atmospheric temperature throughout, irrespective of the process temperature, if it should be so desired: certainly temperatures well below the limiting one for creep are easily maintained. Thus it should be possible to accomplish two aims: (*a*) remove the limit upon process temperature normally imposed by available structural materials, (*b*) recover virtually all the heat which normally would escape through the walls of such a vessel. In addition, a chemically active reactant which at high temperatures would be highly corrosive may be almost neutral at atmospheric temperatures, greatly simplifying the problem of surface protection.

The method employed is simple. The vessel is lined with an envelope of fluid-permeable material adapted to the temperature gradients to be expected, the temperatures to be attained in the various parts, and the chemical action to be expected. Two cases may be distinguished: (*a*) The permeating fluid is either a reactant in the process or an acceptable diluent thereof; (*b*) the permeating fluid is one other than the reactants and does not mingle with the reactants or products. In cases (*a*) and (*b*), the permeating fluid is distributed over the surface of the permeable envelope, adjacent to the vessel wall which is to be kept cold, at a temperature substantially atmospheric, and at a mass rate of flow such as may be required. The fluid permeates through the permeable envelope from the outer surface to, and through the inner surface thereof.

In the case of (*a*), the fluid leaving the inner surface mingles with the reactants and products; it may be a portion of one of the reactant fluids. In case (*b*), an impermeable envelope is placed inside the permeable one so that the fluid in the space between the two envelopes may be collected and removed. Such fluid being highly heated may be passed through a heat exchanger adapted to recovering the heat therein, either by heating a portion of the reactants or in some other way. Very broadly, any sort of construction or process may be handled by one or other of the methods described.

The laws of the temperatures in the permeable bodies involved have been investigated theoretically and by tests. It is hoped a paper thereon may be presented to the Society. For determining the possibilities, a cracking chamber for oils operating at 850 F and 750 psi, 6 ft diam × 40 ft long, was investigated. Using commercially available permeable bodies, it was found that such a method would be quite practicable. The wall thickness of the vessel was 4 in., in the original design. With cold walls, only 2 in. was necessary. The original and very necessary stainless lining would be desirable but far from essential in the cold-wall design. The fluid flow through the permeable body would be of the order of 1 per cent of the reactants processed. Virtually no heat would be lost from the process, hence no additional heat would be required in the reactants.

It should be remembered that at moderate rates of permeation, the permeating fluid passing through the inner surface of the permeable envelope is at process temperature. If control of the exothermic heat of a reaction is required, the rate of flow of fluid may be increased to lower the reaction temperature.

Later on the partners applied the methods to a furnace with controlled combustion.[4] As the alleged date of the "sweat-cooling" conception is given as September, 1944, it will be seen that the patent specifications of the partners fully anticipated such a development as has been carried on under Navy Project SQUID.

The partners have gone further, applying related methods to very compact heat exchangers, cooling systems, the protection of gas-turbine blading and furnace parts, and have even shown how heat exchange may be secured between fluids, one at high, the other at atmospheric pressure, while avoiding much of the creep effect which must be considered in usual designs. For example, it is possible to build steam generators and furnaces, therefore, for 3500 psi and 1300 F sufficiently compact to generate nearly 200 lb per cu ft of space per hr (as compared with a more usual figure of 12 lb per cu ft per hr for 750 F, 1400 psi), while using materials readily available and of moderate cost. The weight of equipment would be about 10 per cent of the more usual, based upon steam generators for modern ore vessels.

The primary purpose of this discussion, then, is to point out that much work has been done toward the more scientific design of equipment for high temperatures and pressures for the twin purposes of economizing heat, and removing temperature limitations to the process which have been set hitherto by the properties of structural materials. It is impossible to do more than hint at the methods to be used. Obviously, many other useful effects may be secured. Only very limited amounts of material for special purposes should require the application of the methods given in the paper.

