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Discussion

Discussion: "A Numerical Study of Thermal Dispersion in Porous Media" and "Numerical Determination of Thermal Dispersion Coefficients Using a Periodic Porous Structure"

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In the two papers [1–2], Kuwahara et al. investigated thermal dispersion in convective flow in porous media by using a two-dimensional periodic model of porous structure. In their study, a macroscopically uniform flow is assumed to pass through a collection of square rods placed regularly in an infinite space, where a macroscopic temperature gradient is imposed perpendicularly and in parallel to the macroscopic flow direction, respectively. Due to the periodicity of the model, only one structure unit is taken for a calculation domain to resolve an entire domain of porous medium. Exhaustive numerical computations were conducted for thermal dispersion conductivity. The transverse thermal dispersion conductivity results were correlated as the following expression [1]:

$$\frac{(k_{dis})_{yy}}{k_f}=0.022\frac{\mathrm{Pe}_D^{1.7}}{(1-\varepsilon)^{1/4}}\quad \text{for } (\mathrm{Pe}_D<10) \tag{26a}$$

and the longitudinal thermal dispersion conductivity results were correlated as [2]

$$\frac{(k_{dis})_{xx}}{k_f}=0.022\frac{\mathrm{Pe}_D^{2}}{(1-\varepsilon)}\quad \text{for } (\mathrm{Pe}_D<10) \tag{16a}$$

$$\frac{(k_{dis})_{xx}}{k_f}=2.7\frac{\mathrm{Pe}_D}{\varepsilon^{1/2}}\quad \text{for } (\mathrm{Pe}_D>10) \tag{16b}$$

where $\mathrm{Pe}_D=\mathrm{Pe}(1-\varepsilon)^{1/2}$. Equation (26*a*) was also summarized together with their longitudinal thermal dispersion conductivity results, Eq. (16), in their later paper [2]. The parameters used in this discussion are defined as the same as those applied by Ref. [1–2]; see also the *Nomenclature* in this discussion.

However, correlations (26*a*), (16*a*) and (16*b*) are questionable. Here is my detailed analysis.

According to the definition on the Peclet number given by Refs. [1–2], the Peclet number Pe should be expressed as

$$\mathrm{Pe}=(\rho C_p)_f u_m H/k_f \tag{D1}$$

However, the generally applied definition on the Peclet number is [3–7]

$$\mathrm{Pe}_g=(\rho C_p)_f u_m D/k_f \tag{D2}$$

If the generally applied definition on the Peclet number, Eq. (*D*2), is applied to Eqs. (26*a*), (16*a*) and (16*b*), Eqs. (26*a*), (16*a*) and (16*b*) will present the unreasonable results due to the structural unit applied by Kuwahara et al. [1–2]. The structural unit applied by Kuwahara et al. [1–2] in their numerical experiments is shown in Fig. 1.

According to Fig. 1, the total volume of the structural unit is

$$V_t=H^2 \tag{D3}$$

The pore volume of the unit is

$$V_p=H^2-D^2 \tag{D4}$$

So, the porosity of the unit can be expressed as

$$\varepsilon=V_p/V_t=1-D^2/H^2 \tag{D5}$$

From Eq. (*D*5), we have

$$D/H=(1-\varepsilon)^{1/2} \tag{D6}$$

or

$$H=D/(1-\varepsilon)^{1/2} \tag{D7}$$

Equation (*D*7) indicates that H is a function of porosity. Inserting Eq. (*D*7) into Eq. (*D*1) yields

$$\mathrm{Pe}=(\rho C_p)_f u_m D/[k_f(1-\varepsilon)^{1/2}] \tag{D8}$$

Due to the generally applied definition for the Peclet number Eq. (*D*2), Eq. (*D*8) can be written as

$$\mathrm{Pe}=\mathrm{Pe}_g/(1-\varepsilon)^{1/2} \tag{D9}$$

Equation (*D*9) establishes the relation between the Peclet number, Eq. (*D*1), defined by Refs. [1–2] and the generally applied Peclet number, Eq. (*D*2) [3–7].

According to $\mathrm{Pe}_D=\mathrm{Pe}(1-\varepsilon)^{1/2}$ (defined by Refs. [1–2]) and $\mathrm{Pe}=\mathrm{Pe}_g/(1-\varepsilon)^{1/2}$ (i.e. Eq. (*D*9)), we obtain

$$\mathrm{Pe}_D=\mathrm{Pe}_g \tag{D10}$$

Inserting Eq. (*D*10) into Eqs. (26*a*), (16*a*) and (16*b*) results in

$$\frac{(k_{dis})_{yy}}{k_f}=0.022\frac{\mathrm{Pe}_g^{1.7}}{(1-\varepsilon)^{1/4}}\quad \text{for } (\mathrm{Pe}_D<10) \tag{D11}$$

$$\frac{(k_{dis})_{xx}}{k_f}=0.022\frac{\mathrm{Pe}_g^{2}}{(1-\varepsilon)}\quad \text{for } (\mathrm{Pe}_D<10) \tag{D12a}$$

$$\frac{(k_{dis})_{xx}}{k_f}=2.7\frac{\mathrm{Pe}_g}{\varepsilon^{1/2}}\quad \text{for } (\mathrm{Pe}_D>10) \tag{D12b}$$

where $\mathrm{Pe}_g=(\rho C_p)_f u_m D/k_f$ is the generally defined definition for the Peclet number given by Eq. (*D*2).

From the above analysis, it is seen that the equations (*D*11), (*D*12*a*) and (*D*12*b*) are equivalent to Eqs. (26*a*) and (16*a*) and (16b). Equations (*D*11) and (*D*12) are transverse and longitudinal thermal dispersion conductivities, respectively, which are ex-

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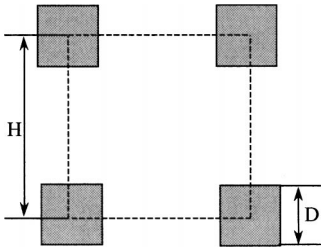

Fig. 1 The structural unit applied by Refs. [1–2] in their numerical experiments

pressed in terms of porosity and generally defined Peclet number Pe_g. However, Eqs. (*D*11) and (*D*12) present unreasonable results. For example, the thermal dispersion conductivities given by Eqs. (*D*11) and (*D*12*a*) increase with the increase of porosity. If porosity is 1, Eqs. (*D*11) and (*D*12*a*) present the infinite values for transverse and longitudinal thermal dispersion conductivities, while Eq. (*D*12*b*) presents nonzero longitudinal thermal dispersion conductivity. These are inconsistent with physical situation because the thermal dispersion should be reduced with the increase of porosity, and when porosity is 1, there is no dispersion and thermal dispersion conductivity should be zero.

In conclusion, it seems to me that the transverse and longitudinal thermal dispersion conductivity correlations, Eq. (26*a*), Eq. (16*a*) and (16b) given by Refs. [1–2], are questionable because they present unreasonable results.

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Nomenclature

D = size of square rod or size of particle
H = size of structural unit
k = thermal conductivity
Pe = Peclet number based on H and the macroscopically uniform velocity defined by Refs. [1–2]
Pe_g = generally defined Peclet number given by Eq. (*D*2)
u_m = the macroscopically uniform velocity
V = volume
ε = porosity

Subscripts

dis = dispersion
f = fluid
xx = longitudinal direction
yy = transverse direction
g = generally applied
t = total
p = pore

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