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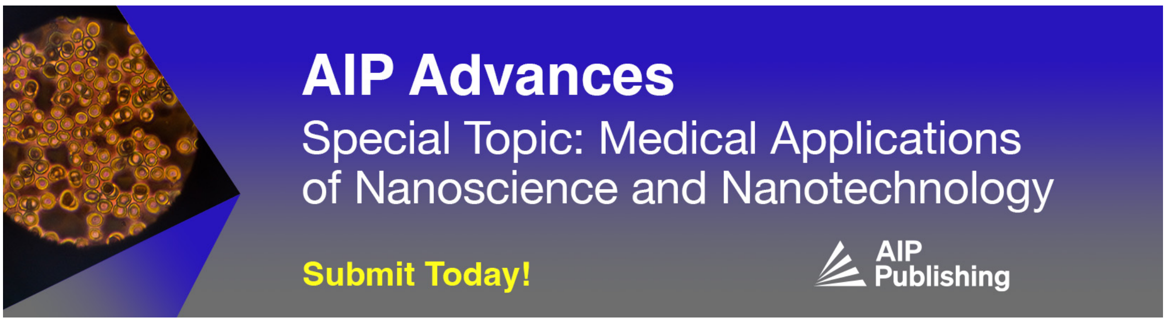

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In-plane and out-of-plane free vibration of stepped straight and curved beams with different materials

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This paper presents a comprehensive investigation of the free vibrations of stepped straight and curved beams with different shapes and different materials. The beams are assumed to be Euler-Bernoulli type, and Finite Element Displacement Method (FEDM) is used as a computational approach. In-plane and out-of-plane vibration analyses are handled with stepped straight and curved beams at different end conditions. Material pairs of the stepped curved beam are considered as (i) steel-steel, (ii) steel-aluminum, (iii) steel-brass and (iv) steel-araldite. Results are given in tabular form and compared with those in literature and computations obtained by Ansys. The effects of beam shape and different material type on the vibration characteristics are also investigated. https://doi.org/10.1063/1.5038722

I. INTRODUCTION

Dynamics of stepped beams have been commonly investigated by many researchers due to their importance of industrial applications in many engineering areas such as the automotive world or nautical world. Since the stepped beams are used in many structures and engineering applications, their vibration characteristics have been of great interest to researchers. One one hand, there have been many studies on the straight stepped beams. Some of them are presented as follows. Kisa and Gurel[1] present a novel numerical technique to analyze the free vibration of uniform and stepped cracked beams with a circular cross-section. Mao and Pietrzko[2] investigate free vibration of a stepped Euler-Bernolli beam consisting of two uniform sections by using the Adomian Decomposition Method (ADM). Mao[3] states that ADM offers an accurate and effective method of free vibration analysis of multiple-stepped beams with arbitrary boundary conditions. Suddoung et al.[4] study on free vibration response of stepped beams made from functionally graded materials. It is indicated that the differential transformation method is useful for solving the governing differential equations of such beams. Lee[5] applies a Chebyshev-tau method based on Euler-Bernoulli and Timoshenko beam theories to the free vibration analysis of stepped beams. Lin and Ng[6] introduce a novel numerical method to the prediction of vibration modes of general stepped beams with arbitrary steps and general elastic supports. Özyiğit et al.[7] work on the out-of-plane vibrations of curved uniform and curved tapered beams.

On the other hand, there are rare studies for stepped curved beams in literature. One of them is presented by Noori et al.[8] where they investigate damped, and undamped transient response of in-plane and out-of-plane loaded stepped curved rods with circular cross-sections. The novelty of this study is to address the vibration characteristics of a curved beam with different materials regarding stepped wise condition. In-plane and out-of-plane free **v**ibration**s** of stepped straight and curved beams are investigated. The vibration analysis consists of two main parts: (i) A stepped straight beam is taken into account concerning different boundary conditions. The results are compared with

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the literature. (ii) A stepped curved beam is taken into consideration regarding different boundary conditions and different materials.