Authors' Closure

Mr. Richardson's remarks are of interest. However, they bear little relation to the technical aspects of the problem treated by the authors, and it is felt that no comments regarding them are necessary.

Three-Dimensional Solution for Stress Concentration Around a Circular Hole in a Plate of Arbitrary Thickness[1]

M. M. Frocht.[2] This paper has at least two important aspects. It is important because it extends the scope of the mathematical theory of elasticity, in which it is the three-dimensional generalization of Kirsch's plane solution. It is equally important on account of its conclusions regarding the stress concentrations in thick bars with circular holes in tension.

The following remarks are confined to the latter aspects of the paper:

In 1942 the writer, jointly with M. M. Leven, made a photoelastic study[3] of the factors of stress concentration in tensile bars of finite width with circular holes. Our attention at that time was confined to the primary or to the largest principal stress and we were not concerned with the remaining principal stresses.

It is of some interest to compare the conclusions arrived at in 1942, on the basis of optical studies with those arrived at by the authors via the analytical route.

1 *Stress Variation.* In 1942 we found "that in thick bars with circular holes the inner layers carry a somewhat greater stress than the outer layers." The authors find that "if $\Delta \neq 0$

[4] U. S. Patent No. 2,311,350, "Method and Apparatus for Controlling Combustion," issued to E. A. Richardson, Feb. 16, 1943.

[1] By E. Sternberg and M. A. Sadowsky, published in the Journal of Applied Mechanics, March, 1949, Trans. ASME, vol. 71, pp. 27–38.

[2] Research Professor of Mechanics, Illinois Institute of Technology, Chicago, Ill. Mem. ASME.

[3] "On the State of Stress in Thick Bars," by M. M. Frocht and M. M. Leven, *Journal of Applied Physics*, vol. 13, 1942, pp. 308–313.

the stress-concentration factor is reduced at the faces of the plate and increased in the interior" ($\Delta = t/D$).

2 *Magnitude of Stress Variation and Its Significance.* In 1942 we stated, "the variation in the stresses are, however, small and do not exceed 12 per cent. . . . For practical design purposes the factors of stress concentration for thick plates may be assumed to be equal to the corresponding two-dimensional factors.[4] The authors now find that "regardless of the thickness ratio, the reduction of maximum stress σ_θ does not exceed 10 per cent. . . .," and that, "The foregoing conclusions with regard to σ_θ . . . lend further support to the general assertion that factors of stress concentrations based upon two-dimensional analysis apply to plates of arbitrary thickness ratio."

3 *Vose's Conclusions.* In 1942 we stated that "the results published by Vose . . . are believed to be in error." Commenting on the same results the authors say: "By neglecting σ_z . . . R. W. Vose arrived at the erroneous conclusion that the factor of stress concentration depends sensitively upon the thickness of the plates."

4 *State of Plane Strain.* Although guided by the different evidence, the same conclusions were also reached with regard to the thickness ratio Δ at which a state of plane strain is developed. Thus, in 1942, we found that "no deviation from the state of plane stress (or strain) can be found in bars with central circular holes when r/t is of the order of magnitude of about $^1/_4$ in. (This corresponds to $\Delta = 2$.)

The authors state: "It is noteworthy that the maximum values of σ_θ are practically attained already at $\Delta = 2$. (This maximum value of σ_θ is the value for plane strain.)"

It is now well established that in two-dimensional problems the results from photoelasticity and the theory of elasticity are generally in excellent agreement. The results obtained by the authors bear on the analogous question in the three-dimensional field. It should, however, be kept in mind that the problem solved by the authors deals with a plate of infinite width, whereas the plates studied photoelastically were of finite width with a minimum ratio of diameter of hole to width of plate of 0.14.

One aspect of the paper deserves special mention. The authors show that in three-dimensional problems Poisson's ratio may have a considerable influence on the state of stress. Fortunately, this influence is confined to the secondary stresses as is to be anticipated. In any event, photoelasticians will have to be cognizant of the influence of this factor.

In closing, the discusser wishes to express his admiration for the excellent piece of analytical work done by Drs. Sternberg and Sadowsky.

Authors' Closure

Professor Frocht's comments on experimental corroboration of some of the results obtained in the paper are greatly appreciated. The authors wish to take this opportunity to make reference to a master's thesis[5] in which E. Reissner's method was applied to the determination of three-dimensional corrections for the plane problem of the half-plane under sinusoidal normal tractions along the edge. In view of this application, the statement made on page 28 of the paper, requires modification. At the same time the authors call attention to an alternative solution by A. E. Green[6] of the problem under discussion, which appeared after our work had been submitted for publication.

[4] The same effect was subsequently found in thick slotted plates in tension.

[5] "On Generalized Plane Stress," by R. A. Clark, MS thesis, Massachusetts Institute of Technology, Cambridge, Mass., 1946.

[6] "Three-Dimensional Stress Systems in Isotropic Plates, I," by A. E. Green, *Philosophical Transactions*, Royal Society of London, England, series A, vol. 240, April, 1948, pp. 560–597.

A Strain-Energy Expression for Thin Elastic Shells[1]

W. T. Koiter.[2] The author indicates some inaccuracies in Love's theory of thin shells and particularly in Love's formula for the energy of bending. The arguments put forward to discredit Love's expressions are indeed convincing. Therefore it may be of interest to observe that the corrections indicated by the author are only of small *practical* importance. In effect, the differences between Love's (linear) components of curvature and the linear parts of the author's components of curvature can be shown to amount to

$$\frac{A_2}{E} + \kappa_1 = \text{linear part of } \frac{A_1}{Er_1}$$

$$\frac{B_2}{G} + \kappa_2 = \text{linear part of } \frac{B_1}{Gr_2}$$

$$\frac{C_2}{\sqrt{EG}} + \tau = 0$$

$$\frac{C'_2}{\sqrt{EG}} + \tau = \text{linear part of } \frac{2C_1}{\sqrt{EG}}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

As has been shown elsewhere,[3] it is inherent to the assumptions used in the engineering theory of thin shells that contributions to the strain components of the order zA_1/rE, etc., may be neglected.

Therefore in all cases in which the engineering theory (based upon the assumptions that normals of the undeformed middle surface, remain normals of the deformed middle surface, and direct stresses on planes parallel to the middle surface may be neglected) is applicable, the author's corrections cannot affect the bending energy appreciably.

On the contrary, it may be considered allowable to add to the components of curvature A_2/E, etc., *arbitrary terms* of the order A_1/Er_1, etc., in order to simplify the bending energy as far as possible.

Author's Closure

The author has asserted that Love's formula does not correctly account for the effects of the tangential displacements u and v upon the energy due to bending, since it contains the derivatives u_x and v_y which have no effect upon bending, and it omits the derivative u_y, which, by reason of symmetry, must have an effect similar to that of the derivative v_x. Dr. Koiter has shown that the effects of the derivatives of u and v upon the energy of bending are, in any case, negligible compared to the effects of these derivatives upon the energy of stretching of the middle surface. Consequently, Love's theory may be expected to lead to solutions of special problems which do not differ significantly from solutions derived from the author's equations. Nevertheless, the author's equations have the advantage that they are symmetrical, and that all of the terms in these equations can be explained "physically." Love's equations do not meet these requirements, since they are unsymmetrical with respect

[1] By H. L. Langhaar, published in the June, 1949, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 71, pp. 183–189.

[2] Professor of Applied Mechanics, Technische Hogeschool, Delft, Holland.

[3] "Over de stabiliteit van het elastisch evenwicht" ("On the Stability of Elastic Equilibrium"), by W. T. Koiter, Thesis, Technische Hogeschool, Delft, 1945, chapt. 5 (Theory of Shells for Finite Displacements).