during the calibration. The performance curve of the meter for operation on any gas of known density and viscosity may be located on the chart by determining the \( m/\mu \) values corresponding to selected \( \rho/\mu m^2 \) ratios within the range of interest for this particular operating condition. The circles define such a performance curve which could be obtained with a gas of low density and high viscosity. Another performance curve
corresponding to operation on a high-density fluid is defined by the
trigones.

This illustrative example has used the parameters \( m/\mu \) and \( \rho/\mu^2 \). Similar, but not identical, charts could be obtained in the
same manner by plotting

\[
\frac{\omega}{C} = \frac{\phi}{(Qp/\mu, 1/Qp)}
\]

or

\[
\frac{\omega}{C} = \frac{\phi}{(Qp/\mu, 1/Qp)}
\]

These relations provide methods of using the alternate quantities
rotor speed or volume flow in those applications in which mass
flow is not a convenient parameter.

As stressed throughout the paper, the analysis is based upon
the assumptions that nonfluid retarding torques are constant
under all operating conditions, and the exact magnitude of their
influence may be predicted from an experimental observation of
the starting flow rate. If these assumptions are found to be true
under those conditions commonly encountered during the metering
of gases, then the procedures described provide a very convenient
method of predicting the exact effect of fluid density upon rotor
performance.

Considering turbine meters of 2-in. size and smaller, practical
problems may arise in regard to the determination of the influcence
of nonfluid retarding torques by the starting flow-rate
procedure. Nearly all of these smaller turbine meters generate an
a-c voltage in a stationary pickoff coil through the varying reluctance of a magnetic circuit as influenced by the blades of the
turbine. In such meters there is a definite, magnetic locking action under the condition of zero rotor speed. Thus, starting
flow rate is usually appreciably larger, 25 percent or more, than
the flow rate at which the rotor ceases to turn under a decreasing
flow condition. In applications involving these turbine meters,
perhaps the stopping flow rate is a more logical quantity to use
for the evaluation of nonfluid retarding torques. However, it is
possible that even this may be influenced by magnetic forces.

Also, the rotors of the smaller-size turbine meters are not always balanced. As these meters are normally operated in a horizontal
position, any unbalance will have an appreciable influence upon
the starting and stopping flow rates. Therefore, the possibility of
rotor unbalance should be considered when the procedures de-
scribed in this paper are used for the experimental determination
of the influence of nonfluid retarding torques.

Leroy H. Smith, Jr. 6

Although the present paper presents much useful information,
it is found to contain the same fundamental inconsistency that
discussed pointed out in his discussion of an earlier paper [6]
by one of the present authors. The difficulty centers around the
meaning of \( M_r \) in equation (2). In order for equation (2) to be
correct, \( M_r \) must be the total fluid torque on the blading plus
the torque caused by fluid shear on the annulus walls. The annulus
wall shear is small and is not pertinent to this discussion; it will
be neglected. The total fluid torque on the blading is transmitted
into the hub through the blade roots; this goes mainly into \( M_h \),
but it contains also any fluid friction torque inside the hub, which
I will call \( M_h \). Thus

\[
M_r = M_r + M_h
\]

This equation is in conflict with equation (9), since \( M_r \) in the
paper is made up at least partly of fluid forces on the blades and
\( M_h \) is not. Thus, either equation (2) or equation (9) has to be
modified.

It is suspected because of the name given to \( M_r \) by the authors
(retarding torque) that it was their intention that it include drag
forces on the blading but not lift forces. If this is the case, then

6 Manager, Compressor Aerodynamic Development Unit, Advanced Engine and Technology Department, Flight Propulsion
Division, General Electric Company, Cincinnati, Ohio.
radial length of the blades, this average condition being assumed to exist at the root mean square radius \( \bar{r} \). This simplifying assumption reduces the problem to a two-dimensional one; i.e., only the tangential direction and axial direction of the rotor are to be considered.

