Closure to Discussion: "A Numerical Study of Thermal Dispersion in Porous Media" (Kuwahara, F., Nakayama, A., and Koyama, H., 1996, ASME J. Heat Transfer, 118, pp. 756–761) and "Numerical Determination of Thermal Dispersion Coefficients Using a Periodic Porous Structure" (Kuwahara, F., and Nakayama, A., 1999, ASME J. Heat Transfer, 121, pp. 160–163)

A. Nakayama

F. Kuwahara
Dept. of Mechanical Engineering, Shizuoka University, 3-5-1 Johoku, Hamamatsu, 432-8561 Japan

We thank Dr. Yu for his discussion on two papers [1,2] of ours on thermal dispersion in porous media that appeared in the Journal of Heat Transfer. Dr. Yu rewrites our correlations established performing a series of numerical experiments as follows:

For the transverse dispersion:

$$\frac{(k_{dis})_{yy}}{k_f} = 0.022\frac{\mathrm{Pe}_D^{1.7}}{(1-\varepsilon)^{1/4}} \quad \text{for } (\mathrm{Pe}_D < 10) \qquad (26a)$$

$$\frac{(k_{dis})_{yy}}{k_f} = 0.052(1-\varepsilon)^{1/2}\,\mathrm{Pe}_D \quad \text{for } (\mathrm{Pe}_D > 10) \qquad (26b)$$

For the longitudinal dispersion:

$$\frac{(k_{dis})_{xx}}{k_f} = 0.022\frac{\mathrm{Pe}_D^{2}}{(1-\varepsilon)} \quad \text{for } (\mathrm{Pe}_D < 10) \qquad (16a)$$

$$\frac{(k_{dis})_{xx}}{k_f} = 2.7\frac{\mathrm{Pe}_D}{\varepsilon^{1/2}} \quad \text{for } (\mathrm{Pe}_D > 10) \qquad (16b)$$

Dr. Yu assumes that the thermal dispersion decreases with the increase in the porosity, for it vanishes when the porosity asymptotically approaches unity. (Incidentally, he did not give Eq. (26*b*) in his discussion, perhaps because it satisfies the foregoing asymptotic behavior with which he is concerned.) He is correct for the case of a high Peclet number since it should decrease with the increase in ε (see Eqs. (26*b*) and (16*b*)), but incorrect for the case of low Peclet number. Dr. Yu should note that our low Peclet number correlations given by (26*a*) and (16*a*) are consistent with the following analytical expressions reported by Koch and Brady [3] for the dispersion in fixed beds (also see Kaviany [4]):

For the transverse dispersion:

$$\frac{(k_{dis})_{yy}}{k_f} = \frac{\sqrt{2}}{240}\frac{\mathrm{Pe}_D^{2}}{(1-\varepsilon)^{1/2}} \quad \text{for } (\mathrm{Pe}_D \ll 1)$$

For the longitudinal dispersion:

$$\frac{(k_{dis})_{xx}}{k_f} = \frac{\sqrt{2}}{60}\frac{\mathrm{Pe}_D^{2}}{(1-\varepsilon)^{1/2}} \quad \text{for } (\mathrm{Pe}_D \ll 1)$$

Kaviany [4] explains the denominator $(1-\varepsilon)^{1/2}$, stating that, for the case of sufficiently low Peclet number, convection extends a distance of $D/3\sqrt{2}(1-\varepsilon)^{1/2} \sim H$, resulting in the spread of the heat much beyond a distance of a particle diameter associated with the pure conduction contribution.

Like these expressions, our expressions (26*a*) and (16*a*) fail as the porosity approaches unity. They may also fail under the other limiting condition, namely, $\varepsilon \to 0$, since the permeability–porosity relationship is different there. The porosity range in which our correlations are valid may be judged by reexamining Fig. 8 in our paper [1], where the coefficients are plotted against the abscissa variable $(1-\varepsilon)$ to find possible correlations. The errors become large as the porosity approaches zero. Also, in the range close to the origin (i.e. $\varepsilon = 1$), the coefficients reduce to values too small to elucidate a functional relationship. Thus, we presume that our correlations given by (26*a*), (26*b*), (16*a*) and (16*b*) are valid only for a limited porosity range, say, $0.2 < \varepsilon < 0.8$. It should also be noted that neither (26*b*) nor (16*b*) can be used beyond this range, since neither of them has been designed to account for the asymptotic behavior that Dr. Yu presumed in his discussion.

As in Figs. 9 in Ref. [1] and Fig. 4 in Ref. [2], the high Peclet number correlations are found in good accord with available experimental data. This substantiates the validity of our correlations. However, sufficient experimental data are not available for the low Peclet number range with which Dr. Yu is mostly concerned. For this range, we simply repeat that the correlations are consistent with those for packed beds reported by Koch and Brady [3].

References

[1] Kuwahara, F., Nakayama, A., and Koyama, H., 1996, "A Numerical Study of Thermal Dispersion in Porous Media," ASME J. Heat Transfer, **118**, pp. 756–761.
[2] Kuwahara, F., and Nakayama, A., 1999, "Numerical Determination of Thermal Dispersion Coefficients Using a Periodic Structure," ASME J. Heat Transfer, **121**, pp. 160–163.
[3] Koch, D. L., and Brady, J. F., 1985, "Dispersion in Fixed Beds," J. Fluid Mech., **154**, pp. 399–427.
[4] Kaviany, M., 1995, *Principles of Heat Transfer in Porous Media*, 2nd edition Springer-Verlag, New York, p. 209.