II. MODELLING AND FORMULATION

The in-plane elastic and kinetic energy equations of straight (s) and curved (c) beam can be expressed as follows (Fig. 1) where E, I and A are the modulus of elasticity, mass moment of inertia and cross-section area of beams, respectively. ρ is the density of the material.

$$U_s = \frac{1}{2}E\int_x \left[A\varepsilon_s^2 + I\kappa_s^2\right]dx \tag{1}$$

$$T_s = \frac{1}{2}\rho A\int_x \left(\dot{U}_s^2 + \dot{V}_s^2\right)dx \tag{2}$$

$$U_c = \frac{1}{2}E\int_s \left[A\varepsilon_c^2 + I\kappa_c^2\right]ds \tag{3}$$

$$T_c = \frac{1}{2}\rho\int_s \left[A\left(\dot{U}_s^2 + \dot{V}_s^2\right) + I\dot{\beta}_c^2\right]ds \tag{4}$$

Strain, cross-sectional rotation, and curvature change of straight and curved beams are

$$\varepsilon_s = \frac{\partial u_s}{dx} \quad \beta_s = \frac{\partial v_s}{\partial x} \quad \kappa_s = \frac{\partial \beta_s}{\partial x} = \frac{\partial^2 v_s}{\partial x^2} \tag{5}$$

$$\varepsilon_c = \frac{\partial u_c}{\partial s} + \frac{v_c}{R} \quad \beta_c = \frac{\partial \mathrm{v_c}}{\partial \mathrm{s}} - \frac{u_c}{R} \quad \kappa_c = \frac{\partial \beta_c}{\partial s} = \frac{\partial^2 v_c}{\partial s^2} - \frac{1}{R}\frac{\partial u_c}{\partial s} \tag{6}$$

The out-of-plane elastic and kinetic energy equations of curved (c) beam can be expressed as follows;[7]

$$U_{cout} = \frac{1}{2}EI\int_S \kappa_{cout}^2 ds + \frac{1}{2}GJ\int_S \varphi_c^2 ds \tag{7}$$

$$T_{cout} = \frac{1}{2}\rho A\int_s \dot{\omega}_c^2 ds + \frac{1}{2}\rho I\int_s \dot{\Psi}_c^2 ds + \frac{1}{2}\rho J\int_s \dot{\Phi}_c^2 ds \tag{8}$$

where G is the modulus of shear and J is the polar moment of inertia, respectively. The term $(\dot{\ })$ denotes differentiation with respect to time t. Out-of-plane curvature change, torsion, and slope terms are;

$$\kappa_{cout} = \frac{\Phi_c}{R} - \frac{\partial^2 w_c}{\partial s^2}, \quad \varphi_c = \frac{\partial \Phi_c}{\partial s} + \frac{1}{R}\frac{\partial w_c}{\partial s}, \quad \Psi_c = \partial w_c / \partial s \tag{9}$$

where Φ_c is the torsional displacement of the curved element.

By following the finite element procedure, the stiffness and inertia matrices are obtained for straight and curved beam elements for in-plane and out-of-plane vibrations.

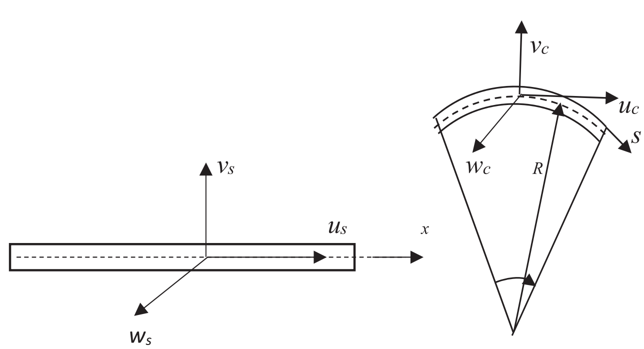

FIG. 1. Straight and curved beam elements.

III. FREE VIBRATION ANALYSIS

Matrix equation for the free vibrations of beam starts with an equation form

$$[K]\{V\}+[M]\left\{\frac{d^2V}{dt^2}\right\}=\{0\} \tag{10}$$

where $\{V\}$ denotes global displacement vector, $[K]$ and $[M]$ are global stiffness and inertia matrices, respectively.[7] The solution of Eq. (7) is assumed as

$$\{V\}=\left\{\overline{V}\right\}e^{j\omega_n t} \tag{11}$$

where $j=\sqrt{-1}$, ω_n is natural frequency and $\{\overline{V}\}$ is displacement amplitude vector of all nodes. Then, one obtains the eigenvalue equation giving the natural frequencies for in-plane and out-of-plane vibrations

$$\left|\,[K]-\omega_n^2[M]\,\right|=0 \tag{12}$$

IV. VIBRATIONS OF STRAIGHT BEAM

In this part, natural frequencies are obtained for the clamped side(s) of clamped-clamped (C-C) and clamped-free (C-F) boundary conditions corresponding to the following equation:

$$u_s=v_s=\frac{\partial v_s}{\partial x}=0 \tag{13}$$

Vibration behavior of stepped straight beam is analyzed by considering natural frequencies. Results are compared with those in literature as presented in Tables I–III. One can say that there is a good agreement between results of the present study and those in the literature.[5,6]

TABLE I. Natural frequencies (Hz) of a single-stepped beam under C-F boundary condition.

