



Book Reviews

The Geometry of Involute Gears, J. R. Colbourne, Springer-Verlag, New York, 1987, 532 pages.

Reviewed by Robert Errichello¹

This book is a true textbook on involute gears, the first since the classic works of Buckingham and Merritt. It will be extremely useful to graduate students, practicing gear engineers, and gear researchers. Those who wish to develop computer software for gear geometry will find the presentation style, algorithms, and example problems very helpful. Professor Colbourne has a skill for describing complicated gear geometry in a straight-forward manner that impresses the reader with its clarity. He derives all equations from first principles such as the Law of Gearing, and reduces the equations to useful design algorithms.

While written for involute gears, much of the material is applicable to noninvolute, conjugate gears. The chapter on the geometry of noninvolute gears is especially helpful for defining the shape of the root fillets of involute gears.

The author has chosen to cover spur gears in Part 1 (Chapters 1–12) and helical gears in Part 2 (Chapters 13–17), believing that gear geometry is more easily understood if one studies the simpler geometry of a spur gear first and follows with the more general case of a helical gear. A potential difficulty with this arrangement is that there may be cases where the reader is left wondering whether a particular result derived for a spur gear applies to a helical gear.

Vector theory is used to describe the three-dimensional geometry of helical gears. Students should feel comfortable with the vector algebra, while some older engineers may be somewhat dismayed by the apparent complexity of the equations. However, vector theory is carefully and gradually introduced and used mainly in derivations and proofs. With a little review of vector mechanics, most engineers should not be overwhelmed.

The equation symbols generally agree with those used in North America except the author has refined his notation so that it is consistent and very easy to follow. Metric module is used to describe tooth size as is the practice in Europe and Japan. However, the relation between diametral pitch and module is explained and several of the examples are given in terms of diametral pitch.

The book begins with a description of the requirements for a constant angular velocity ratio and a definition of the Law of Gearing. The relationship of the path of contact for conjugate profiles and the basic rack is explained. The properties of the involute and the geometry of involute gear teeth are treated next. Equations for the basic parts of gear teeth are given; pitch, tooth thickness, addenda, dedenda, etc. It is demonstrated that the base pitch and pressure angle of the

basic rack are fundamental, and sufficient to define a system of involute gear teeth.

The kinematics of a pinion meshed with a rack or another external gear are described and equations are given for calculating the displacement and velocity at any point in the meshing cycle. The influence of variation in center distance on the operating conditions are explored, and the important advantage of involute gear teeth is highlighted: any pair of involute gears can mesh correctly provided their base pitches are equal, regardless of small changes to the center distance.

There is a chapter on calculating contact ratio and backlash which also gives methods for checking for gear tooth interference.

The chapter on gear cutting is excellent. It covers form cutting with milling cutters, shaping with pinion cutters or rack cutters, and hobbing.

One of the features of the involute system is its extreme flexibility through the use of addendum modification using standard gear cutters. Professor Colbourne prefers the term "profile shift" rather than "addendum modification" and his profile shift, "*e*," is the actual distance between the rack cutter (or hob) reference line and the cutting pitch circle of the work piece. With this definition, the profile shift, *e*, includes the small adjustment of cutter position to thin the teeth of the work piece for backlash. This is in contrast to European practice where the addendum modification coefficients are chosen independent of backlash considerations. Gear designers will appreciate the algorithm for designing profile-shifted gears for standard or nonstandard center distances.

Methods are given for calculating important geometry such as the radii to the highest and lowest points of single-tooth contact, limit circles, form circles, and the geometry of the root fillet of undercut gears. There is detailed treatment of the various methods for measuring gear tooth thickness such as Vernier caliper, span, and wire (pin) measurements.

With the material given in the chapter on noninvolute gears, the reader will be able to calculate the entire shape of both the profile and the root fillet of gear teeth generated by either rack cutters or pinion cutters. The fillet shape of an undercut gear and profile modification (tip and root relief) are also discussed in this chapter.

The Euler-Savary equation is derived and used to develop expressions for the radius of curvature of both the involute profile and the root fillet. These radii are used in the following chapter in the calculation of Hertzian contact stress and root fillet bending stress. The author's formulation of the stress equations is different from that of AGMA 218.01 [1], but the resulting equations are based on the same theories, and he gives formulas for converting his results to AGMA geometry factors I and J.

Part 1 of the textbook concludes with a chapter on internal spur gears. The development of the material is very similar to earlier chapters on external gears except the additional checks that the gear designer must make are emphasized. Fillet in-

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terference, tip interference, axial and radial assembly and manufacturing problems such as tip trimming and cutter rubbing are covered in depth. The calculations necessary to obtain the shape and radius of curvature of the root fillet of an internal gear are shown. A design procedure is given which avoids interference while maximizing working depth. Readers will also be interested in the author's technical paper [2] which presents the design procedure and interference checks in a concise algorithm.

Part 2 of the book covers most aspects of helical gears including geometric design, gear cutting by shaping and hobbing, and gear tooth inspection. Manufacturing engineers will be especially interested in the discussion of hobbing machines with gear trains or differentials.

There is a comprehensive chapter on crossed helical gears, especially important because the meshing geometry of a crossed helical gear pair forms the theoretical basis of the hobbing process.

The book concludes with a chapter on tooth stresses in helical gears. The author's analysis of stresses differs significantly from AGMA methods and he suggests several

possible improvements for the AGMA procedures. Stress analysts will also find the author's technical paper [3] on fillet stresses informative.

An important feature of the book is its many numerical examples which gear designers should find helpful for checking their work and validating computer programs. I would have liked to see some discussion of protuberance for prehave or pregrind cutters, but perhaps this can be added in the next edition. This book contains much material that is relevant to the work of design and manufacturing engineers. Indeed, it is recommended to all those who study or practice the art of gear design and manufacturing.

References

- 1 AGMA 218.01, *AGMA Standard for Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth*, Dec. 1982.
- 2 Colbourne, J. R., "The Geometric Design of Internal Gear Pairs," AGMA Technical Paper No. 87 FTM 2, Oct. 1987.
- 3 Colbourne, J. R., "Effect of Oblique Loading on the Fillet Stress in Helical Gears," AGMA Technical Paper No. 86 FTM 6, Oct. 1986.