Current-Voltage Characteristics for a Tunnel Junction Composed of a Conductor with a CDW and Superconductor

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Current-voltage \((J-V)\) and differential-conductivity-voltage \((dJ/dV-V)\) characteristics are analytically calculated at zero temperature for a tunnel junction: a superconductor (S) and a Peierls conductor (P) separated by an insulator (I). Here P is a conductor with a charge density wave (CDW). The Hamiltonians of S and P are treated in the mean field approximation. To calculate the current \(J\) analytically, \(\Delta_S = \Delta_P \equiv \Delta\) is assumed, where \(\Delta_S\) and \(\Delta_P\) are the energy gaps of S and P, respectively. When the second order of the perturbation theory in the tunnel Hamiltonian is considered in the conventional tunnel Hamiltonian approach, the \(J-V\) and \(dJ/dV-V\) characteristics depend on only the CDW phase \(\varphi\). The current \(J\) has a discontinuous jump at \(eV = 2\Delta\) for \(\varphi \neq 0\). The differential conductivity \(dJ/dV\) has a singularity at \(eV = 2\Delta\) for \(\varphi \neq 0\). The relation \(J(V,\varphi) = -J(-V,\varphi + \pi)\) is obtained.

§1. Introduction

A charge density wave (CDW) can be characterized by a complex order parameter. In particular, the phase \(\varphi\) is very important, and the fluctuation of the phase corresponds to a sliding motion, which produces remarkable behavior, like non-Ohmic conductivity \(^1\) and narrow band noise. \(^2\) The fluctuation of the phase in bulk systems has already received much attention in the last few decades. \(^3\)

By contrast, CDW tunnel junctions have been investigated little in the mean field approximation. To this time, the dependence of the current on the CDW phase \(\varphi\) has been investigated for \(P_1-I-P_2\) and \(P-I-N\) junctions, where P, I, and N denote a Peierls conductor, insulator and normal metal, respectively. \(^4\) - \(^8\) A Peierls conductor is a conductor with a CDW. In 1983 and 1984, Artemenko and Volkov \(^4\), \(^5\) treated \(P_1-I-P_2\) and \(P-I-N\) junctions by using the Keldysh technique. Their results correspond to those for quasiparticle tunneling across \(S_1-I-S_2\) and \(S-I-N\) junctions, respectively, where S denotes a superconductor. In 1996, Tanaka et al. \(^8\) treated \(P-I-N\) junction by solving the Bogoliubov-de Gennes equation for CDW. Their results show that, unlike the \(S-I-N\) junction, the current is dependent on the CDW phase \(\varphi\), and Andreev reflection \(^9\) does not occur.

In this paper, we investigate the current-voltage \((J-V)\) and differential-conductivity-voltage \((dJ/dV-V)\) characteristics for a \(P-I-S\) junction in the conventional tunnel Hamiltonian approach. Both P and S can be characterized by the phases of the order parameters, but the CDW phase \(\varphi\) has a different physical meaning from the S phase \(\phi\), because the uncertainty relations are different. The CDW phase \(\varphi\) connects with the momentum, while the S phase \(\phi\) connects with the total particle number. By

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comparing the junction with the S\(_1\)-I-S\(_2\) junction, we investigate how the difference in the physical meaning appears in the \(J\)-\(V\) and \(dJ/dV\)-\(V\) characteristics.

This paper is organized as follows. In §2, the general expression for the tunnel current \(J\) is presented for a P-I-S junction in the conventional tunnel Hamiltonian approach. The current \(J\) is expressed by using the Green’s functions of S and P. First, their Hamiltonians are given in the mean field approximation. Second, the Green’s functions are presented. Finally, the general expression for the tunnel current \(J\) is presented for the junction by using these Green’s functions. In §3, the \(J\)-\(V\) and \(dJ/dV\)-\(V\) characteristics are analytically calculated at zero temperature for the junction. The results are discussed in §4. In §5, the conclusions are presented.

§2. General expression for tunnel current

Here, the general expression for the tunnel current \(J\) is presented for P-I-S junction (see Fig. 1) in the mean field approximation. The current \(J\) is expressed by using the Green’s functions of S and P. In this paper, the conventional tunnel Hamiltonian approach\(^{10,11}\) is used. For simplicity, the right- and left-hand electrodes of the junction include no impurities, and a one-dimensional system is treated.