Mode	Present Study	Lee[5]
1	10.909	10.91
2	58.263	58.26
3	161.167	161.17
4	313.904	313.90

TABLE II. Natural frequencies (Hz) of a two-stepped beam under C-C boundary condition.

Mode	Present Study	Lee[5]
1	16.126	16.13
2	41.010	41.01
3	78.677	78.68
4	130.549	130.55
5	195.017	195.01

TABLE III. Natural frequencies (Hz) of a stepped beam with four-segment under simple boundary condition.

Mode	Present Study	Lin and Ng[6]
1	0.4336923	0.4335073
2	1.8027555	1.8028559
3	4.4147033	4.4147511
4	9.5413348	9.5412683
5	13.266108	13.266013
6	19.358906	19.358889
7	25.760441	25.760378
8	35.004538	35.004206
9	43.219404	43.218561
10	55.663627	55.662552

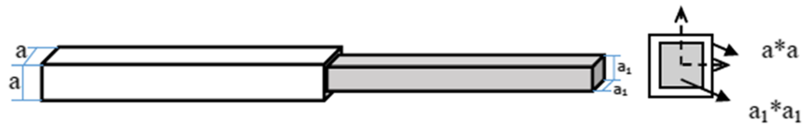

FIG. 2. Single-stepped beam with square cross-section.

TABLE IV. Natural frequencies (Hz) of a stepped steel beam under C-F boundary condition.

Mode	a_1=a	a_1=0.9a	a_1=0.8a	a_1=0.7a	a_1=0.6a	a_1=0.5a
1	16.360	17.838	19.489	21.234	22.815	23.590
2	102.53	98.320	92.319	84.766	76.413	68.635
3	287.07	272.768	259.578	246.245	229.631	205.104
4	562.55	531.257	494.373	454.395	416.560	387.344
5	929.94	884.277	838.197	784.455	712.008	616.218

Then, a single-stepped steel beam with a square cross-section is regarded as shown in Figure 2. The beam is 1 m in length, and it is stepped at mid-point. (a=20 mm, E=$2^{*}10^{11}$ Pa, and ρ=7800 kg/m^3). The thin part of the beam is connected centrally to the right side of the thick part. The boundary condition of the beam is assumed to be clamped-free (namely C-F, clamped end at left and the free end at the right side). Results are given in Table IV because there is a smaller cross-section at the right side of the beam. It is noted that (i) fundamental frequencies increase with decreasing cross-section area, (ii) the other frequencies, however, decrease with decreasing cross-section area.

V. IN-PLANE VIBRATIONS OF STEPPED CURVED BEAMS

In this part, in-plane natural frequencies are obtained for the clamped side(s) of C-C and C-F boundary conditions corresponding to the following equation:

$$u_c = v_c = \frac{\partial v_c}{\partial s} = 0 \tag{14}$$

A. Quarter circle stepped beam

The similar analysis is performed for a stepped curved steel beam. The stepped curved beam is shown in Figure 3 with 90^0 arc angle (α). Cross-sectional and material properties are assumed to be the same as the previous part for the straight stepped beam. The quarter circle beam is 1 m in length, and stepped at mid-point. The cross-section of the beam is still square (i.e., a=20 mm at left part with decreasing ratio to the right part) and the connection between two cross-sections is central. The results of in-plane natural frequencies are obtained under C-C and C-F boundary conditions as given in Tables V and VI respectively.

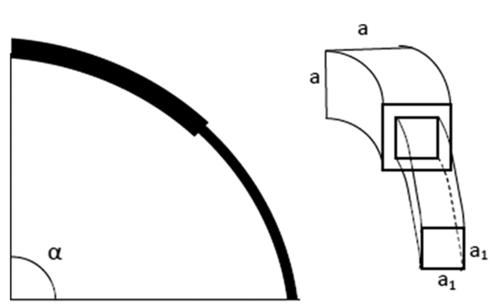

FIG. 3. Quarter circle stepped curved steel beam.

