We propose a simple model of the neutrino mass matrix that can explain the solar and atmospheric neutrino problems in a $3(\nu_L+\nu_R)$ framework. Assuming that only two right-handed neutrinos are heavy and a Dirac mass matrix has a special texture, we construct a model with four light neutrinos. The favorable structure of flavor mixings and mass eigenvalues required by these neutrino deficits is realized as a result of the seesaw mechanism. It is possible that bi-maximal mixing structure can be obtained in this scheme. Since it contains a light sterile neutrino, it may be capable of explaining the LSND result successfully.

We consider an embedding of this scenario for the neutrino mass matrix into the $SU(5)$ grand unification scheme using the Froggatt-Nielsen mechanism based on $U(1)_{F1} \times U(1)_{F2}$. In this case, only a large mixing angle MSW solution is obtained for the solar neutrino problem as a consistent solution.

§1. Introduction

Recently, the existence of non-trivial lepton mixing has been strongly suggested by atmospheric and solar neutrino observations whose results can be explained by assuming neutrino oscillations. The predicted flavor mixing is much larger than that of the quark sector. The explanation of this feature is a challenging problem for the construction of a satisfactory grand unified theory (GUT) and a great number of studies have been carried out. In most of such studies, the smallness of the neutrino mass is explained by the celebrated seesaw mechanism, and the flavor mixing structure is considered to be controlled by the Froggatt-Nielsen mechanism. There are many works in which the Abelian flavor symmetry is discussed. There is another experimental suggestion regarding neutrino oscillation by the Liquid Scintillator Neutrino Detector (LSND). If we require a simultaneous explanation of its result and of the atmospheric and solar neutrino deficits, it is well known that three different values of the squared mass difference are necessary. Then four light neutrinos, including a sterile neutrino ($\nu_s$), are required. Various models of the sterile neutrino can be found in Refs. 10) – 14). Following the recent Super-Kamiokande analysis of the solar neutrino, the explanation of the solar neutrino problem based on the $\nu_e-\nu_s$ oscillation seems to be disfavored. This suggests that the $(3+1)$-neutrino spectrum might be a more favored scenario for the neutrino mass hierarchy than the $(2+2)$-scheme.

In this paper we consider a neutrino mass matrix in a $3(\nu_L+\nu_R)$ framework using the seesaw mechanism. However, in contrast to the ordinary seesaw models, our model contains a light right-handed neutrino as a result of the special texture.
of a right-handed Majorana neutrino mass matrix.\(^*\) Although there are other similar works along this direction, in most of them it is necessary to introduce the Majorana masses for the left-handed neutrinos in order to simultaneously obtain the required values of the mass eigenvalues and the flavor mixing angles, as can be found, for example, in Refs. 13) and 14). This implies that the introduction of a new triplet Higgs field might be necessary. In the present model we only need the Dirac neutrino masses and the right-handed Majorana neutrino masses if we assume a special but simple texture in both of them at tree level. The model seems to have fewer parameters than the previous models.

One of the interesting points of our model is that the large mixing angle MSW solution for the solar neutrino problem can be consistently accommodated in the same way as other solutions.\(^{17,18}\) It might also be possible to explain the LSND result if we use an appropriate solution for the solar neutrino problem.\(^{18}\) Moreover, it is interesting that this scenario for the neutrino mass matrix can also be embedded into the GUT scheme by introducing a suitable flavor symmetry. Such an example in the SU(5) model is constructed in this paper by fixing the charge assignment of quarks and leptons for that symmetry.

The organization of this paper is as follows. In §2 we define our model and discuss its various phenomenological features in the case that the charged lepton mass matrix is diagonal. In §3 we consider the embedding of the scenario into the SU(5) GUT scheme. We discuss the realization of the required form of the mass matrix on the basis of the Froggatt-Nielsen mechanism. The flavor structure in the quark sector is also discussed here. Section 4 is devoted to a summary.

§2. A model of the neutrino mass matrix

We consider a model defined by the following neutrino mass terms, which are different from those of the usual seesaw model in the 3(\(\nu_L+\nu_R\)) framework:

\[
-\mathcal{L}_{\text{mass}} = \sum_{\alpha} \sum_{p=2,3} m_{p\alpha} N_p \nu_\alpha + \sum_{p=2,3} m_{p1} N_p N_1 + \frac{1}{2} \sum_{p=2,3} M_p N_p N_p + \text{h.c.}, \tag{2.1}
\]

where \(\nu_\alpha\) is an active neutrino (\(\alpha = e, \mu, \tau\)) and \(N_P\) (\(P = 1\,\text{--}\,3\)) is a charge conjugated state of the right-handed neutrino. We make the following assumption for the mass parameters in Eq. (2.1):

\[
m_{2e} = m_{2\mu} = m_{2\tau} \equiv \hat{\eta}, \quad m_{3e} \equiv \bar{\eta}_1, \quad m_{3\mu} = m_{3\tau} \equiv \bar{\eta}_2,
\]

\[
\hat{\eta} \sim \bar{\eta}_1 \sim \bar{\eta}_2 < m_{21} \sim m_{31} \ll M_2 \sim M_3, \tag{2.2}
\]

\(^*\) There is a scenario known as the singular seesaw mechanism\(^{16}\) in which a similar right-handed Majorana neutrino mass matrix is used. However, the features of that mass matrix seem to be very different from those of the present one. In such a scenario, its determinant is zero, and for this reason the (2+2)-scheme tends to be obtained. Moreover, the mass scale of the Dirac masses is \(O(1)\) eV. On the other hand, in our model the mass matrix with zero determinant is the Dirac mass matrix, not the right-handed Majorana mass matrix. As a result, the (3+1)-scheme can be obtained, and the Dirac mass scale can be \(O(1)\) GeV, as in the ordinary seesaw case.
where the mass parameters should be understood as their absolute values, although this is not expressed explicitly. A crucial difference from the usual seesaw model is that one of the right-handed neutrinos is assumed to be very light and also to have very small mixings with other heavy right-handed neutrinos. We assume $M_{23} = 0$ in the Majorana mass matrix of $N_p$ here, for simplicity. (The following arguments are not changed significantly if we introduce a non-zero $M_{23}$.) Under this assumption, we can integrate out heavy right-handed neutrinos $N_p$ and get the following 4 x 4 matrix as a result of the seesaw mechanism:

$$m_{\nu} = \begin{pmatrix} A & B & B & D \\ B & C & C & E \\ B & C & C & E \\ D & E & E & F \end{pmatrix}.$$  \hspace{1cm} (2.3)