The Hamiltonians of S and P are given in the mean field approximation. The Hamiltonian of S (\(H_S\)) has the form\(^{11}\)

\[
H_S = \sum_{k\sigma} \xi_k a_{k\sigma}^\dagger a_{k\sigma} - \sum_k (a_{k\uparrow}^\dagger a_{-k\downarrow}^\dagger \Delta_S e^{i\phi} + \text{h.c.},)
\]

(2.1)

where \(\Delta_S\) and \(\phi\) are the amplitude and phase of the order parameter of S, respectively. The first term is the free-electron Hamiltonian. The operators \(a_{k\sigma}^\dagger\) (\(a_{k\sigma}\)) are the creation (annihilation) operators of an electron with wave number \(k\) (\(\hbar = 1\)) and spin projection \(\sigma\).

Fig. 1. The geometry for a P-I-S junction, where P, S, and I denote a Peierls conductor, superconductor, and insulator, respectively. A charge density wave (CDW) appears along the \(x\) axis, and an insulator I exists at \(x = 0\).
The Hamiltonian of P ($H_P$) has the form\(^\text{12}\)
\[
H_P = \sum_{k\sigma} \xi_k a_{k\sigma}^\dagger a_{k\sigma} - \sum_{k\sigma} (a_{k+2k_F}\sigma e^{i\varphi} + \text{h.c.}),
\]
where $\Delta_P$ and $\varphi$ are the amplitude and phase of the order parameter of P, respectively, and $k_F$ is the Fermi wavenumber.

From the above Hamiltonians, the Green’s functions of S and P can be obtained at finite temperature ($T \neq 0$). The Green’s functions of S have the form\(^\text{11}\)
\[
G(k, \tau) = \frac{ip_n + \xi_k}{(ip_n)^2 - (\xi_k^2 + \Delta_S^2)},
\]
\[
F(k, \tau) = \frac{\Delta_S e^{i\varphi}}{(ip_n)^2 - (\xi_k^2 + \Delta_S^2)},
\]
where $p_n = (2n + 1)\pi T$ ($k_B = 1$), $n = 0, \pm 1, \pm 2, \ldots$. The Green’s functions $G(k, \tau)$ and $F(k, \tau)$ are the Fourier transforms of $G(k, \tau) = -\langle T_\tau a_{k\uparrow}(\tau)a_{k\downarrow}^\dagger \rangle_S$ and $F(k, \tau) = -\langle T_\tau a_{k\downarrow}(\tau)a_{-k\uparrow} \rangle_S$, respectively where $a_{k\sigma}(\tau) = \exp(\tau H_S) a_{k\sigma}^\dagger \exp(-\tau H_S)$, and the operator $T_\tau$ denotes the Wick time-ordering operator. Here we have defined the average
\[
\langle \cdots \rangle_S = \frac{\text{Tr}\{e^{-H_S/T} \cdots \}}{\text{Tr}\{e^{-H_S/T}\}}.
\]

The Green’s functions of P have the form\(^\text{13}\)
\[
G_{++}(k, \tau) = \frac{ip_n + \xi_{k+k_F}}{(ip_n)^2 - (\xi_{k+k_F}^2 + \Delta_P^2)},
\]
\[
G_{+-}(k, \tau) = \frac{\Delta_P e^{i\varphi}}{(ip_n)^2 - (\xi_{k+k_F}^2 + \Delta_P^2)}.
\]
The Green’s functions $G_{++}(k, \tau)$ and $G_{+-}(k, \tau)$ are the Fourier transforms of $G_{++}(k, \tau) = -\langle T_\tau a_{k+k_F}(\tau)a_{k-k_F}^\dagger \rangle_P$ and $G_{+-}(k, \tau) = -\langle T_\tau a_{k-k_F}(\tau)a_{k+k_F}^\dagger \rangle_P$, respectively, where we have abbreviated the spin projection $\sigma$.

The current $J$ can be derived by using these Green’s functions. Here the conventional tunnel Hamiltonian approach is used. The total Hamiltonian $H$ is expressed as
\[
H = H_R + H_L + H_T.
\]
The terms $H_R$ and $H_L$ describe the right- and left-hand electrodes of the junction. The tunnel Hamiltonian $H_T$ has the form
\[
H_T = \sum_{k,p,\sigma} T_{k,p} a_{k\sigma}^\dagger a_{p\sigma} + \text{h.c.},
\]
where the $T_{k,p}$ are the tunnel matrix elements. Hereafter, $k$ and $p$ denote the wave numbers on the right- and left-hand side, respectively.
The general expression for the current $J$ obtained at second order in the perturbation theory applied to $H_T$ becomes