TABLE V. Natural frequencies (Hz) of a stepped curved steel beam under C-C boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	259.404	258.33	233.865	232.99	198.319	197.16	131.46	130.48
2	484.897	481.19	430.471	427.19	372.314	370.19	342.403	341.02
3	893.721	883.00	806.169	797.85	687.767	680.15	490.233	485.56
4	1099.46	1093.40	1036.10	1027.4	924.134	916.39	769.461	762.10
5	1449.35	1432.50	1343.22	1333.1	1265.14	1256.7	1110.41	1097.4

TABLE VI. Natural frequencies (Hz) of a stepped curved steel beam under C-F boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	17.198	17.201	20.485	20.474	23.940	23.879	23.197	23.063
2	82.874	82.794	77.977	77.783	67.298	67.109	58.831	58.798
3	261.380	260.570	239.697	239.01	215.652	214.68	162.379	161.23
4	533.402	530.220	470.325	467.27	395.400	393.26	347.706	346.41
5	894.767	886.050	805.563	798.51	688.170	681.35	490.994	486.45

Table V says that in-plane natural frequencies of all modes decrease due to the thinner right side part of the stepped beam. However, results of the beam under C-F boundary condition in Table VI indicate that there is an increase for the first mode up to a_1=0.5a.

B. Half circle two-stepped curved beam

In this part, computational analysis is carried to half circle steel beams which are symmetrically stepped at two different arc angle types. As shown in Figures 4 and 5, arc angles of parts are considered 45-90-45 (A) and 60-60-60 (B) in degrees, respectively. While the cross-section at the bottom is still 20×20 mm^2 at both sides, the cross-section of the mid-part of the curved beam is less than 20×20 mm^2. The length of the central axis of half circled beam is 1 m. The mid-part is connected to left and right sides centrally.

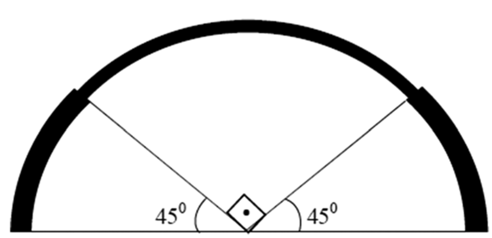

FIG. 4. Two-stepped half-circle beam (A).

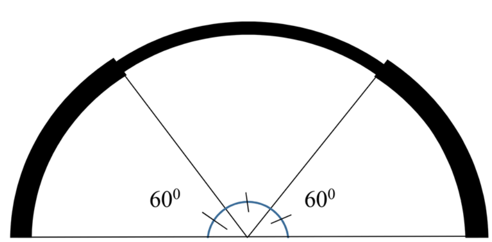

FIG. 5. Two-stepped half-circle beam (B).

TABLE VII. Natural frequencies (Hz) of symmetrical two-stepped half circle curved steel beam A under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	201.007	194.088	189.554	200.482
2	440.110	417.436	354.884	273.483
3	817.547	760.987	667.114	478.206
4	1238.97	1141.74	1050.38	835.164
5	1798.54	1597.08	1393.73	1245.27

TABLE VIII. Natural frequencies (Hz) of a symmetrical two-stepped half circle curved steel beam B under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	201.007	197.152	176.876	152.058
2	440.110	445.407	435.570	334.834
3	817.547	754.859	708.157	695.743
4	1238.97	1136.08	967.923	825.508
5	1798.54	1689.16	1524.148	1137.06

On the one hand, results in Table VII say that the fundamental frequencies decrease moderately at first and then increase due to the thinner mid-part of the beam (A). However, other frequencies decrease consistently. On the other hand, results in Table VIII indicate that a consistent decrease is observed at modes 1, 3, 4 and 5. However, mode 2 shows an increase at the beginning and later a decrease due to the thinner mid-part of the beam.

1. Beam with different materials

In this part, different materials are considered for mid-part of the beams (A) and (B) (see Figures 4 and 5). Brass and aluminum are chosen as a material type of the mid-part ($E=10^{11}$ Pa, $\rho=8000$ kg/m^3 for brass and $E=0.7^*10^{11}$ Pa, $\rho=2700$ kg/m^3 for aluminum). Results are presented in Tables IX and X for the steel-brass-steel beam (A) and the steel-brass-steel beam (B), respectively.