The matrix elements $A - F$ are expressed in terms of the model parameters in (2.2) as

$$A = \frac{\hat{\eta}^2}{M_2} + \frac{\hat{\eta}_1^2}{M_3}, \quad B = \frac{\hat{\eta}^2}{M_2} + \frac{\hat{\eta}_1 \eta_2}{M_3}, \quad C = \frac{\hat{\eta}^2}{M_2} + \frac{\eta_2^2}{M_3},$$

$$D = \frac{\hat{m}_{21}}{M_2} + \frac{\hat{m}_{31}}{M_3}, \quad E = \frac{\hat{m}_{21}^*}{M_2} + \frac{\hat{m}_{31}^*}{M_3}, \quad F = \frac{m_{21}^2}{M_2} + \frac{m_{31}^2}{M_3}. \hspace{1cm} (2.4)$$

If we define the diagonalization matrix $U$ of the matrix (2.3) as $m_{\nu}^{\text{diag}} = U^T m_{\nu} U$, we find that $U$ can be written as

$$U = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & -\sin \theta \sin \delta + \cos \theta \sin \gamma \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ -\frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ -\sin \gamma & -\sin \delta & 0 & 1 \end{pmatrix}, \hspace{1cm} (2.5)$$

where $|\sin \gamma|, |\sin \delta| \ll 1$ is assumed, and mixing angles are defined by

$$\tan 2\theta = \frac{2\sqrt{2}B}{A - 2C}, \quad \sin \gamma \simeq \frac{D \cos \theta + \sqrt{2}E \sin \theta}{F}, \quad \sin \delta \simeq -\frac{D \sin \theta + \sqrt{2}E \cos \theta}{F}. \hspace{1cm} (2.6)$$

The mass eigenvalues of $m_{\nu}$ are expressed as

$$m_1 \simeq A \cos^2 \theta + \sqrt{2}B \sin 2\theta + 2C \sin^2 \theta,$$

$$m_2 \simeq A \sin^2 \theta - \sqrt{2}B \sin 2\theta + 2C \cos^2 \theta,$$

$$m_3 = 0, \quad m_4 = F. \hspace{1cm} (2.7)$$

\footnote{It should be noted that the number of light sterile neutrinos is restricted at most to one in the present scenario. We obtain a 3 x 3 matrix if all of $N_p$ are heavy. Even in such a case, as long as we assume a proportional relation between $(m_{1\alpha})$ and $(m_{2\alpha})$ as vectors whose components are labeled by $\alpha$, the texture for the active neutrinos is the same as that in Eq. (2.3). Then this texture can be applied to the explanation of the solar and atmospheric neutrino problems in the same way as in the following discussion. This texture is essentially the same as that discussed in Ref. 19), although it is derived in a different context.}
where we neglect the contribution from the fourth row and column of $m_\nu$ to $m_1$ and $m_2$, taking account of facts such as $A \gtrsim \frac{D_1^2}{F}$, $B \gtrsim \frac{D_2E}{F}$ and $C \gtrsim \frac{E_2}{F}$. Here we should note that in this model the violation of the proportional relation between $(m_{2a})$ and $(m_{3a})$ as vectors is crucial to restrict the number of zero mass eigenvalues to one and to control the mixing structure. There is a degree of freedom in the choice of the two elements of $(m_{3a})$ which are taken to be equal in (2.2). As long as we consider the case in which the charged lepton mass matrix is diagonal, this is not important. But when we consider a different situation, it might become crucial for the consideration of oscillation phenomena. If the charged lepton mass matrix is diagonal, the above mixing matrix $U$ is just the flavor mixing matrix $V^{(MNS)}$ that controls the neutrino oscillation. We assume this in the charged lepton sector, and we also assume no $CP$ violation in the lepton sector. At this stage, we cannot determine to which flavor each $\nu_\alpha$ corresponds, so that we use the Roman numerals for the subscript $\alpha$ temporarily. Next, we study the features of the oscillation phenomena in the model in order to fix the neutrino flavor.

It is well known that the transition probability due to the neutrino oscillation $\nu_\alpha \rightarrow \nu_\beta$ after the flight length $L$, using the matrix elements of (2.5), is

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right), \quad (2.8)$$

where $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$, and the weak eigenstate $\nu_\alpha$ is related to the mass eigenstate $\tilde{\nu}_i$ by $\nu_\alpha = U_{\alpha i} \tilde{\nu}_i$ in the basis that the charged lepton mass matrix is diagonal. In Table I we list the contribution to each neutrino transition mode $(\alpha, \beta)$ from a sector $(i, j)$ of the mass eigenstates. As a phenomenologically interesting case, we consider the situation that the mass eigenstates $\tilde{\nu}_1$ and $\tilde{\nu}_2$ are almost degenerate and the hierarchy ($m_3 \ll m_1 \sim m_2 \ll m_4$) among the mass eigenvalues holds. This corresponds to a well-known reversed hierarchy scenario for the atmospheric and solar neutrino problems in the $(3+1)$-neutrino spectrum.\(^{20}\)

The absolute value of each mass eigenvalue is smaller than that in the ordinarily considered scenario because $m_3 = 0$. In this case, no neutrino could be considered as a possible form of hot dark matter. If we apply this mass hierarchy to explain the atmospheric and solar neutrino data, the squared mass difference should be taken as\(^1, 2\)

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{13}^2 \simeq \Delta m_{23}^2 \lesssim 6 \times 10^{-3} \text{ eV}^2, \quad (2.9)$$

$$10^{-10} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 1.5 \times 10^{-4} \text{ eV}^2. \quad (2.10)$$

Table I. The contributions to each neutrino transition process $\nu_\alpha \rightarrow \nu_\beta$ from each sector $(i, j)$ of the mass eigenstates.