$$J = 2e\text{Im}\{X(\omega_n \rightarrow -eV + i\delta)\}, \quad (2.9)$$

where $\delta$ is positive and infinitesimal and $\omega_n = 2n\pi T, n = 0, \pm 1, \pm 2, \cdots$. The voltage $eV (e > 0)$ can be expressed as the difference between the chemical potentials $\mu_R$ and $\mu_L$, i.e., $eV = \mu_L - \mu_R$, where $\mu_R$ and $\mu_L$ correspond to the right- and left-hand sides of the junction, respectively. The correlation function $X(\omega_n)$ takes the form

$$X(\omega_n) = \sum_{k,p,\sigma} \sum_{k',p'} T_{k,p} T_{k',p'}^{\ast} \int_0^{1/T} d\tau e^{i\omega_n \tau} \langle T\tau a_{k'\sigma}^{\dagger} a_{k\sigma}(\tau) \rangle_R \langle T\tau a_{p\sigma}(\tau) a_{p'\sigma}^{\dagger} \rangle_L, \quad (2.10)$$

where $a_{k\sigma}^{\dagger}(\tau) = \exp(\tau H_R) a_{k\sigma}^{\dagger} \exp(-\tau H_R)$ and $a_{p\sigma}(\tau) = \exp(\tau H_L) a_{p\sigma} \exp(-\tau H_L)$. Here we have defined the averages

$$\langle \cdots \rangle_R = \frac{\text{Tr}\{e^{-H_R/T} \cdots \}}{\text{Tr}\{e^{-H_R/T}\}}, \quad \langle \cdots \rangle_L = \frac{\text{Tr}\{e^{-H_L/T} \cdots \}}{\text{Tr}\{e^{-H_L/T}\}}. \quad (2.11)$$

For P-I-S junction ($H_L = H_P$ and $H_R = H_S$), the correlation function can be obtained by using the Green’s functions of S and P. We have

$$X_{P-I-S}(\omega_n) = 2T \sum_{k,p} \sum_{\nu_{m}} \left[ T_0^2 \{G_{++}(p, \nu_m) G^{\ast}(k + \nu_{m}, \nu_{m} - \omega_n) + G_{+-}(p, \nu_m) G(k + \nu_{m}, \nu_{m} - \omega_n) \} 
- T_0 T_Q \{G_{++}^{\ast}(p, \nu_m) G^{\ast}(k + \nu_{m}, \nu_{m} - \omega_n) - G_{+-}(p, \nu_m) G(k + \nu_{m}, \nu_{m} - \omega_n) \} 
+ G_{+-}(p, \nu_m) G^{\ast}(k + \nu_{m}, \nu_{m} - \omega_n) - G_{++}^{\ast}(p, \nu_m) G(k + \nu_{m}, \nu_{m} - \omega_n) \} 
- T_Q^2 \{G_{++}(p, \nu_m) G^{\ast}(k + \nu_{m}, \nu_{m} - \omega_n) + G_{+-}^{\ast}(p, \nu_m) G(k + \nu_{m}, \nu_{m} - \omega_n) \} \right], \quad (2.12)$$

where the asterisk are complex conjugation. The tunnel matrix elements $^4$ are given by

$$\begin{pmatrix} T_{k+k_F,p+k_F} & T_{k+k_F,p-k_F} \\ T_{k-k_F,p+k_F} & T_{k-k_F,p-k_F} \end{pmatrix} \equiv \begin{pmatrix} T_0 & T_Q \\ T_Q & T_0 \end{pmatrix}, \quad (2.13)$$

where $T_0$ and $T_Q$ are the tunnel matrix elements without transfer and with transfer from one side of the Fermi surface to the other, respectively.

In §3, by using the above expression, the current $J$ is calculated analytically at zero temperature for the junction.

§3. Results

The current $J$ can be derived analytically at zero temperature by calculating the correlation function for the P-I-S junction. In the analytical calculation, $\Delta_S = \Delta_P \equiv \Delta$ is assumed. Using the analytical expression, we investigate the $J-V$ and $dJ/dV-V$ characteristics.
For the voltage $V > 0$, the current $J$ takes the form

$$J = J_1 + J_2,$$

(3.1a)

$$J_1 = \frac{2\sigma_0}{e} \theta(eV - 2\Delta) \left[ \frac{(eV)^2}{eV + 2\Delta} K(\alpha) - \frac{(eV + 2\Delta)}{eV + 2\Delta} \{K(\alpha) - E(\alpha)\} \right],$$