TABLE IX. Natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-brass-steel beam A under C-C boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	169.467	168.96	166.047	165.63	170.956	170.73	185.815	185.87
2	371.819	369.15	336.970	334.55	281.079	279.15	235.136	233.83
3	692.901	684.38	634.052	626.68	520.738	514.75	359.842	356.55
4	1018.03	1001.40	963.435	949.63	862.677	850.47	615.200	607.82
5	1463.57	1432.20	1314.89	1291.3	1198.87	1182.4	983.675	970.27

TABLE X. Natural frequencies (Hz) of symmetrical two-stepped (B) half circle curved steel-brass-steel beam B under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	182.186	172.165	154.194	142.672
2	398.004	403.754	372.267	261.913
3	702.922	665.038	648.522	613.012
4	1084.91	965.634	839.499	782.639
5	1611.81	1485.35	1241.89	914.673

TABLE XI. Natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-aluminum-steel beam A under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	229.836	221.590	222.160	231.396
2	479.923	416.773	333.944	272.056
3	870.945	798.346	658.493	449.664
4	1315.52	1128.67	1111.70	815.747
5	1785.62	1585.25	1439.93	1321.48

TABLE XII. Natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-aluminum-steel beam B under C-C boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	213.803	213.06	193.104	192.49	167.430	167.01	151.384	151.19
2	500.906	496.67	483.292	479.15	412.827	408.89	266.827	264.78
3	826.106	817.19	778.784	770.81	758.075	750.41	754.254	746.48
4	1185.98	1165.5	1045.01	1029.1	895.996	884.38	823.034	813.77
5	1835.90	1791.2	1742.65	1702.2	1510.95	1480.1	1068.90	1053.9

Moreover, results are presented in Table XI and Table XII for the steel-aluminum-steel beams (A) and (B) respectively.

As it is seen in Tables VII–X, one can conclude that the replacement of mid-part with brass causes a decrease at all frequencies for beam A and B. However, when noting Tables VII, VIII, XI, and XII, one can observe that replacing the steel part (mid-part) with aluminum causes an increase for the beam (A), but a decrease for the beam (B) at fundamental frequencies. On the other hand, for the upper modes, lower or higher values are obtained as natural frequencies for different cross-sections of mid-parts.

VI. OUT-OF-PLANE VIBRATIONS OF STEPPED CURVED BEAMS

The work in Sec. V is reconsidered concerning out-of-plane vibration of a half circle stepped beam under clamped side(s) of C-C and C-F boundary conditions such that

$$\Phi_c = w_c = \frac{\partial w_c}{\partial s} = 0 \tag{15}$$

Results of the out-of-plane natural frequencies for steel beams (A) and (B) are presented in Tables XIII and XIV ($G=0.84^*10^{11}$ Pa).

Tables XIII and XIV indicate that (i) the fundamental natural frequencies increase because of the thinner mid-part of the beam, (ii) other frequencies of the modes shows a decrease trend.

TABLE XIII. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel beam A under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	83.834	96.411	112.474	120.317
2	241.903	230.069	217.938	217.993
3	507.261	454.876	375.995	287.847
4	869.127	785.137	673.763	485.505
5	1324.76	1181.92	1052.711	852.904

TABLE XIV. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel beam B under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	83.834	93.066	104.871	120.822
2	241.903	227.410	198.835	162.490
3	507.261	475.515	442.966	367.280
4	869.127	795.644	726.124	689.287
5	1324.76	1212.71	1025.09	835.545

A. Beam with different materials

At last, three different materials are taken into account for mid-part of the beams (A) and (B) namely Brass, Aluminum and Araldite (G=0.39*10^{11} Pa for Brass, G=0.26*10^{11}Pa for Aluminum, E=979*10^6 Pa, ρ=1000 kg/m^3, G=623*10^6 Pa for Araldite). Results of the out-of-plane vibration are presented in Table XV and Table XVI for the steel-brass-steel beam (A) and (B) respectively.

The out-of-plane analysis states that (i) the steel-brass-steel beam shows similar frequency behavior with the steel beam, (ii) when comparing Tables XIII and XIV with Tables XV and XVI respectively, one can see that the decrease of natural frequencies is due to the brass effect in mid-part of the curved beam.

Results of the out-of-plane natural frequencies for the steel-aluminum-steel beams A and B are presented in Tables XVII and XVIII.