$$\begin{array}{ccc}
(\alpha, \beta) & (i, j) & -4U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j}(= A) \\
(I, II) & (1, 2) & \frac{1}{7} \sin^2 2\theta \quad (A) \\
(I, III) & (1, 2) & \frac{1}{7} \sin^2 2\theta \quad (B) \\
(II, III) & (1, 3) & \sin^2 \theta \quad (C) \\
 & (2, 3) & \cos^2 \theta \quad (D) \\
 & (1, 2) & -\frac{1}{4} \sin^2 2\theta \quad (E) \\
\end{array}$$

\(^{20}\) The absolute value of each mass eigenvalue is smaller than that in the ordinarily considered scenario because $m_3 = 0$. In this case, no neutrino could be considered as a possible form of hot dark matter. If we apply this mass hierarchy to explain the atmospheric and solar neutrino data, the squared mass difference should be taken as $^{1, 2}$

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{13}^2 \simeq \Delta m_{23}^2 \lesssim 6 \times 10^{-3} \text{ eV}^2, \quad (2.9)$$

$$10^{-10} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 1.5 \times 10^{-4} \text{ eV}^2. \quad (2.10)$$
A suitable value of $\Delta m_{12}^2$ should be chosen within the above range, depending on which solution is adopted for the solar neutrino problem.

By inspecting Table I, we find that the simultaneous explanation of the deficits of both the atmospheric neutrino and the solar neutrino is possible if we identify the weak eigenstates of neutrinos ($e, \mu, \tau$) with (I, II, III). Under this identification, a 3x3 submatrix of (2.5) is recognized as the correctly arranged MNS mixing matrix. If we note that $m_3 = 0$ and $\Delta m_{33}^2 \simeq \Delta m_{23}^2$ are satisfied, we find that the atmospheric neutrino can be understood in terms of $\nu_\mu \rightarrow \nu_\tau$, obtained as the combination of (C) and (D) in Table I. This description is independent of the value of $\sin \theta$. On the other hand, it is believed that solar neutrinos can be described by $\nu_e \rightarrow \nu_\mu$ (A) and also $\nu_e \rightarrow \nu_\tau$ (B). In both processes, the amplitude $A(= -4 \sum U_{\alpha i} U_{\beta i} |U_{\alpha j} U_{\beta j}|)$ is $\frac{1}{2} \sin^2 2\theta$. Thus if $\sin^2 2 \theta \sim 10^{-2}$, the small mixing angle MSW solution (SMA) is realized.\(^{18}\) In the case of $\sin^2 2 \theta \sim 1$, it can yield a large mixing angle MSW solution (LMA), a low mass MSW solution (LOW), and a vacuum oscillation solution (VO),\(^{18}\) depending on the value of $\Delta m_{12}^2$. The CHOOZ experiment\(^{21}\) constrains a component $U_{e3}$ of the MNS mixing matrix.\(^{22}\) This results from the fact that the amplitude $A$ of the contribution to $\nu_e \rightarrow \nu_x$ with the squared mass differences $\Delta m_{13}^2$ or $\Delta m_{23}^2$ always contains $U_{e3}$. The model is free from this constraint since $U_{e3} = 0$ is satisfied independently of the value of $\sin \theta$.

In order to see the viability of the scenario in a more quantitative way it is useful to estimate numerically what kind of tuning of the primary parameters in (2.1) and (2.2) is required to realize a suitable value for the oscillation parameters. For convenience we introduce the following parameterization for the three light states:

$$
\frac{\hat{\eta}_1}{\sqrt{M_2}} \equiv \mu^{\frac{1}{2}}, \quad \frac{\hat{\eta}_1}{\sqrt{M_3}} \equiv \epsilon_1 \mu^{\frac{1}{2}}, \quad \frac{\hat{\eta}_2}{\sqrt{M_3}} \equiv \epsilon_2 \mu^{\frac{1}{2}}.
$$

Also, for simplicity, we assume $M_2 = M_3$. Then, the overall mass scale is determined by $\mu$, and the hierarchy among the mass eigenvalues is controlled by $\epsilon_1$ and $\epsilon_2$. When $\epsilon_1 = \sqrt{2}$ and $\epsilon_2 = -\frac{1}{\sqrt{2}}$, the two mass eigenvalues $m_1$ and $m_2$ are equal. Using this fact, we can estimate a typical scale of $\mu$ from the condition (2.9) as $\mu \simeq 1.8 \times 10^{-2}$ eV. This value corresponds to $M_2 \sim 5.6 \times 10^{10}$ GeV for $\hat{\eta} \sim 1$ GeV. The deviation from these values of $\epsilon_1$ and $\epsilon_2$ determines the difference between $m_1$ and $m_2$ and also the value of $\sin \theta$. In Fig. 1, we give a scatter plot of the possible solutions for both the atmospheric and solar neutrino problems in the $(\epsilon_1, \epsilon_2)$ plane. In this figure solutions for both signs of $\sin 2\theta$ are contained. Since we consider the reversed hierarchy here, both possibilities are allowed. In particular, in the SMA case we should note that $|m_1| < |m_2|$ has to be satisfied in order for the matter effect to exist. From the figure we find that the SMA, the LOW and the VO indicate a finer tuning of the parameters than the LMA to realize the required values of the squared mass difference and $\sin^2 2\theta$.

In the present model we have a light sterile neutrino. Therefore it may be possible to explain the LSND result, if $m_4 \sim O(1)$ eV is satisfied. We can check whether all of mass eigenvalues and various mixing angles quoted above can be consistent with the LSND explanation by using the required relations (2.4), (2.6) and (2.7). If we take $\epsilon_1 \sim 1.41$ and $\epsilon_2 \sim -0.71$ as a typical example in Fig. 1 and also assume
$M_2 = M_3$ and $m_{21} = m_{31}$, we obtain

$$\bar{\eta}_1 \sim 1.41 \hat{\eta}, \quad \bar{\eta}_2 \sim -0.71 \hat{\eta}, \quad \sin \gamma \sim 1.21 \frac{\hat{\eta}}{m_{31}}, \quad \sin \delta \sim 0.21 \frac{\hat{\eta}}{m_{31}}.$$  \hspace{1cm} (2.12)

In order to see the feature related to the LSND, we note that the relevant amplitude $A_{\text{LSND}}$ can be written by using the unitarity of $V^{(\text{MNS})}$ and the relation $|m_{1,2,3}| \ll |m_4|$ as

$$A_{\text{LSND}} = 4(V^{(\text{MNS})}_{e4})^2 (V^{(\text{MNS})}_{\mu 4})^2.$$ \hspace{1cm} (2.13)