(3.1b)

$$J_2 = -\frac{8\sigma_0}{e} \theta(eV - 2\Delta) \frac{T_0 T_Q}{T_0^2 + T_Q^2} \frac{\Delta eV}{eV + 2\Delta} K(\alpha) \cos \varphi,$$

(3.1c)

$$\sigma_0 = 4\pi e^2 \left( T_0^2 + T_Q^2 \right) N_R N_L, \quad \alpha = \frac{eV - 2\Delta}{eV + 2\Delta},$$

(3.2)

where $K(\alpha)$ and $E(\alpha)$ are the complete elliptic integrals of the first and second kind, respectively. The function $\theta(x)$ is the Heaviside step function. Here $N_R$ and $N_L$ are the densities of states at the Fermi levels on the right- and left-hand side, respectively.

From the result, we note four points. First, $J_i \neq 0$ ($i = 1, 2$) for $eV > 2\Delta$. Second, the first term $J_1$ corresponds to a quasiparticle current in the $S_1-I-S_2$ junction. Third, the second term $J_2$ is proportional to $\cos \varphi$. Fourth, the current $J$ is independent of the $S$ phase $\varphi$.

To investigate the dependence of the $J$-$V$ characteristics on the CDW phase $\phi$, we consider the simpler case of $T_0 = T_Q$. The current $J$ takes the form

$$J = \frac{2\sigma_0}{e} \theta(eV - 2\Delta) \left[ \frac{(eV)^2}{eV + 2\Delta} K(\alpha) - \frac{(eV + 2\Delta)}{eV + 2\Delta} \{K(\alpha) - E(\alpha)\} \right] - \frac{2\Delta eV}{eV + 2\Delta} K(\alpha) \cos \varphi,$$

(3.3)

The current $J$ is a periodic function of $\varphi$ with period $2\pi$. The CDW-phase dependence is plotted in Fig. 2. From this figure, we note two points. First, the current $J$ has a discontinuous jump at $eV = 2\Delta$ for $\varphi \neq 0$ $(0 \leq \varphi \leq \pi)$. The jump depends on the CDW phase $\varphi$ as

$$J(2\Delta^+) = \frac{\sigma_0}{e} \pi \Delta \{1 - \cos \varphi\},$$

(3.4)

where $2\Delta^+ = 2\Delta + \delta$. Second, the current $J$ increases as the CDW phase $\varphi$ $(0 \leq \varphi \leq \pi)$ increases.

Next, we investigate the dependence of the $dJ/dV$-$V$ characteristics on the CDW phase $\varphi$ with the above conditions: $V > 0$ and $T_0 = T_Q$. The differential conductivity $dJ/dV$ is obtained as

$$\frac{dJ}{dV} = 2\sigma_0 \theta(eV - 2\Delta) \left[ \frac{4\Delta}{(eV + 2\Delta)(eV - 2\Delta)} K(\alpha) \{\Delta \cos \varphi - (eV - \Delta)\} - \frac{1}{eV(eV - 2\Delta)} E(\alpha) \{\Delta eV \cos \varphi - (eV)^2 + 2\Delta^2\} \right].$$

(3.5)

The CDW-phase dependence is displayed in Fig. 3. From this figure, we note two
Fig. 2. The dependence of the \( J-V \) characteristics on the CDW phase \( \varphi \) for a P-I-S junction. The current \( J \) has a discontinuous jump at \( eV = 2\Delta \) for \( \varphi \neq 0 \) \((0 \leq \varphi \leq \pi)\). It increases as the CDW phase \( \varphi \) \((0 \leq \varphi \leq \pi)\) increases.

Fig. 3. The dependence of the \( \frac{dJ}{dV} \)-\( V \) characteristics on the CDW phase \( \varphi \) for P-I-S junction. The differential conductivity \( \frac{dJ}{dV} \) has a singularity at \( eV = 2\Delta \) for \( \varphi \neq 0 \) \((0 \leq \varphi \leq \pi)\). It increases as the CDW phase \( \varphi \) \((0 \leq \varphi \leq \pi)\) increases.
points. First, the differential conductivity \( dJ/dV \) has a singularity at \( eV = 2\Delta \) for \( \varphi \neq 0 \) (\( 0 \leq \varphi \leq \pi \)). For \( eV = 2\Delta^+ \) and \( \varphi = 0 \), it becomes

\[
\frac{dJ}{dV} = 3\pi\sigma_0 < +\infty.
\]

Second, the differential conductivity \( dJ/dV \) increases as the CDW phase \( \varphi \) \( (0 \leq \varphi \leq \pi) \) increases.