TABLE XV. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-brass-steel beam A under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	78.790	89.952	100.954	94.584
2	204.606	196.735	194.526	201.607
3	425.998	371.881	305.604	257.175
4	735.210	651.635	528.778	365.414
5	1094.90	988.652	863.295	629.386

TABLE XVI. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-brass-steel beam B under C-C boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	79.551	78.824	88.783	87.767	101.541	100.11	117.494	115.76
2	217.829	215.27	199.978	198.08	173.155	171.81	150.274	148.83
3	455.270	447.78	425.056	417.42	380.150	373.15	281.810	277.78
4	750.149	736.53	692.567	679.63	651.592	637.42	608.394	594.92
5	1158.77	1130.6	1024.19	1001.6	868.568	849.11	776.596	756.33

TABLE XVII. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-aluminum-steel beam A under C-C boundary condition.

	a_1=a		a_1=0.8a		a_1=0.6a		a_1=0.4a	
Mode	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS	Present Study	ANSYS
1	123.999	122.72	138.770	137.22	151.589	149.88	138.894	137.47
2	263.454	261.1	248.570	246.63	240.020	238.16	240.507	240.55
3	496.091	488.38	428.357	422.86	343.220	339.31	277.834	272.68
4	882.962	863.74	802.306	786.38	667.396	656.73	461.704	456.60
5	1341.95	1307.3	1236.11	1205.4	1131.45	1106.4	869.467	857.65

TABLE XVIII. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-aluminum-steel beam B under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	110.502	118.224	127.420	136.985
2	236.510	211.579	179.506	155.040
3	518.077	502.484	471.733	364.525
4	866.905	802.840	759.775	744.192
5	1290.14	1135.34	942.645	820.519

TABLE XIX. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-araldite-steel beam A under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	71.976	59.261	45.0611	30.195
2	182.743	162.251	127.152	85.884
3	242.805	240.494	232.898	170.681
4	289.807	269.475	259.190	248.987
5	452.322	363.105	286.995	269.962

TABLE XX. Out-of-plane natural frequencies (Hz) of symmetrical two-stepped half circle curved steel-araldite-steel beam B under C-C boundary condition.

Mode	a_1=a	a_1=0.8a	a_1=0.6a	a_1=0.4a
1	116.675	115.828	100.712	70.159
2	144.225	144.188	145.221	146.202
3	210.218	174.713	153.766	148.930
4	485.702	399.030	301.236	201.006
5	741.715	712.275	581.617	392.216

On one hand, Tables XVII and XVIII present that (i) for the first mode, the effect of material (aluminum) on frequency shows mostly an increasing tendency due to the smaller sizes of the mid-part of the beam, (ii) conversely, for the other modes, aluminum effect on frequency indicates fully a decreasing tendency. On the other hand, when comparing Tables XVII and XVIII with Tables XIII and XIV respectively, mode three possesses lower frequency values for the beam (A) and modes second and fifth possess lower frequency values for the beam (B).

Results of the out-of-plane natural frequency for the steel-araldite-steel beams A and B are presented in Tables XIX and XX, respectively. One can note that (i) making mid-part thinner reduces the frequencies at almost all modes, (ii) frequency gap between sequential modes becomes smaller than that of other bimaterial beams.

VII. DISCUSSIONS

In order to obtain high potential functionality of a beam under the dynamic condition, bimaterial approach is one of the solutions to improve damping, energy absorption, and structural stability characteristics. Consequently, investigation of vibration characteristics of a curved beam with different materials regarding stepped wise condition is a key point to understand how to design a beam to resist or enhance in-plane and out-of-plane free vibrational behavior. As engineering materials, one may say that brass and araldite are useful materials for these purposes. While the former material may be chosen for improvement of in-plane free vibrational behavior, the latter may be selected for that of out-of-plane free vibrational behavior. One may also note that there is a critical point for araldite usage. That is, because modes of steel-araldite-steel beam have close frequency values, there may occur catching resonance.

VIII. CONCLUSIONS

Despite there are many studies about curved beams and stepped beams separately, this study provides a practical solution for vibrations of beams which are stepped and curved together. In-plane and out-of-plane free vibrations of half circle stepped beams are investigated because they are rare in literature. Stepped parts of the beams are also considered concerning different engineering materials, e.g., steel, brass, aluminum, and araldite. Both geometrical and material effects on natural frequencies are determined and interpreted. Results conclude that this finite element solution proposed here is suitable to investigate the vibrations of stepped and curved bimaterial beams.

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