Then, the amplitude can be written by using Eq. (2.5) as

$$A_{\text{LSND}} \simeq 2(\cos \theta \sin \gamma - \sin \theta \sin \delta)^2 (\cos \theta \sin \delta + \sin \theta \sin \gamma)^2.$$ \hspace{1cm} (2.14)

The LSND data require this to be near $1.2 \times 10^{-3}$ for $\Delta m^2_{\text{LSND}} \sim 1$ eV$^2$. Here we should recall that $|\sin \gamma|, |\sin \delta| \ll 1$ should be satisfied under our assumption (2.2). If we use the large mixing angle solutions for the solar neutrino problem, we obtain $A_{\text{LSND}} \sim \frac{1}{2}(\sin^2 \gamma - \sin^2 \delta)^2$. Here we require $A_{\text{LSND}}$ to take the above value, we find $m_{31} \sim 5.4\hat{\eta}$, and then $\sin \gamma \sim 0.23$ and $\sin \delta \sim 0.04$ by using Eq. (2.12). Moreover, $m_4$ can take a suitable value for the LSND result such as $m_4 \sim \frac{2m^2_{31}}{M_2} \sim 1$ eV.

On the other hand, if we adopt the SMA solution and then $\cos \theta \sim 1$, we have $A_{\text{LSND}} \sim 2 \sin^2 \gamma \sin^2 \delta$. Then if we require $A_{\text{LSND}}$ to take a suitable value, we find $m_{31} \sim 3.2\hat{\eta}$ and $m_4 \sim 0.4$ eV, which is too small for the explanation of the LSND data. Taking account of these analyses, we find that the inclusion of the LSND result restricts our model to large mixing angle solutions with respect to the solution for the solar neutrino problem. This feature of the model might be favorable if we take seriously the recent Super-Kamiokande analysis of the solar neutrino.  

Fig. 1. The scatter plot of possible solutions for the atmospheric and solar neutrino problems in the $(\epsilon_1, \epsilon_2)$ plane.
However, even in this case we should comment on the influence on the big-bang nucleosynthesis (BBN) of the oscillation processes $\nu_{e,\mu,\tau} \to \nu_s$ in the early universe. The BBN predicts that the effective neutrino species during the primordial nucleosynthesis should be less than 3.6. This fact severely constrains the mixing angle $\gamma, \delta$ and the squared mass difference $\Delta m^2$ between a sterile neutrino ($\nu_s$) and left-handed active neutrinos which mix with it.\(^{23}\) The restriction on the number of the effective neutrino species during the primordial nucleosynthesis gives a condition on the $\nu_{s2}-\nu_{e,\mu,\tau}$ sector. Here, it is sufficient to consider the most stringent condition. When $m_{\nu_{s2}}^2 > m_{\nu_{e,\mu,\tau}}^2$ is satisfied, this can be formulated as\(^{23}\)

$$\Delta m^2 \sin^4 2\theta \lesssim 3 \times 10^{-6} \text{ eV}^2 \text{ for } (\nu_{\mu,\tau}, \nu_s). \quad (2.15)$$

In the present model the relevant $\Delta m^2$ is $m_4^2 \sim O(1)$ eV$^2$. The values of $\sin \gamma$ and $\sin \delta$ required to account for the LSND data induce these processes at a large rate. In fact, if we estimate the value corresponding to the left-hand side of Eq. (2.15) by using these values, we obtain $O(10^{-3})$ eV$^2$. Thus, the known BBN bound on $\nu_{\mu,\tau} \to \nu_s$ cannot be satisfied unless we assume something new, for example, the presence of a large lepton number asymmetry at the BBN epoch.\(^{24}\) We might need more information regarding this point to consider the model with light sterile neutrinos as a realistic possibility.

In Table I the only remaining contribution (E) to $\nu_{\mu} \to \nu_{\tau}$ does not provide any effect in the short-baseline experiment, even in the case $\sin^2 2\theta \simeq 1$, since $\Delta m^2_{12}$ is too small. However, this mode may be relevant to the long-baseline experiment in the case of $\Delta m^2_{12} \sim 10^{-4}$ eV$^2$, which corresponds to the LMA solution of the solar neutrino deficit. We show the effect of the mode (E) on the $P(\nu_{\mu} \to \nu_x)$ in Fig. 2. The dashed line comes from the modes (C) and (D), which correspond to the ordinary two flavor oscillation $\nu_{\mu} \to \nu_{\tau}$. The thick solid line is that which is obtained by taking account of the contribution of (E). In the thin solid line, the contribution of (A) which corresponds to $\nu_x = \nu_e$ is also taken into account. This shows that it may be possible to discriminate our model from others in long-baseline experiments with the flight length $L \gtrsim 2000$ km. Moreover, the present model may be expected to have another experimental signature in the neutrinoless double $\beta$-decay.\(^{25}\) Using Eq. (2.5), the effective mass parameter which appears in the formula of the rate of neutrinoless double $\beta$-decay can be estimated as

$$|m_{ee}| \equiv \sum_j |U_{ej}|^2 e^{i\phi_j} m_j = (m_1 \cos^2 \theta + m_2 \sin^2 \theta) \sim m_1, \quad (2.16)$$

because of the fine degeneracy between $m_1$ and $m_2$. Thus $|m_{ee}|$ takes a value in the range of $0.04 - 0.08$ eV that is independent of the value of $\sin \theta$ and then independent of the solution of the solar neutrino problem. This value seems to be within the reach in the near future experiments.

Before closing this section we make some short comments. In the above analysis, we assumed the reversed mass hierarchy in the (3+1) scheme. As another hierarchy among the mass eigenvalues, we can consider the usual one, that is, $m_3 \lesssim m_2 \ll$
Fig. 2. The transition probability $P(\nu_\mu \rightarrow \nu_\tau(\neq \mu))$ as a function of the flight length $L$ km. We assume $E = 1$ GeV, $\Delta m_{13}^2 = 3.5 \times 10^{-3}$ eV$^2$ and $\Delta m_{12}^2 = 10^{-4}$ eV$^2$.