For the voltage \( V < 0 \), the current \( J \) is derived by using the relation \( J(V, \varphi) = -J(-V, \varphi + \pi) \) we have obtained.

### §4. Discussion

Here we discuss the results obtained in §3, i.e., the dependence of the \( J-V \) and \( dJ/dV-V \) characteristics on the CDW phase \( \varphi \) for the P-I-S junction, by comparing with S\(_1\)-I-S\(_2\) junction. First, we discuss the \( J-V \) characteristics. The first term \( J_1 \) is generated by the terms \( \text{Re}\{G_{++}(p, ip_m)G(k + k_F, ip_m - i\omega_n)\} \) and \( \text{Re}\{G^*_{++}(p, ip_m)G(k + k_F, ip_m - i\omega_n)\} \), and it corresponds to the quasiparticle current in S\(_1\)-I-S\(_2\) junction. The second term \( J_2 \) is generated by the term \( \text{Re}\{G_{+-}(p, ip_m)\text{Im}\{G(k + k_F, ip_m - i\omega_n)\}\} \), and it does not exist in S\(_1\)-I-S\(_2\) junction, because \( G_{+-} \) corresponds to \( F \), and the current cannot be generated by the term \( GF \) for S\(_1\)-I-S\(_2\) junction. The current \( J \) is independent of the S phase \( \phi \), because the current \( J \) cannot be generated by the Green’s function \( F \) to second order in the perturbation theory for \( H_T \). To investigate the dependence of the \( J-V \) characteristics on the S phase \( \phi \), we must treat the two electron tunneling, which appears at fourth order in the perturbation theory for \( H_T \). The terms \( J_i \) \( (i = 1, 2) \) have discontinuous jumps at \( eV = 2\Delta \) because of the mutual action of the singular densities of states on both sides of the barrier.

We have obtained the relation \( J(V, \varphi) = -J(-V, \varphi + \pi) \). Here we consider \( T_0 = T_Q \). The current \( J \) is a periodic function of \( \varphi \) with period \( 2\pi \). For \( \varphi = 0 \), the current \( J \) is continuous at \( eV = 2\Delta \), because the jump in \( J_2 \) has opposite sign and totally compensates for the jump in \( J_1 \). For \( \varphi \neq 0 \) \( (0 \leq \varphi \leq \pi) \), the current \( J \) has a discontinuous jump at \( eV = 2\Delta \), and the jump depends on the CDW phase \( \varphi \); because the current \( J \) includes the second term \( J_2 \), which is proportional to \( \cos \varphi \). For the S\(_1\)-I-S\(_2\) junction, a jump also exists at \( eV = 2\Delta \), but it is independent of the S phase \( \phi \).

Next, we discuss the \( dJ/dV-V \) characteristics. From the relation \( J(V, \varphi) = -J(-V, \varphi + \pi) \), we have obtained the relation

\[
\frac{d}{dV}J(V, \varphi) = \frac{d}{d(-V)}J(-V, \varphi + \pi).
\]

For \( \varphi = 0 \), the differential conductivity \( dJ/dV \) has a discontinuous jump at \( eV = 2\Delta \), while for \( \varphi \neq 0 \) \( (0 \leq \varphi \leq \pi) \), the differential conductivity \( dJ/dV \) has a singularity at \( eV = 2\Delta \), whose singularity is dependent on the CDW phase \( \varphi \), because the differential conductivity \( dJ/dV \) includes the term \( dJ_2/dV \), which is proportional to \( \cos \varphi \).
§5. Conclusions

We have investigated the $J$-$V$ and $dJ/dV$-$V$ characteristics at zero temperature for a P-I-S junction. When the second order of the perturbation theory for $H_T$ is considered, the $J$-$V$ and $dJ/dV$-$V$ characteristics depend on only the CDW phase $\varphi$. The current $J$ has a discontinuous jump at $eV = 2\Delta$ for $\varphi \neq 0$. The differential conductivity $dJ/dV$ has a singularity at $eV = 2\Delta$ for $\varphi \neq 0$. We have obtained the relation $J(V, \varphi) = -J(-V, \varphi + \pi)$.

In this paper, we have considered a system in which Peierls conductor $P$ includes no impurities. This means that the CDW phase $\varphi$ is independent of the position in the Peierls conductor, i.e., that $\varphi$ is constant. Therefore, in the case that there are impurities in $P$, the results obtained in this paper are not applicable. In the future, we would like to consider the effect of the impurities.

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References