Consider the torque equilibrium condition of the rotor in the tangential direction when the rotor is in steady-state rotation and refer to Fig. 14. The difference between the total lift force \( M_L \) on the rotor due to the tangential component of lift force \( L \) and the total blade drag torque \( M_D \) due to the tangential component of blade drag force \( D \) equals the total external retarding torque \( M' \), resisting the rotation of the rotor; i.e.,

\[ M' = M_L - M_D \]

To establish the relationship between the total external retarding torque \( M' \) just defined and the total retarding torque \( M_L \) used in the paper, it is seen that \( M' \) is defined as the total retarding torque which resists the rotation of the rotor and includes the total blade drag torque \( M_D \). Referring to equation (9), the total blade drag torque \( M_D \) constitutes a part of \( M' \), the total retarding torque due to fluid forces, which in turn constitutes a part of \( M' \). Therefore

\[ M' = M_L - M_D \]

Equation (40) may be expressed as

\[ M' = M_L - M_D = (\frac{\pi n L}{\cos \bar{a}_t}) - (\frac{\pi n D}{\sin \bar{a}_t}) \]

where

- \( n \) = number of rotor blades
- \( L \) = lift force on one blade
- \( D \) = blade drag force on one blade

Lift force \( L \) on one blade of blade area \( A_b \) can be expressed in terms of a corresponding lift coefficient \( C_L \) defined as

\[ L = \frac{1}{2} \rho W_1C_L A_b \]

But

\[ W_1 = \frac{V_1}{\cos \bar{a}_t} \]

\[ A_b = \phi (r_a - r_b) \]

For small angle of attack \( \bar{a} \), the lift coefficient may be expressed as

\[ C_L = C_1 \sin \bar{a} \]

where \( C_1 \) is the lift constant, independent of \( \bar{a} \). By means of equations (43), (44), (45), and (46), equation (42) becomes

\[ M' = M_L - M_D = \left[ \frac{1}{2} \rho \bar{V}_1 \bar{V}_2 (r_a - r_b) n \cos \bar{a}_t \left( \frac{\sin \bar{a}}{\cos \bar{a}_t} \right) \right] - (\frac{\pi n D}{\sin \bar{a}_t}) \]

Both Rubin, et al., and this paper are referring to helically twisted blades of constant width \( b \) (Fig. 14). Therefore

\[ \bar{a} = \frac{b}{\cos \bar{a}} \]

It can be shown (see Appendix 1) that

\[ \frac{\sin \bar{a}}{\cos \bar{a}_t} = \cos \bar{a}_t (\bar{V}_1 \tan \bar{a} - r_a) \]

But \( \bar{V}_1 \tan \bar{a} = r_1 \bar{a} \) by equation (1) and \( \Delta \omega = (\omega_1 - \omega) \) by equation (5a) in this paper; equation (49) is thus

\[ \frac{\sin \bar{a}}{\cos \bar{a}_t} = \cos \bar{a}_t (\bar{V}_1 \tan \bar{a} - r_a) \]

Also

\[ \bar{V}_1 = Q/A = Q/(r_a^2 - r_b^2) \] for thin blades

Define

\[ q = \frac{r}{r_a} \]

Using equations (48) through (52), equation (47) becomes after simplification

\[ M' = M_L - M_D = \left[ \frac{\pi n D}{(\pi + q)} \right] \]

Equation (53), which is derived from the two-dimensional airflow approach, shows that the blade drag torque \( M_D \) does contribute to the rotor slip \( \Delta \omega \).

Assume that the outlet relative velocity \( V_2 \) is parallel to the blade. Then, from Fig. 14:

\[ \Delta \omega = \omega_1 - \omega = \frac{\bar{V}_1}{n} \]

Equation (53) can also be expressed in terms of \( \bar{V}_1 \) instead of \( \Delta \omega \):

\[ M' = M_L - M_D = \left[ \frac{\pi n D}{(\pi + q)} \right] \]

It is noted that \( \bar{V}_1 \) in equation (54) is the tangential component of the absolute outlet fluid velocity \( \bar{V}_2 \) for fluid particles near the outlet pressure side of the blade only. Since \( \bar{V}_1 = 0 \), \( \bar{V}_2 \) also represents the fluid deflection in the tangential direction for fluid particles near the outlet pressure side of the blade only. \( \bar{V}_2 \) for fluid particles at other positions along the tangential direction at blade outlet section are different from \( \bar{V}_1 \) given by equation (54) except in the case of full fluid guidance as explained in the following text.

As described clearly, equation (53) or (54) from the airfoil approach would suggest that, at a given rotor slip, twice the lift torque \( M_L \) could be obtained by either doubling the number of blades \( n \) or doubling blade area (i.e., \( b \)). This is essentially true when the solidity parameter \( nb/2a \) of the rotor blading is low and the interference between blades is small. However, the increase of lift torque \( M_L \) with increase of the solidity parameter \( nb/2a \) cannot go on indefinitely. There must be an upper limit. This upper limit can be seen to occur at the point when the rotor has a sufficient number of blades of the required area (i.e., a high enough solidity parameter) to cause practically all of the fluid particles to deflect by about the same amount that the rotor slips. This condition can be designated as the condition of “full fluid guidance.” Further increase of the solidity parameter \( nb/2a \) beyond that required for full fluid guidance cannot increase the lift torque \( M_L \) available from the fluid at a given rotor slip; however, it will continue to increase the blade drag torque \( M_D \).

Now consider the case when the meter solidity is high enough to attain the condition of full fluid guidance under operating condition. Then, practically all of the fluid particles will deflect by almost the same amount that rotor slips. The values of \( \bar{V}_1 \) for all the fluid particles are practically the same and equal to \( \bar{V}_2 \) of fluid particles near the pressure side at the blade outlet which is given by equation (54). For full fluid guidance, the lift
torque $M_L$ is therefore obtained from the practically uniform deflection $V_a = r\Delta a$ at radius $r$ of all the fluid $pQ$ involved.

From steady-flow angular momentum consideration:

$$M_L = \tau pQ V_a$$  \hspace{1cm} (56)

For full fluid guidance, the lift torque term of equation (55) or (56) should be replaced by its upper limit value of equation (60):

$$M' = M_L - M_D = \left(\tau pQ V_a\right) - \left(\tau n D \sin \alpha_i\right)$$  \hspace{1cm} (57)

A comparison of this equation (57), which is obtained from the two-dimensional airfoil approach when applied to condition of full fluid guidance, with equations (2) and (9) in this paper seems in order. Equation (2) gives

$$\tau pQ V_a = M_r$$  \hspace{1cm} (2)

Equation (9), together with equation (41), yields

$$M_r = M_f + M_a = M' + M_D$$  \hspace{1cm} (58)

Equating (2) and (58):

$$\tau pQ V_a = M' + M_D$$  \hspace{1cm} (59)

Or

$$M' = \left(\tau pQ V_a\right) - M_D = \left(\tau pQ V_a\right) - \left(\tau n D \sin \alpha_i\right)$$  \hspace{1cm} (60)

It is seen that equation (60), obtained from equation (2) and equation (9) in the paper, is identical to the equation (57) obtained from the two-dimensional airfoil approach when applied to condition of full fluid guidance, thus verifying the validity of the results of analysis based on equation (2) and equation (9) in this paper. Together with equation (41), equation (57) from the two-dimensional airfoil approach may also be expressed as

$$M_L = \tau pQ V_a = M' + M_D = M_f$$  \hspace{1cm} (61)

This particular form of equation (61) suggests a practical method of attack on the given problem which could be used in practice to obtain the same end results as from the two-dimensional airfoil approach when applied to condition of full fluid guidance. This practical method of attack solves the problem by means of a one-dimensional momentum approach but treats the blade drag torque $M_D$ as a part of the total external retarding torque $M'$. This method has the advantage of reducing the consideration of a two-dimensional problem to a one-dimensional problem, thus making it much simpler and straightforward. However, it has the weakness of being empirical; it can only be justified by the fact that it yields the same end results as does the two-dimensional airfoil approach when applied to the condition of full fluid guidance.

It is interesting to compare the minimum value of the solidity parameter $(nb/2r_0)_{\min}$ for full fluid guidance from the two-dimensional airfoil approach described here with that given by Rubin,4 where the third (i.e., radial) dimension is also considered. $(nb/2r_0)_{\min}$ can be readily obtained by equating the lift torque term of equation (55) and the lift torque term of equation (57) and is found to be

$$\left(\frac{nb}{2r_0}\right)_{\min} = \frac{\pi(1 + q)}{C_t} \hspace{1cm} (62)$$

The corresponding solidity $(c/S)$ of the rotor blading evaluated at blade tip position can be shown to be

$$\left(\frac{c}{S}\right)_t = \frac{nb}{2r_0} \frac{1}{\pi \cos \beta_t} = \frac{1 + q}{C_t \cos \beta_t} \hspace{1cm} (63)$$

where $\beta_t$ is the blade angle at blade tip position.

For a numerical comparison, use the same values given by Rubin, et al.4

$$C_t = 2.0, \quad q = 0.5, \quad \beta_t = 45 \, \text{deg}$$

Equations (62) and (63) give, respectively:

$$\left(\frac{nb}{2r_0}\right)_{\min} = 2.36 \text{ versus } 2.53 \text{ from Rubin}^4$$

$$\left(\frac{c}{S}\right)_t = 1.06 \text{ versus } 1.14 \text{ from Rubin}^4$$

The rather small differences are due to the simplifying assumption used here that the average condition exists at $r$ instead of integrating along the radial direction of the blade as done in Rubin.4

Although the minimum values of solidity for full fluid guidance are also somewhat affected by the total retarding load on the rotor and other geometric parameters of the meter as described in Rubin,4 a turbine meter with a solidity not much greater than unity and blade angle less than 50 deg will attain, or at least approach, the condition of full fluid guidance under most operating conditions.

It is also interesting to point out that the 6-in. gas turbine meter of Fig. 9 illustrated in this paper has the values of $(nb/2r_0) = 2.56$ and $(c/S)_t = 1.19$, which are slightly greater than their respective minimum values required to attain full fluid guidance. With a large fluid retarding torque exerted on the protruding part of the blades by the fluid in the annular recess of the otherwise straight flow passage, this meter runs with a 9 percent rotor slip in its most operating range, as shown in Fig. 10 in the paper.

The authors wish to thank Mr. Fox and Dr. Smith for their contributions toward clarifying the foregoing concepts.

The authors are also glad to see that Mr. Fox questions the justification of the assumption and the method of experimental determination of the density effect term $(1 + \eta)M_r$, described in the paper. Because of its lengthy and involved nature, the authors could not explain it in detail in the paper because of limitation of space.

The deviation factor $\eta$ is defined by equation (4): $\eta = \Delta \omega / \omega_i$.

In physical terms, $\eta$ can be considered as the percentage loss of fluid guidance available for a given rotor slip due to the finite fluid guidance capacity of the rotor blading. For a rotor with full fluid guidance, $\Delta \omega = 0$, see Fig. 3. From equation (4), $\eta = 0$ if $\Delta \omega = 0$ for any practical value of $V_a$. As shown by equation (36), $\eta$ depends primarily upon the geometric parameters of the rotor blading and has no such influence as does the actual rotor slip. From equations (4), (5a), and Fig. 3, it can be readily shown

$$\frac{\Delta \omega}{\omega_i} = \frac{\omega_i - \omega_f}{\omega_i} = \frac{\omega_f + \Delta \omega}{\omega_i} = \frac{(1 + \eta) \omega_f}{\omega_i} \hspace{1cm} (64)$$

For purpose of illustration, let $\eta = 0.11$ for a certain given rotor blading as estimated from equation (36) or somehow experimentally determined at a certain rotor slip. Assume $\eta$ is independent of rotor slip. At starting flow, the rotor slip is practically 100 percent, i.e., $\Delta \omega/\omega_i = 1$. From equations (64) and (4)

$$\begin{align*}
\frac{\Delta \omega}{\omega_i} &= \frac{(\Delta \omega/\omega_i)}{(1 + 0.11)} = 0.9 \left(\frac{\Delta \omega}{\omega_i}\right) = 0.9, \\
\frac{\omega_f}{\omega_i} &= 0.9(\omega_i) \\
\Delta \omega &= 0.11(\omega_i) \\
\Delta U &= 0.1(\omega_i)
\end{align*}$$

This indicates that, out of the 100 percent rotor slip, the fluid deflects only 90 of the 100 percent rotor slip (or equal to 99 percent rotor slip) to produce useful lift torque; the remaining 10 of the 100 percent rotor slip (or equal to 10 percent rotor slip) is lost due to the incapability of full fluid guidance of the rotor. At maximum flow, assume the rotor slip is 10 percent, i.e., $\Delta \omega/\omega_i = 0.1$. Then

$$\begin{align*}
\frac{\Delta \omega}{\omega_i} &= \frac{(\Delta \omega/\omega_i)}{(1 + 0.11)} = 0.9 \left(\frac{\Delta \omega}{\omega_i}\right) = 0.9, \\
\frac{\omega_f}{\omega_i} &= 0.9(\omega_i) \\
\Delta \omega &= 0.11(\omega_i) \\
\Delta U &= 0.09(\omega_i)
\end{align*}$$

$$\begin{align*}
\Delta \omega &= 0.11(\omega_i) \\
\Delta U &= 0.09(\omega_i)
\end{align*}$$

$$\Delta \omega = 0.11(\omega_i)$$

$$\Delta U = 0.09(\omega_i)$$
This shows that, out of the 10 percent rotor slip, the fluid deflects only 90 of 10 percent rotor slip (or equal to 9 percent rotor slip) to produce useful lift torque. The remaining 10 of the 10 percent rotor slip (or equal to 1 percent rotor slip) is lost due to the capability of the full fluid guidance of the rotor. These results at the two extreme flow conditions just mentioned do not seem unreasonable. They are based on the assumption that the deviation factor \( \eta \) is essentially a constant for a given rotor as indicated by equation (36) and does not vary significantly with rotor slip. Therefore, this assumption about \( \eta \) does not seem unreasonable.

\( M_x \) is defined as the total retarding torque due to nonfluid forces. Its effect on meter accuracy is given by equation (12):

\[
\frac{\Delta \omega}{\omega} = \left[ \tan \frac{\beta}{4} \right] \frac{M_x}{\rho Q^2}
\]

(12)

Since the question is centered on the effect of rotor speed \( \omega \) or flow rate \( Q \) on the value of \( M_x \), the present discussion can be restricted to a given meter measuring a fluid of given density \( \rho \). Equation (12) can thus be written as

\[
\frac{\Delta \omega}{\omega} = \left[ \frac{1}{\tan \frac{\beta}{4} (1 + \eta)} \right] \frac{M_x}{Q^2}
\]

(65)

where \( H = \frac{1}{\tan \frac{\beta}{4} (1 + \eta)} \), a constant quantity under present consideration.

The total retarding torque due to nonfluid forces \( M_x \) present in a turbine meter consists of that due to the signal generating device and that due to bearing load. For a mechanical device, the friction torque increases only slightly with speed. For an electric device, the retarding torque is essentially independent of speed except when the rotor is stationary. The bearing load can be considered to consist of three different component parts: (a) that which is practically independent of speed (or the Coulomb type friction) and is mainly due to the weight of the rotor system; (b) that which increases almost linearly with speed and is mainly due to viscous drag in the bearing; and (c) that which increases in the square of speed and is mainly due to axial thrust load on the bearing and the load due to dynamic unbalance of the rotor system. Therefore, the total retarding torque due to nonfluid forces \( M_x \) can also be considered to consist of three component parts which vary differently with the rotor speed:

\[
M_x = K_0 + K_\omega + K_{\omega^2}
\]

(66)

\( K_0 \) represents mainly the load due to the signal generating device and the bearing load due to the weight of the rotor system. \( (K_\omega) \) represents mainly the bearing load due to the viscous drag in the bearing. \( (K_{\omega^2}) \) represents mainly the bearing load due to axial thrust and dynamic unbalance of the rotor system. Substituting equation (66) into equation (65):

\[
\frac{\Delta \omega}{\omega} = H \left[ K_0 + K_\omega \frac{\omega}{Q} + K_{\omega^2} \frac{\omega^2}{Q^2} \right]
\]

(67)

Within the operating flow range of a turbine meter, the rotor speed \( \omega \) is in linear proportion to the flow rate \( Q \) through the meter, i.e.,

\[
\omega/Q = 1/J
\]

(68)

where \( J \) is a meter constant or “meter factor” of the meter. Substituting equation (68) into equation (67):

\[
\frac{\Delta \omega}{\omega} = H \left[ \left( \frac{K_0}{Q^2} \right) + \left( \frac{K_\omega}{Q} \right) + \left( \frac{K_{\omega^2}}{Q^2} \right) \right]
\]

(69)

It is noted from equation (69) that the fractional rotor slip \( \Delta \omega/\omega \) due to nonfluid retarding torques \( M_x \) consists of three parts: (a) That \( \omega = (1/Q) \); (b) that \( \omega = (1/Q) \); and (c) that practically independent of \( Q \) or \( \omega \).

In flow measurement, it is the constancy of the fractional rotor slip \( \Delta \omega/\omega \), or the meter factor within the operating flow range, which determines the meter accuracy. The absolute magnitude of \( \Delta \omega/\omega \) has no bearing on the meter accuracy since it can be easily taken care of by using a proper meter factor determined from calibration. Referring to equation (69), a high-accuracy meter with a wide operating flow range from \( Q_{\text{min}} \) down to \( Q_{\text{min}} \) requires that the variation of the magnitude of \( (\Delta \omega)/\omega \) with flow rate \( Q \) within the entire operating flow range must be small. The absolute magnitude of \( (\Delta \omega)/\omega \) is of no importance. As shown by equation (69), that part \( (K_{\omega^2}) \) of \( M_x \) results in a constant amount of fractional rotor slip (independent of \( Q \) or \( \omega \)) within the operating flow range of the meter. Therefore, those loads of \( M_x \), which are proportional to \( \omega \), such as the axial thrust load and the dynamic unbalance load, have practically no effect on the meter accuracy. Their effect of causing a constant amount of fractional rotor slip (equal to \( HK_\omega/J^2 \)) thus changing the absolute value of the correct meter factor, is taken care of by calibration of the meter.

It is seen from equation (69) that both the component part \( K_{\omega^2} \) and the component part \( K_0 \) of \( M_x \) cause the fractional rotor slip \( \Delta \omega/\omega \) to increase with the decrease of flow rate \( Q \) or rotor speed \( \omega \). However, for a given change in \( Q \), the percentage change in \( \Delta \omega/\omega \) due to the part \( K_{\omega^2} \) is much less than that due to the part \( K_0 \) of \( M_x \). For example, for a turbine meter with an operating flow range \( Q_{\text{min}}/Q_{\text{max}} = 10 \), \( \Delta \omega/\omega \) due to the part \( K_0 \) at \( Q_{\text{min}} \) is 10 times that at \( Q_{\text{max}} \), whereas \( \Delta \omega/\omega \) due to the part \( K_{\omega^2} \) at \( Q_{\text{min}} \) is 100 times that at \( Q_{\text{max}} \). Moreover, it is noted that the part \( K_0 \) of \( M_x \) represents mainly the viscous drag in the bearings, and thus the constant \( K_1 \) of \( K_{\omega^2} \) is essentially in linear proportion to the viscosity of the fluid in the bearings. In gas turbine meters to which this paper is restricted, the bearings, either ball or journal, are usually not externally lubricated. Under this circumstance, the fluid in the bearings is the gas itself. As the viscosity of most gases is very low, the value of \( K_1 \) is very small, thus resulting in low load of \( K_2 \). For these two reasons, \( \Delta \omega/\omega \) due to the part \( K_{\omega^2} \) of \( M_x \) is usually much smaller than that due to the part \( K_0 \) of \( M_x \) in determining the minimum operating flow \( Q_{\text{min}} \) of the meter and can thus be neglected in most applications. From this reasoning, it follows that, in gas turbine meters where the bearings are not externally lubricated, it is only the component part \( K_0 \) of \( M_x \) that has an important effect on the meter accuracy at low flows and has to be determined. \( K_0 \) is the speed-independent part of \( M_x \) and represents mainly the load due to the signal generating device and that part of the bearing load which is due to the weight of the rotor system. At “starting flow,” the rotor speed \( \omega \) is almost zero and \( M_x \) is almost equal to \( K_0 \). Therefore, the value of \( M_x \) determined by the starting flow procedure as described in the paper is that of \( K_0 \) only. This conclusion that only the speed-independent part \( K_0 \) of \( M_x \) has an important effect on meter accuracy and needs to be considered is indicated without explanation in the paper by the definition of \( M_x \) following equation (9): “If \( M_x = \) total retarding torque due to nonfluid forces, mainly bearing loads from weight of the rotating system and pick (or register) load.” The speed-dependent part of \( M_x \) either has only a small effect on meter accuracy or changes only the absolute magnitude of the meter factor, the effect of which is taken care of by calibration.

In case of gas turbine meters with externally lubricated bearings, equation (69) indicates that the proper lubricant used should have the lowest viscosity consistent with other requirements. The viscous drag in the bearings with lubricant of high viscosity could reduce considerably the operating flow range of a turbine meter in high-accuracy flow measurement, especially in low pressure ranges.

The authors concur with Mr. Fox that the results from this paper should be viewed with caution. Because of the complexity of the fluid mechanics involved in several simplifying assumptions and restrictions are necessary to obtain useful results. The degree of accuracy in the prediction of meter performance by means of the method described depends greatly on how closely these assumptions and restrictions are met. However, it is hoped that the experimental determination of many important quantities from the calibration data could account for many secondary
effects which are otherwise too complicated to be considered analytically. This experimental determination could significantly improve the accuracy of meter performance prediction.

The authors also agree fully with Mr. Fox that, with the present technique in gas meter calibration, it is very difficult to obtain enough actual data with sufficient accuracy and reliability to test the degree of accuracy of the method described in the paper. It is the authors' sincere hope that this paper will stimulate further research and investigation in this field.

The authors are happy to see that Mr. Shafer, by means of dimensional analysis and the techniques described in the paper, derives a family of performance curves (Fig. 13) from which the meter performance can be predicted for any fluid of known density and viscosity. While the performance curves presented in the paper are more convenient to use in actual application, the curves presented by Mr. Shafer are more general.

The authors are in complete agreement with Mr. Shafer that the "stopping flow rate" instead of the so-called "starting flow rate" should be used for the evaluation of nonfluid retarding torques $M_n$. Not only can the stopping flow rate be more closely determined experimentally than the starting flow rate, it also gives the correct value of $M_n$ which depends upon the coefficient of kinetic friction rather than the coefficient of static friction. It should be noted that the "starting flow rate" is defined in the paper as "the minimum flow rate $Q_0^*$ at which the meter just starts to turn or turns with $\omega/\omega_i \to 0." Therefore, the authors really intend to mean the "stopping flow rate" rather than "the starting flow rate."

For high-accuracy turbine meters, the rotors should be balanced at least statically. Test results show that no difficulty is encountered to obtain consistent values of stopping flow rate for statically balanced rotors.