It is found that such a hierarchy cannot be consistent with the experimental data in the present scenario if we note that $\nu_e$ cannot be identified with the state I to explain the solar neutrino deficit. We should also note that the above result depends crucially on the assumption regarding the charged lepton sector, for which we assumed the mass matrix to be diagonal. As long as the flavor mixing in the charged lepton sector is small, the above result seems to be applicable. However, if it is large, the result can be greatly changed. When we consider the GUT, such situations exist. In the next section we study this issue.

§3. Embedding into $SU(5)$

We consider an embedding of our neutrino model into the supersymmetric $SU(5)$ GUT. In this case, the charged lepton mass matrix can be related to the neutrino mass matrix through a group theoretical constraint. Thus we cannot assume a small flavor mixing in the quark sector independently of the lepton sector. The result obtained in the previous section may be modified if we embed our scenario into the GUT scheme. In order to control the flavor mixing structure, we adopt the Froggatt-Nielsen mechanism and introduce Abelian flavor symmetries $U(1)^F_1 \times U(1)^F_2$. The symmetries are assumed to be broken by small parameters $\lambda$ and $\epsilon$, so that Yukawa couplings inducing the fermion masses are suppressed by both powers of $\lambda$ and $\epsilon$. As a result, the fermion mass hierarchy is produced. We consider a model discussed in Ref. 5) as such a typical example and modify it to embed our scenario for the neutrino mass into it.

In the $SU(5)$ GUT, quarks and leptons are embedded into the representations of $SU(5)$ as

$$
\begin{aligned}
10 &\ni (q, u^c, e^c), \\
5^* &\ni (d^c, \ell), \\
1 &\ni \nu^c.
\end{aligned}
$$

We assign the charge of $U(1)^F_1 \times U(1)^F_2$ to each representation as $5)$

$$
\begin{aligned}
10 &: (3, 2, 0), & (0, 0, 0), \\
5^* &: (1, 0, 0), & (0, 0, 0), \\
1 &: (0, 0, 0), & (\alpha, \beta, \beta),
\end{aligned}
$$
where the numbers in the parentheses represent the charges given to each generation, and \( c, \alpha \) and \( \beta \) are non-negative integers. The ordinary doublet Higgs fields \( H_1 \) and \( H_2 \) in the minimal supersymmetric standard model are assumed to have no charge of \( U(1)_{F_1} \times U(1)_{F_2} \). In addition to these fields we introduce \( SU(5) \) singlet fields \( S_1 \) and \( S_2 \) which have charges \((-1,0)\) and \((0,-1)\) for the flavor symmetries, respectively.\(^*\)

The symmetries control the flavor mixing structure by regulating the number of fields \( S_1 \) and \( S_2 \) contained in each non-renormalizable term. If the singlet fields \( S_1 \) and \( S_2 \) have vacuum expectation values \( \langle S_1 \rangle \) and \( \langle S_2 \rangle \), the above-mentioned suppression factors for the Yukawa couplings can be realized as the power of \( \lambda = \frac{\langle S_1 \rangle}{M_{pl}} \) and \( \epsilon = \frac{\langle S_2 \rangle}{M_{pl}} \). Here, \( M_{pl} \) is the Planck scale.

Using the Abelian flavor charges introduced above, we can obtain the quark and lepton mass matrices in the forms

\[
M_u \sim \begin{pmatrix}
\lambda^6 & \lambda^5 & \lambda^3 \\
\lambda^5 & \lambda^4 & \lambda^2 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \langle H_2 \rangle, \quad M_d \sim \begin{pmatrix}
\lambda^{3+c} & \lambda^{2+c} & \lambda^c \\
\lambda^3 & \lambda^2 & 1 \\
\lambda^3 & \lambda^2 & 1
\end{pmatrix} \langle H_1 \rangle,
\]

\[
M_\nu \sim \begin{pmatrix}
\lambda^{c+\alpha} & \lambda^c & \lambda^a \\
\lambda^c & \lambda^\beta & \lambda^\beta \\
\lambda^c & \lambda^\beta & \lambda^\beta
\end{pmatrix} \langle H_2 \rangle, \quad M_e \sim \begin{pmatrix}
\lambda^{3+c} & \lambda^3 & \lambda^3 \\
\lambda^{2+c} & \lambda^2 & \lambda^2 \\
\lambda^c & 1 & 1
\end{pmatrix} \langle H_1 \rangle,
\]

\[
M_R \sim \begin{pmatrix}
\epsilon^{2\alpha} & \epsilon^{\alpha+\beta} & \epsilon^{\alpha+\beta} \\
\epsilon^{\alpha+\beta} & \epsilon^{2\beta} & \epsilon^{2\beta} \\
\epsilon^{\alpha+\beta} & \epsilon^{2\beta} & \epsilon^{2\beta}
\end{pmatrix} M. \quad (3.3)
\]

Here, \( M \) is the mass scale relevant to the origin of the right-handed Majorana neutrino mass. Dirac mass matrices are written in the basis of \( \psi_R m_D \bar{\psi}_L \). We do not consider the \( CP \) phases here. (In the mass matrices (3.3) we omit order 1 coupling constants. This is the reason we use “\( \sim \)” here.) We should note that \( M_\nu \) and \( M_R \) in (3.3) can have a texture similar to that defined by Eqs. (2.1) and (2.2), up to implicit coefficients of order 1, as long as \( \alpha \gg \beta \) is satisfied.\(^*\)* At least in the case \( c = 0 \) and 1, which is assumed in the following discussion, we can make \( M_\nu \) satisfy the condition (2.2) by tuning the order 1 coefficients. Thus we can have a mass matrix similar to that in Eq. (2.3) as a result of the seesaw mechanism, although there is a non-zero element \( M_{23} \) in \( M_R \), unlike in that one defined by Eq. (2.1). Its diagonalization matrix \( U \) can be considered to have a form similar to that of Eq. (2.5). Their difference results only from the definition of the matrix elements \( A - F \), as we see below.

In the quark sector, the mass eigenvalues and the CKM matrix elements can be found after some inspection as

\[
m_u : m_c : m_t = \lambda^6 : \lambda^4 : 1, \quad m_d : m_s : m_b = \lambda^{3+c} : \lambda^2 : 1, \quad (3.4)
\]

\(^*\) We may need to introduce some fields to cancel the chiral anomaly of the flavor symmetry if it is a non-anomalous gauge symmetry. However, we do not go further into this issue in the present paper.

\(^*\) In this context, it may be allowable to consider the coefficients assumed here to be order 1 as being near \( (\sqrt{c}, \frac{1}{\sqrt{c}}) \) if \( \epsilon < \lambda \) for \( M_\nu \).
In the charged lepton sector, we can determine the mass eigenvalues by noting \( SU(5) \) relations such as \( M_e^T = M_d \). The ratio of the mass eigenvalues is the same as that for the down quark sector, and hence

\[
m_e : m_\mu : m_\tau = \lambda^{3+c} : \lambda^2 : 1.
\]

This result has some different features from those presented in Ref. 5) in the down quark and charged lepton sectors. These result from the charge assignment for which is needed to realize the Dirac neutrino masses defined by Eqs. (2.1) and (2.2). If we assume \( \lambda \sim 0.22 \), these results seem to describe the experimental data in a qualitatively favorable way, except for \( m_e \) and \( m_u \), which are predicted to be too large, in particular, in the case of \( c = 0 \). This is a common problem known to exist in schemes based on the Abelian flavor symmetry and charge assignments similar to that given in (3.2). We cannot overcome this problem without something new.

We define the diagonalization matrix \( \hat{U} \) of the charged lepton mass matrix in a basis that \( \hat{U}^T M_e \hat{M}_e \hat{U} \) is diagonal. Then \( \hat{U} \) can be approximately written as

\[
c = 0 : \quad \hat{U} = \begin{pmatrix}
\frac{1}{\sqrt{2}} \cos \xi - \frac{1}{\sqrt{6}} \sin \xi & \frac{1}{\sqrt{3}} \sin \xi + \frac{1}{\sqrt{6}} \cos \xi & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{2}} \cos \xi - \frac{1}{\sqrt{6}} \sin \xi & -\frac{1}{\sqrt{3}} \sin \xi + \frac{1}{\sqrt{6}} \cos \xi & 0 & \frac{1}{\sqrt{3}} \\
\frac{2}{\sqrt{6}} \sin \xi & -\frac{2}{\sqrt{6}} \cos \xi & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
\]

\[
c = 1 : \quad \hat{U} = \begin{pmatrix}
\cos \xi & 0 & \sin \xi & 0 \\
-\frac{1}{\sqrt{2}} \sin \xi & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \xi & 0 \\
-\frac{1}{\sqrt{2}} \sin \xi & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \xi & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.
\]

The hierarchical structure (3.6) of the mass eigenvalues requires the mixing angle \( \xi \) to satisfy \( \sin \xi \sim O(\lambda) \). In the neutrino sector, we need to determine the finer structure of the Dirac neutrino mass matrix to be suitable for the charged lepton mass matrix given in (3.3) for the purpose of accounting for various data for the neutrino oscillations. With this in mind, we should recall that there is a degree of freedom in the choice of two elements of the Dirac neutrino masses \( m_{3\alpha} \), which are taken to be equal by tuning the order 1 coefficients. After some investigation, we find that it seems to be favorable to take \( m_{3e} = m_{3\mu} = \bar{\eta}_1, \quad m_{3\tau} = \bar{\eta}_2 \), instead of the assignment given in Eq. (2.2). Under this assumption, the mass matrix of the light neutrinos can be written as

\[
m_\nu = \begin{pmatrix}
A & A & B & D \\
A & A & B & D \\
B & B & C & E \\
D & D & E & F
\end{pmatrix},
\]

\(^{*}\) This implies that \( \bar{\eta} \sim \bar{\eta}_1 \sim \lambda^c e^3 \sim e^3 \) and \( \bar{\eta}_2 \sim e^3 \). The difference among these results from the order 1 coefficients.
and the matrix elements $A - F$ are defined by

$$
A = \frac{\eta^2}{M_3} + \frac{\eta_1^2}{M_2} - 2\frac{\eta_1 \eta 1_2}{M_{23}}, \quad B = \frac{\eta^2}{M_3} + \frac{\eta_1 \eta 2}{M_2} - \frac{\eta(\eta_1 + \eta 2)}{M_{23}},
$$

$$
C = \frac{\eta^2}{M_3} + \frac{\eta_2^2}{M_2} - 2\frac{\eta_1 \eta 2}{M_{23}}, \quad D = \frac{\eta m_{21}}{M_3} + \frac{\eta_1 m_{31}}{M_2} - \frac{\eta m_{31} + \eta_1 m_{21}}{M_{23}},
$$

$$
E = \frac{\eta m_{21}}{M_3} + \frac{\eta_2 m_{31}}{M_2} - \frac{\eta m_{31} + \eta_2 m_{21}}{M_{23}}, \quad F = \frac{m_{21}^2}{M_3} + \frac{m_{31}^2}{M_2} - 2\frac{m_{21} m_{31}}{M_{23}},
$$

where $M_a^{-1} = M_a/(M_2 M_3 - M_{23}^2)$ and we partially use the notation in Eq. (2.5) because of the change in the choice of $\eta_1$ and $\eta_2$. Using the modified $U$ and Eqs. (3.7) and (3.8), the MNS matrix of the lepton mixing defined by $V^{(\text{MNS})} = \hat{U}^T U$ is calculated for both values of $c$ as

$$
V^{(\text{MNS})}_{c=0} \simeq \begin{pmatrix}
-\frac{f^{(1)}}{\sqrt{3}} \sin \xi & -\cos \xi & \frac{f^{(2)}}{\sqrt{3}} \sin \xi & a_1 \\
\frac{f^{(1)}}{\sqrt{3}} \cos \xi & -\sin \xi & \frac{f^{(2)}}{\sqrt{3}} \cos \xi & a_2 \\
\frac{f^{(1)}}{\sqrt{3}} & 0 & \frac{f^{(2)}}{\sqrt{3}} & a_3 \\
-\sin \gamma & 0 & -\sin \delta & 1
\end{pmatrix}, \quad (3.12)
$$

$$
V^{(\text{MNS})}_{c=1} \simeq \begin{pmatrix}
\frac{\cos \theta}{\sqrt{2}} \cos \xi + \frac{f^{(1)}}{2} \sin \xi & -\frac{1}{\sqrt{2}} \cos \xi & -\frac{\sin \xi}{\sqrt{2}} & \frac{f^{(2)}}{2} \sin \xi & a_1 \\
\frac{f^{(1)}}{2} \sin \xi & \frac{1}{2} & \frac{f^{(2)}}{2} & a_2 \\
\frac{\cos \theta}{\sqrt{2}} \sin \gamma + \frac{f^{(1)}}{2} \cos \xi & \frac{1}{2} \cos \xi & -\frac{\sin \xi}{\sqrt{2}} & \frac{f^{(2)}}{2} \cos \xi & a_3 \\
-\sin \gamma & 0 & -\sin \delta & 1
\end{pmatrix}, \quad (3.13)
$$

where we use definitions

$$
a_i = v_{i1} \sin \gamma + v_{i3} \sin \delta, \quad f^{(1)}_{\pm} = \cos \theta \pm \sqrt{2} \sin \theta, \quad f^{(2)}_{\pm} = \sqrt{2} \cos \theta \pm \sin \theta,
$$

and $v_{ij}$ represents the $ij$-element of the corresponding $V^{(\text{MNS})}$. To derive these expressions we use $|\sin \gamma|, |\sin \delta| \ll 1$ and ignore higher-order terms. The mixing angles $\theta, \gamma$ and $\delta$ in this case are defined as

$$
\tan 2\theta = \frac{2\sqrt{2} B}{2A - C}, \quad \sin \gamma \simeq \frac{\sqrt{2} D \cos \theta + E \sin \theta}{F}, \quad \sin \delta \simeq -\frac{\sqrt{2} D \sin \theta + E \cos \theta}{F}.
$$

(3.15)

Now we study the oscillation phenomena in both cases in more detail. First we consider the case of $c = 0$. Taking account of the fact that $m_2 = 0$ and $V_{\tau_2}^{(\text{MNS})} = 0$, the reversed hierarchy scenario cannot be adopted from the viewpoint
of the atmospheric neutrino problem. We must assume the normal hierarchy ($|m_2| \lesssim |m_1| \ll |m_3| \ll |m_4|$) in the $(3+1)$-scheme in order to realize $\Delta m_{12}^2 \simeq \Delta m_{\text{solar}}^2$ and $\Delta m_{23}^2 \simeq \Delta m_{13}^2 \simeq \Delta m_{\text{atm}}^2$. Using Eq. (3.12), we find that the amplitude for $\nu_\mu \to \nu_\tau$ is

$$A = -4V_{\mu 1}^{(\text{MNS})}V_{\tau 1}^{(\text{MNS})}V_{\mu 3}^{(\text{MNS})}V_{\tau 3}^{(\text{MNS})}$$

$$= 4 \left( \sqrt{2} - \frac{1}{2} \tan 2\theta \right)^2 \cos^2 2\theta \cos^2 \xi.$$  \hspace{1cm} (3.16)

After some investigation, we find that this suggests that $\tan^2 \theta \leq 0$ and $\cos^2 \theta \sim 1$ should be satisfied for the explanation of the atmospheric neutrino problem. We should also recall the fact that $\sin \xi \sim O(\lambda)$. Although a large value of $|\sin \theta|$ (such as 0.95) can satisfy the bound from the atmospheric neutrino, it seems to be disfavored by the solar neutrino data. Thus, we should try to explain the solar neutrino problem by the SMA due to $\nu_e \to \nu_\mu$ corresponding to $\Delta m_{12}^2$. The SMA solution requires $|m_1| < |m_2|$ to make the matter effect exist. However, this condition cannot be satisfied, because $m_2 = 0$. A consistent solution cannot be found in this case. The situation for the solar neutrino problem is completely different from that in the case considered in the previous section. Only the SMA solution is applicable in the present case, since both $V_{\tau 2}^{(\text{MNS})} = 0$ and $m_2 = 0$ make the contribution of $\nu_e \to \nu_\tau$ to solar neutrino deficit zero. Thus, although we must use $\nu_e \to \nu_\mu$, $m_2 = 0$ makes the matter effect non-existent. The situation that only the SMA solution is applicable originally comes from the non-diagonal structure of the charged lepton mixing matrix (3.7). Since the present neutrino mass matrix induces a large mixing between $\nu_e$ and $\nu_\mu$ by itself, we need a small mixing in the corresponding place of the charged lepton sector to realize the large mixing solution for the solar neutrino. However, this condition for the charged lepton sector is not satisfied in the present case.

Next, we treat the case $c = 1$. Also in this case, the reversed hierarchy cannot induce a sufficiently large amplitude $A$ for $\nu_\mu \to \nu_\tau$ using the modes with $\Delta m_{12}^2$ and $\Delta m_{23}^2$. We need adopt the normal mass hierarchy also in this case. The relevant amplitude for $\nu_\mu \to \nu_\tau$ is estimated as

$$A = \sum_{i=1,2} -4V_{\mu i}^{(\text{MNS})}V_{\tau i}^{(\text{MNS})}V_{\mu 3}^{(\text{MNS})}V_{\tau 3}^{(\text{MNS})}$$

$$= \left( \cos^2 \theta - \frac{\sin^2 \theta}{2} \right) \left( \frac{\cos^2 \theta}{2} - \sin^2 \theta + \frac{1}{2} \right),$$  \hspace{1cm} (3.17)

where we take account of $\sin \xi \ll 1$ in the estimation. If $\cos^2 \theta \sim 1$ is satisfied, the above amplitude can be suitable for the atmospheric neutrino problem. The contribution to the solar neutrino deficits comes from $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ with $\Delta m_{12}^2$. Their combined amplitude is almost equal to 1, and the large mixing angle solution is realized. The value of $\cos^2 \theta$ is fixed to generate a large mixing for the explanation of the atmospheric neutrino, and it also results in a large mixing between $\nu_e$ and $\nu_\mu$. On the other hand, in the charged lepton sector, $c = 1$ causes the mixing...
between $e$ and $\mu$ to be small, so that we can have a large mixing angle solution for the solar neutrino. Also in this case, we give the scatter plot of the LMA solutions in Fig. 3 by assuming $M_{23} = 0$ and $M_2 = M_3$ and using the parametrization (2.11). It seems difficult to realize the LOW and VO solutions, since the required value of $\Delta m^2_{12}$ should be much smaller than that in the LMA. Using Fig. 3 we can again find typical values of the primary parameters of the model. As an example, if we take $\epsilon_1 \sim 1.75$ and $\epsilon_2 \sim -0.75$, we can obtain

$$\tilde{\eta}_1 \sim 1.75 \hat{\eta}, \quad \tilde{\eta}_2 \sim -0.75 \hat{\eta}, \quad \tan 2\theta \sim -0.13, \quad \sin \gamma \sim 1.9 \frac{\hat{\eta}}{m_{31}}, \quad \sin \delta \sim 0.26 \frac{\hat{\eta}}{m_{31}}.$$  

These fix the MNS matrix in the present case as

$$V^{(\text{MNS})}_{e1} = \begin{pmatrix} 0.62 & -0.79 & -0.10 & 0.62 \sin \gamma - 0.10 \sin \delta \\ 0.55 & 0.50 & -0.67 & 0.55 \sin \gamma - 0.67 \sin \delta \\ 0.58 & 0.35 & 0.73 & 0.58 \sin \gamma + 0.73 \sin \delta \\ - \sin \gamma & 0 & - \sin \delta & 1 \end{pmatrix}. \quad (3.19)$$

We can see that the CHOOZ constraint on $V^{(\text{MNS})}_{e3}$ is satisfied in (3.19). In order to see the possibility to explain the LSND result, we stipulate that $\mathcal{A}_{\text{LSND}} \sim 1.2 \times 10^{-3}$. Then, using Eq. (2.13) and $\mu \sim 7.7 \times 10^{-3}$ eV, we obtain $m_{31} \sim 7.8 \hat{\eta}$ and $m_4 \sim 0.93$ eV, which seem to be in a suitable region for the LSND. We also have $\sin \gamma \sim 0.25$ and $\sin \delta \sim 0.03$, which are consistent with our assumption for $\sin \gamma$ and $\sin \delta$. These analyses show that this case can account for all neutrino oscillation data, including the LSND. It is also interesting that the effective mass $|m_{ee}|$ for the neutrinoless double $\beta$-decay can have rather large value because of the $m_4$ contribution. In fact,
using the above numerical values, it can be estimated as $|m_{ee}| \sim |U_{e4}|^2 m_4 \sim 0.02 \text{ eV}$, which seems to be a realistic value from the experimental viewpoint.

It is also useful to note that the above values of the primary parameters of the model are realized through the suitable charge assignment of $\alpha$ and $\beta$. In order to consider such an example, we take $\langle H_2 \rangle \sim 100 \text{ GeV}, M \sim 4 \times 10^{15} \text{ GeV}$ and $\epsilon \sim 10^{-2}$. Then, if we set $\alpha = 6$ and $\beta = 0$,\footnote{The more general condition is $\epsilon^\alpha = 10^{-12}$. Since $\epsilon$ can be taken as a very small value, the coefficients that are considered to be of order 1 can consistently take rather large values. On this point, see the footnote below Eq. (3-3), too.} we can check that all required quantities are realized in the suitable range discussed with regard to numerical computation above, up to the order 1 factors.

Finally, we should note that in our scheme it might not be necessary for the required equality among $(m_{2\alpha})$ and also among $(m_{3\alpha})$ in Eq. (2.2) to be satisfied exactly. The allowed deviation should be quantitatively investigated, since it is related to the estimation of the magnitude of the order 1 coefficients.

§4. Summary

We have proposed a scenario for neutrino masses and mixing based on the seesaw mechanism in the $3(\nu_L + \nu_R)$ framework. By assuming a special texture for the right-handed Majorana neutrino mass matrix and the Dirac mass matrix, we obtained a model with four light neutrino states, including a sterile neutrino. One active neutrino is massless, and the others can have masses that are capable of accounting for the atmospheric and solar neutrino deficits, and also the LSND result. We studied two different cases corresponding to a diagonal charged lepton mass matrix and a non-diagonal one, which is obtained by embedding of our neutrino mass matrix into the $SU(5)$ GUT scheme.

In the former case, the so-called reversed mass hierarchy scenario was adopted. Every known solution for the solar neutrino problem can be realized by tuning the Dirac mass matrix of the neutrinos. It is interesting that the large mixing angle MSW solution is most easily realized. Moreover, if we impose the explanation of the LSND on this case, the solution for the solar neutrino problem is restricted to those with the large mixing angle. The difference from the two-flavor oscillation may be observable in $\nu_\mu \rightarrow \nu_\tau$ using a long-baseline experiment with a flight length of more than 2000 km. The neutrinoless double $\beta$-decay might also be accessible if the experimental bound is improved to a level of $|m_{ee}| \sim 0.04 - 0.08 \text{ eV}$.

In the latter case, the Froggatt-Nielsen mechanism has been applied to control the flavor mixing. We introduce the Abelian flavor symmetries $U(1)_{F_1} \times U(1)_{F_2}$ whose factor groups are assumed to have a different breaking scale. The non-trivial charge assignment of $U(1)_{F_1}$ is used only for the 10 and 5* fields of $SU(5)$, and the right-handed neutrino 1 is assumed to have only the charge of $U(1)_{F_2}$. In this setting, we studied the features of the mass and the mixing in both quark and lepton sectors for the two types of the charge assignment. We found that these charge assignments could generate the mass eigenvalues and the flavor mixings for the quark
sector in a qualitatively satisfactory way. If we give a suitable flavor charge to the right-handed neutrinos, our neutrino scenario can also be embedded into the $SU(5)$ scheme consistently. Although the reversed mass hierarchy is disfavored in both charge assignments, the ordinary mass hierarchy presents a consistent explanation of all data of known neutrino oscillation observations in the one charge assignment. In this case, only the LMA solution is allowed for the solar neutrino problem and the LSND result can also be explained.

In this paper we assumed that the tuning of order 1 coefficients is always allowed. Although in our scenario a mild tuning of order 1 coefficients is crucial to the success of the model, we cannot say anything about its origin or justification at the present stage.

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References

8) For example, see the following articles and references therein:
16) For example, see the following articles: