\( \Delta(1232) \) Isobar Probability in Frozen and Hot Neutron, Nuclear and \( \beta \)-Stable Matter

M. Modarres\(^{1,2,*}\) and H. R. Moshfegh\(^{1,3}\)

\(^1\) Physics Department, Tehran University, North Kargar Ave, 14394 Tehran, Iran\(^{**}\)
\(^2\) Center for Theoretical Physics and Mathematics, P. O. Box 11365-8486, Tehran, Iran
\(^3\) Institute for Studies in Theoretical Physics and Mathematics, Tehran, Iran

(Received August 13, 2001)

In this article we calculate the probability of having \( \Delta(1232) \) resonance in frozen and hot neutron, nuclear and \( \beta \)-stable matter. We use the temperature dependence correlation functions that are generated through a lowest order constrained variational calculation with the \( \Delta \)-Reid potential. The \( NN \rightarrow N\Delta \) transition is built in through a two-pion exchange interaction. The electrons and muons are treated relativistically in the total Hamiltonian at given temperature and density, in order to make the fluid electrically neutral and stable with respect to \( \beta \)-decay. We ignore the weak interaction. It is seen that the \( \Delta \) probability in neutron matter is much larger than in nuclear and \( \beta \)-stable matter at a given temperature and density. As we increase the temperature, the \( \Delta \) probability decreases in nuclear and neutron matter. However, this decrease is not significant in case of \( \beta \)-stable nuclear matter. There is overall agreement between our \( \Delta \) probability calculation and the recent experiments performed on \(^3\)He up to \(^{208}\)Pb nuclei. It is concluded that the isobar degrees of freedom could make the equation of state of neutron star matter harder at finite temperature and suppress the numbers of protons and leptons in the proto-neutron stars.

§1. Introduction

It has been noted in several works\(^1\)\(-\)\(^4\) that the many-body calculations for nuclear matter with phenomenological interactions that fit the two-nucleon data yield substantially too strong binding and saturate at too high densities. The work of Green and co-workers\(^2\),\(^5\) and Day and Coester\(^3\) indicates that explicit inclusion of \( \Delta(1232) \) in the two-nucleon scattering amplitude leads to a significant loss of binding and lowering of the saturation density. Specifically, the diagram in Fig. 1 contributes to the two-nucleon scattering amplitude and produces a significant attraction. When considering a two-nucleon system, the intermediate \( N \) and \( \Delta \) are plane-wave states, and there is a summation over all intermediate states propagating in the mean field of the other particles (dispersion effect). Also, the intermediate nucleon must be excluded from the already occupied Fermi sea (Pauli effect). The magnitude of these effects for neutron matter, nuclear matter and finite nuclei were calculated\(^5\)\(-\)\(^7\) by improving the \( N\Delta \) propagators of the second order diagram in Fig. 1. These effects were simply added to the results in which these effects were not included. The importance of \( \Delta \) isobars has also been investigated in the framework of the chiral-invariance \( \pi N \) Lagrangian\(^8\) (see Ref. 9 for details), the Faddeev method,\(^10\)

\(^*\) Corresponding author.
\(^**\) Permanent address.
and the one-boson-exchange (OBE) Brueckner-Hartee-Fock (BHF) approach.\textsuperscript{11}

In 1979,\textsuperscript{12} we included a $NN \rightarrow N\Delta$ transition potential explicitly in the Hamiltonian from the beginning and considered the isobar degree of freedom in our trial variational wave function. The result of the lowest-order constrained variational (LOCV) calculation,\textsuperscript{12,13} using the above transition potential and trial wave function, was that we could obtain reasonable binding and saturation density for nuclear matter.\textsuperscript{13} The LOCV method has also been developed for calculating the various properties of homogeneous nuclear fluids such as hot and frozen neutron, nuclear and $\beta$-stable matter with realistic nucleon-nucleon interactions.\textsuperscript{13,14}

A few sophisticated interactions, such as the $UV_{14}$,\textsuperscript{15} the $AV_{14}$,\textsuperscript{16} and the new argonne $AV_{18}$\textsuperscript{17} potentials, have also been used. These potentials fit the $N-N$ scattering data very well.\textsuperscript{18} Good agreement has been found between the LOCV technique\textsuperscript{19} and the results of more sophisticated methods, such as the variational fermion hypernetted chain (FHNC).\textsuperscript{15,20} The three-body cluster contribution in the nuclear matter energy and the normalization integral $\langle \psi | \psi \rangle$ both at zero and finite temperatures were calculated to test the convergence of the cluster expansion truncation.\textsuperscript{13,14} It was shown that the LOCV technique is capable of treating well-defined phenomenological potentials. However, in most of these works, it has been found that all of the new potentials overbind the nuclear matter at larger density, in comparison with the empirical prediction.

Recently, two new experiments were performed at the 1.3 GeV electron synchrotron of the Institute for Nuclear Study at the University of Tokyo (INS)\textsuperscript{21} and with 500 MeV pions at the Los Alamos Meson Physics Facility (LAMPH).\textsuperscript{22} At INS, the $^3$He($\gamma, \pi^\pm p$) reactions were measured over the photon energy range above the $\Delta$ resonance region.\textsuperscript{21} Corresponding to the measured knockout cross section, the $\Delta NN$ probability in the $^3$He ground state was estimated to have lower and upper limits of $1.5 \pm 0.6 \pm 0.5\%$ and $2.6\%$, respectively. At LAMPH, the cross section for ($\pi^+, \pi^\pm p$) reactions with $^3$He, $^4$He, $^6$Li and $^7$Li targets for quasi-free kinematics at incident beam energy of 500 MeV were measured. The $\Delta NN$ probability was found to vary from 0.5 to 2 percent for these targets.\textsuperscript{22} A new, sophisticated nuclear

![Fig. 1. The contribution to the two-boson exchange $NN$ scattering amplitude.](image-url)
model is needed to calculate the $\Delta$ probability theoretically in order to analyze these experimental data.

The purpose of this work is to calculate the $\Delta$ probabilities in hot and frozen neutron, nuclear and $\beta$-stable matter using the LOCV formalism. The paper is organized as follows. The interaction potential is explained in §2. Section 3 is devoted to a short description of the lowest-order constrained variational method. The coupled differential equation for numerical derivation of the $N\Delta$ correlation function and definition of the $\Delta$ probabilities are discussed in §4. Finally, in §5 we present the results and discussion.

§2. $N$-$\Delta$ transition potential

The transition potential is given in the form\(^\text{12), 23}\)

$$V(NN \rightarrow N\Delta) = \sum_{i>j} V_2(ij),$$  \hspace{1cm} (1)

with

$$V_2 = \sqrt{\frac{2}{6}} \frac{f f^*}{4\pi} \mathbf{r}_1 \cdot \mathbf{T}_2 (S_{12}^0 V(r, T) + \mathbf{r}_1 \cdot \mathbf{T}_2 S_2 V(r, SS)),$$  \hspace{1cm} (2)

where

$$V(r, T) = \sum_{i=1}^{10} \alpha_i \frac{\exp(-\beta_i r)}{(m_i r)^{\gamma_i}} \left[ \frac{\exp(-m_i r)}{m_i r} \left( 1 + \frac{3a_i}{m_i r} + \frac{3b_i}{(m_i r)^2} \right) \right. \left. - \left( \frac{\Lambda}{m_i} \right)^3 \frac{\exp(-Ar)}{Ar} \left( 1 + \frac{3c_i}{Ar} + \frac{3b_i}{(Ar)^2} \right) \left( 1 - \exp(-Ar) \right)^2 \right]$$

$$c_i = \left( \frac{m_i}{\Lambda} \right) a_i + \left[ 1 - \left( \frac{m_i}{\Lambda} \right) \right] b_i,$$  \hspace{1cm} (3)

and

$$V(r, SS) = \sum_{i=1}^{10} \left( \alpha_i \frac{\exp(-\beta_i(1)r)}{(m_i r)^{\gamma_i(1)}} + \alpha_i \frac{\exp(-\beta_i(2)r)}{(m_i r)^{\gamma_i(2)}} \right)$$

$$\times \left[ \left( \frac{\exp(-m_i r)}{m_i r} \right) - \left( \frac{\Lambda}{m_i} \right) \frac{\exp(-Ar)}{Ar} \left( 1 + \frac{A^2 - m_i^2}{2\Lambda \Lambda} \right) \right].$$  \hspace{1cm} (4)

The parameters for $V(r, T)$ and $V(r, SS)$ are given in Tables I and II, respectively. The cutoff $\Lambda$ is chosen to be $7.6\text{fm}^{-1}$, and the coupling constant is chosen such that $f f^*/4\pi = 0.1683$. The $\sqrt{2}$ factor in Eq. (2) reflects the fact that either of the nucleons in the amplitude of Fig. 1 may be excited into a $\Delta$ state. The operator $S_{12}^0$ is the analogue of the usual tensor operator\(^\text{24}\) for the mixed $N\Delta$ channel, and $S_2$ and $T_2$ are the non-square spin and isospin matrices that connect a two-component nucleon spinor to a four-component isobar function.\(^\text{24}\) The transition potential is assumed to act only in the $^1S_0(NN) \leftrightarrow ^5D_0(N\Delta)$ channel. The next possible transition is the $^3P_1(NN) \leftrightarrow ^5P_1(N\Delta)$ channel. The contributions from these channels (i.e. $L \neq 0$) and from the $NN \rightarrow \Delta\Delta$ channels are small.\(^\text{2,3}\) The reasons for this are as follows.
In the LOCV method, we use an ideal Fermi gas-type wave function for the single particle states and variational techniques to derive the wave function of an interacting system \( \text{(12),}\,\text{(19)} \) at finite temperature \( T \),

\[
\psi = \mathcal{F}_T \Phi^T, \tag{6}
\]

\( \psi \) being the wave function of the system at finite temperature, \( \Phi^T \) is the wave function of a single particle at energy \( E \), and \( \mathcal{F}_T \) is the transformation operator that changes the variables from \( E \) to \( T \).
\[ \mathcal{F}_T = S \prod_{i>j} f(ij), \]  
(7)

where \( S \) is a symmetrizing operator. The Jastrow correlation functions \( f(ij) \) are operators, and they are written

\[ f(ij) = \sum_{\alpha,k} f_{\alpha}^{(k)}(ij) O_{\alpha}^{(k)}(ij). \]  
(8)

In above equation, \( \alpha = \{S, L, J, T, M_T; T\} \), \( k = 1-4 \) and

\[ O_{\alpha=1-4} = 1, \left( \frac{2}{3} + \frac{1}{6} S_{12}^I \right), \left( \frac{1}{3} - \frac{1}{6} S_{12}^I \right), S_{12}^I. \]  
(9)

For spin-singlet channels with orbital angular momentum \( L \neq 0 \) and spin-triplet channels with \( L \neq J \pm 1 \), \( k \) is superfluous and is set to unity, while for \( L = J \pm 1 \), it takes values of 2 and 3. The \( L = 0 \) channel, which couples the \( ^1S_0 \) channel to the \( ^5D_0 \) channel, remains. In this case, we set \( k = 1 \) and 4. In this article we focus mainly on \( k = 1 \) and 4 correlation functions.

In general, each of the correlation functions \( f_{\alpha}^{(1)} \), \( f_{\alpha}^{(2)} \) and \( f_{\alpha}^{(3)} \) is required to heal to the modified Pauli function \( f_{p}^{(r)} \), and \( f_{\alpha}^{(4)} \) to 0:

\[
 f_{p}^{(r)} = \left[ 1 - \frac{1}{2} \left( \frac{\gamma_i(r)}{n_B} \right)^2 \right]^{-\frac{1}{2}} \quad n-n \text{ and } p-p \text{ channel} \\
 = 1 \quad n-p \text{ channel},
\]  
(10)

with \( i = p, n \) and

\[
 \gamma_i(r) = \frac{4}{(2\pi)^3} \int n_i(k) J_0(kr) \, dk.
\]  
(11)

Here \( J_0(x) \) are the familiar spherical Bessel functions, and \( n_i(k) \) is defined in Eq. (13).

The total baryon number density \( n_B \) is written as the sum of the proton and neutron number densities, \( n_B = n_p + n_n \), while the condition of electrical neutrality, \( n_p = n_e + n_\mu \), fixes the leptons densities \( n_e \) and \( n_\mu \) (since we have assumed free-neutrino matter). The leptons form two highly relativistic Fermi seas with the kinetic energy

\[
 e_L = (\Omega n_B)^{-1} \sum_{i=e,\mu} \sum_{k,\sigma} \frac{[\hbar^2 k^2 c^2 + m_{\sigma}^2 c^4]^{1/2}}{[\exp((\varepsilon_i(k) - \mu_i)\beta + 1)]},
\]  
(12)

where \( \beta = \frac{1}{k_B T} \), with \( k_B \) the Boltzman factor, \( \varepsilon_i(k) = \{[\hbar^2 k^2 c^2 + m_{\sigma}^2 c^4]^{1/2} - m_{\sigma} c^2 \} \) are the relativistic lepton kinetic energies, and \( \mu_i \) (\( m_i \)) are the chemical potentials (rest masses) of the ith particle pieces. The chemical potentials should satisfy the conditions of \( \beta \)-stability, \( \mu_n - \mu_p + (m_n - m_p - m_e) c^2 = \mu_e \) and \( \mu_e + (m_e - m_\mu) c^2 = \mu_\mu \), as well as the particle number density relation

\[
 n_i(k) = (\Omega^{-1}) \sum_{\sigma, k} [\exp((\varepsilon_i(k) - \mu_i)\beta + 1)]^{-1}.
\]  
(13)
The entropies of leptons and baryons are calculated from the equation given in Ref. 26. The baryonic internal energy is

$$e_B = t_B + e[f],$$

(14)

where the kinetic energy part, $t_B$ has the form

$$t_B = (\Omega n_B)^{-1} \sum_{k,\sigma} \sum_{i=p,n} \left[ \frac{\hbar^2 k_i^2}{2m_i} + m_i c^2 \right] \left[ \exp((\epsilon_i^*(k) - \mu_i)\beta) + 1 \right].$$

(15)

The $\mu_i$ should satisfy an equation similar to that for leptons (Eq. (13)), but with $\epsilon_i^*(k) = \frac{\hbar^2 k_i^2}{2m_i}$, where the effective masses, $m_i^*$, are determined variationally.

The many-body energy term $e[f]$ is calculated by constructing a cluster expansion for the expectation value of our Hamiltonian. Then, we keep only the first two terms in the cluster expansion of the energy functional:

$$e[f] = \frac{1}{A} \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} - e_1 = e_2.$$

(16)

The one-body term $e_1$ is independent of $f$ and is simply $t_B$, while the two-body energy term is defined as

$$e_2 = (2A)^{-1} \sum_{ij} \langle ij | V | ij \rangle_a,$$

(17)

where

$$V(12) = -\frac{\hbar^2}{2m} [f(12), [\nabla_{12}^2, f(12)]] + f(12)V(12)f(12),$$

(18)

and the two-body antisymmetrized matrix elements $\langle ij | V | ij \rangle_a$ are taken with respect to the single-particle functions composing $\Phi^T$, i.e. plane waves. By inserting a complete set of two-particle states twice into Eq. (17) and performing some algebra, we can rewrite the two-body term as

$$e_2 = e_c + e_T + e_{1T},$$

(19)

where

$$e_c = \frac{2}{\pi^4 n_B} \sum_{\alpha \neq 1, S_0, m_{\tau_1}, m_{\tau_2}} |\langle m_{\tau_1} m_{\tau_2} | TM_T \rangle|^2 (2J + 1) \frac{1}{2} \{ 1 - (-1)^{L+S+T} \}$$

$$\times \int_0^\infty dr \frac{\hbar^2}{2m} \left\{ f_\alpha^{(1)^2} + \frac{2m}{\hbar^2} V^c f_\alpha^{(1)^2} \right\} a_{\alpha,m_{\tau_1},m_{\tau_2}}^{(1)}(r),$$

(20)

$$e_T = \frac{2}{\pi^4 n_B} \sum_{JT,m_{\tau_1},m_{\tau_2}} |\langle m_{\tau_1} m_{\tau_2} | T m_{\tau_1} + m_{\tau_2} \rangle|^2 (2J + 1) \frac{1}{2} \{ 1 - (-1)^{J+T} \}$$

$$\times \int_0^\infty dr \left\{ \frac{\hbar^2}{2m} \left( f_\alpha^{(2)^2} + \frac{2m}{\hbar^2} (V^c + 2V^T - V_a^{LS} f_a^{(2)^2}) \right) a_{\alpha,m_{\tau_1},m_{\tau_2}}^{(2)}(r) \right.$$

$$+ \frac{\hbar^2}{2m} \left\{ f_\alpha^{(3)^2} + \frac{2m}{\hbar^2} (V^c - 4V^T - 2V_a^{LS} f_a^{(3)^2}) \right\} a_{\alpha,m_{\tau_1},m_{\tau_2}}^{(3)}(r)$$

$$+ \left\{ r^{-2} (f_\alpha^{(2)^2} - f_\alpha^{(3)^2})^2 + \frac{2m}{\hbar^2} V_a^{LS} f_a^{(2)^2} f_a^{(3)} \right\} b_{\alpha,m_{\tau_1},m_{\tau_2}}^2(r),$$

(21)
and (with \(i\) and \(j\) each standing for either a proton or neutron)

\[
a^{(1)}_{\alpha,i,j}(r,n_B;T) = r^2 I_{ij;J}(r,n_B;T),
\]

\[
a^{(2)}_{\alpha,i,j}(r,n_B;T) = r^2 (2J + 1)^{-1} [(J + 1) I_{ij;J-1}(r,n_B;T) + J I_{ij;J+1}(r,n_B;T)],
\]

\[
a^{(3)}_{\alpha,i,j}(r,n_B;T) = r^2 (2J + 1)^{-1} [(J I_{ij;J-1}(r,n_B;T) + (J + 1) I_{ij;J+1}(r,n_B;T)],
\]

\[
b^{(1)}_{\alpha,i,j}(r,n_B;T) = r^2 (2J + 1)^{-1} [(J I_{ij;J-1}(r,n_B;T) - (J + 1) I_{ij;J+1}(r,n_B;T)],
\]

\[
I_{ij;J}(r,n_B;T) = (2\pi^2 n_B^2)^{-1} \int d\vec{k}_1 d\vec{k}_2 n_i(k_1)n_j(k_2)J^2_{ij}(|\vec{k}_1 - \vec{k}_2|r).
\]

The potential functions \(V^c, V^T\), etc. are given in Ref. 25).

The normalization constraint as well as the coupled and uncoupled differential equations for the \(NN\) channels, coming from the Euler-Lagrange equations, are similar to those described in Refs. 7) and 19). The two-body \(N\Delta\) energy, \(e_T^{11}\), \(N\Delta\) coupled differential equation, and the \(\Delta\) probability integral are discussed in the above references.

§4. Contribution of the \(1^1S_0(NN) \leftrightarrow 5^0D_0(N\Delta)\) transition and the \(\Delta\) probability

The \(NN(1^1S_0)\) plus \(N\Delta(5^0D_0)\) contribution to the internal two-body energy can be written as

\[
e_T^{II} = \frac{2}{\pi^4 n_B} \sum_{m_{r_1},m_{r_2},J=0,T=1} \langle m_{r_1}m_{r_2}|Tm_{r_1} + m_{r_2}\rangle^2 (2J + 1)
\]

\[
\times \frac{1}{2} \int_0^\infty dr \left\{ \frac{\hbar^2}{2\mu} \left\{ f^{(1)}_\alpha r^2 + \frac{\mu}{\mu_\Delta} \left( f^{(4)}_\alpha r^2 + \frac{6}{r^2} f^{(4)}_\alpha \right) \right\} \right. 
\]

\[
\left. + (m_\Delta - m)c^2 f^{(1)}_\alpha r^2 + 2f^{(1)}_\alpha f^{(4)}_\alpha \langle V_2 \rangle + V^c f^{(1)}_\alpha f^{(4)}_\alpha \right\} a^{(1)^2}_\alpha (r),
\]

where \(\langle V_2 \rangle = \langle 5^0D_0|V_2|1^1S_0\rangle\) is the spin-isospin matrix element of the \(V_2\) of Eq. (2), \(\alpha = \{1^1S_0, M_T\}, M_T = m_{r_1} + m_{r_2}\), and \(\mu(\mu_\Delta)\) is the reduce mass of the \(NN(N\Delta)\) channel. The corresponding coupled differential equations are

\[
g^{(1)^\prime\prime}_\alpha (r) - \left[ \frac{a^{(1)^\prime\prime}_\alpha (r)}{a^{(1)}_\alpha (r)} + \frac{2\mu}{\hbar^2} [V^c - \lambda] \right] g^{(1)}_\alpha (r) - \frac{2\mu}{\hbar^2} \langle V_2 \rangle g^{(4)}_\alpha (r) = 0,
\]

\[
g^{(4)^\prime\prime}_\alpha (r) - \left[ \frac{a^{(4)^\prime\prime}_\alpha (r)}{a^{(4)}_\alpha (r)} + \frac{2\mu^*}{\hbar^2} [(m_\Delta - m)c^2 - \lambda] + \frac{6}{r^2} \right] g^{(4)}_\alpha (r) - \frac{2\mu^*}{\hbar^2} \langle V_2 \rangle g^{(1)}_\alpha (r) = 0,
\]

with \(g^{(k)}_\alpha (r) = a^{(k)}_\alpha f^{(k)}_\alpha (r)\). The boundary conditions for solving the above equations are given in the previous section and in Refs. 14) and 19).
Finally, the probability that a pair of nucleons in the \(^1S_0\) state looks like that is a \(N\Delta - ^5D_0\) state is given in terms of the \(f^{(4)}_\alpha(r)\) correlation function i.e.:

\[
P_{N\Delta} = \frac{4}{\pi^4 n_B} \sum_{m_{\tau_1},m_{\tau_2},J=0,T=1} |\langle m_{\tau_1} m_{\tau_2} | Tm_{\tau_1} + m_{\tau_2} \rangle|^2 (2J+1) \frac{1}{2} \int_0^\infty dr f^{(4)}_\alpha(r) a^{(1)}_\alpha(r).
\]  

Then, the \(\Delta\) probability is \(P_\Delta = \frac{1}{2} P_{N\Delta}\). (Note that \(P_{N\Delta}\) plus \(P_{NN}\), which is due to other channels, is normalized to 1).

\[\text{§5. Results and discussion}\]

The calculation was performed following the analysis of §§2-4 for different baryon number densities \(n_B\) and temperatures \(T\) for neutron \((n_p = n_e = n_\mu = 0)\), nuclear \((n_p = n_n = \frac{1}{2} n_B, n_e = n_\mu = 0)\) and \(\beta\)-stable (by minimizing the total free energy with respect to \(n_p\)) matter.

In Fig. 2, we plot \(P_\Delta\) for neutron, nuclear and \(\beta\)-stable matter with respect to \(n_B\) for \(T = 0, 10\) and \(20\) MeV \(k_B^{-1}\). It is seen that in general \(P_\Delta\) increases as we increase the baryon number density. However, the rate of increase decreases at large

\[\text{Fig. 2.} \quad P_\Delta \text{ as a function of } n_B \text{ (fm}^{-3}\text{) for neutron (full), nuclear (dashed) and } \beta\text{-stable matter (dotted). The solid circles are from Ref. 22) for } ^3\text{He } (n_B = 0.1 \text{ fm}^{-3}) \text{ and } ^{208}\text{Pb } (n_B = 0.17 \text{ fm}^{-3}).\]  

\[\text{Fig. 3.} \quad \text{This depletion is increased as a function of } n_B \text{ in } \beta\text{-stable matter with respect to the matter without } \Delta \text{ at various temperature (MeV}k_B^{-1}).\]
Fig. 4. Same as Fig. 3 but for electrons.

Fig. 5. Same as Fig. 3 but for muons.

Fig. 6. $N\Delta$ correlation function in nuclear matter at $T = 5$ and $20\,\text{MeV}k_B^{-1}$.

Fig. 7. Same as Fig. 6 but for neutron matter.
Table III. The $\Delta$ probability found from several experiments, as explained in the text.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Target</th>
<th>Energy</th>
<th>$\Delta$ configuration</th>
<th>$\Delta$ percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emura et al. 27) (1993)</td>
<td>$^3$He $\gamma$</td>
<td>$\gamma(380 - 700 \text{ MeV})$</td>
<td>$NN\Delta$</td>
<td>$\leq 2$</td>
</tr>
<tr>
<td>Amelin et al. 27) (1994)</td>
<td>$^9$Be $p$</td>
<td>$p(1 \text{ GeV})$</td>
<td>$\Delta$</td>
<td>$6 \pm 3 \times 10^{-2}$</td>
</tr>
<tr>
<td>LAMPF 22) (2000)</td>
<td>$^3$He $-^{208}$Pb</td>
<td>$\pi(500 \text{ MeV})$</td>
<td>$\Delta$</td>
<td>0.5 to 3.1</td>
</tr>
<tr>
<td>INS 21) (2000)</td>
<td>$^3$He $\gamma$</td>
<td>$\gamma(380 - 700 \text{ MeV})$</td>
<td>$NN\Delta$</td>
<td>1.5 to 2.6</td>
</tr>
</tbody>
</table>

$n_B$. Also, $P_\Delta$ decreases as the temperature increases. This is expected, since in general as the temperature increases, in order to have a smaller contribution from the repulsive part of the potential, the ranges of the correlation functions become longer. Their derivatives also become smaller, which, in turn, causes the free energy to decrease (see Ref. 14)). This behavior is also seen in Figs. 6 and 7, which we discuss below. The above described temperature dependence is very weak in case of $\beta$-stable matter.

The $\Delta$ probability $P_\Delta$ is much larger for neutron matter than for nuclear matter. This is expected, since in nuclear matter we have a $S^{12}_{11}$ tensor correlation in the triplet spin state, while in the case of neutron matter, this correlation does not exist, i.e., we have only $T = 1$ correlations. Therefore there is more “volume” available for the intermediate $N\Delta$ state via the $S^{11}_{12}$ correlation in neutron matter.

Our results in Fig. 2 should be compared with those in Table III, in which different experimental predictions for the $\Delta$ probabilities are presented. 21), 22), 27) The mean nuclear densities of $^3$He and $^{208}$Pb are 0.1 and 0.17$\text{fm}^{-3}$, respectively. Overall agreement is seen between the experiment and theory.

In Figs. 3, 4 and 5, the ratios of proton, electron and muon densities in $\beta$-stable matter with the $\Delta$ state to the same quantities but calculated without the $\Delta$ state are plotted for various temperatures. It is found that the $\Delta$ causes the $p$, $e$ and $\mu$ abundances to decrease. This depletion is furthered as the temperature increases.

In Figs. 6 and 7 we present the calculated correlation function for the $N\Delta$ state, $-f^{(4)}_\alpha(r)$, for nuclear and neutron matter at $\mathcal{T} = 5$ and $20 \text{MeV}k_B^{-1}$, respectively. The baryon density is 0.15$\text{fm}^{-3}$. It is seen that $-f^{(4)}_\alpha(r)$ has a peak around 1$\text{fm}$ for both nuclear and neutron matter and the variation in temperature does not affect this value. As we pointed out above, $-f^{(4)}_\alpha(r)$ has a longer range and shorter peak at higher temperatures. Also, the peak itself is about 10 percent higher for neutron matter than for nuclear matter. This result confirms our previous conclusion regarding the larger contributions of $\Delta$ in neutron matter.

In Fig. 8 we present the calculated pressure for $\beta$-stable matter with and without the $\Delta$ contribution at $\mathcal{T} = 5$ and $20 \text{MeV}$. It is seen that in general the equations of state are not very sensitive to the temperature, and the inclusion of $\Delta$ does not effect the EOS significantly. As found in our previous works on neutron matter, 14) this figure shows that the $\Delta$ state makes the EOS harder for $\beta$-stable matter. However, since we have worked along an isothermal path rather than an isentropic path, other possibilities exist (see the isentropic EOS of neutron matter in Ref. 19)). We are currently studying this scenario for $\beta$-stable matter.

In this paper, we have investigated the role of $\Delta(1232)$ resonance in the neutron,
nuclear and $\beta$-stable matter. We have considered the $\Delta$ degrees of freedom explicitly in our many-body wave function. The $NN\Delta$ transition potential was used in such a manner that the whole $NN$ interaction fits the nucleon-nucleon scattering data. In this regard, our results should be considered self-consistent, without any free parameters. Our LOCV calculation confirms the idea that the $\Delta$ state could be the most important configuration that modifies the nuclear forces, and it might be fundamental in understanding three-body forces.\(^{28}\)

We have found that the $\Delta$ state in $\beta$-stable matter is insensitive to the temperature of the neutron and nuclear matter, and it is suppressed by the triplet tensor force in nuclear matter. It could be argued that the $\Delta$ state is more easily detected in neutron rich nuclei. It has been shown that our result for $P_\Delta$ is in agreement with present experimental data.

The lepton and proton abundances are affected by the isobar degrees of freedom at high densities in such a way that the lepton and proton densities decrease as the temperature increases. In this work we have only considered neutrino-free matter in $\beta$-equilibrium. Investigation of the lepton abundance is given in Ref. 19). However, our results show that the maximum electron and muon abundances are about 3% and 1% at densities around $0.6\text{ fm}^{-3}$, respectively. As pointed out above, an isentropic $\beta$-stable matter calculation is needed to obtain more realistic results. The existing neutron star models suggest proton concentrations in the range 20-30% at high densities for an isentropic calculation with entropy equal one. It has also been shown that the number of trapped neutrinos plays an important role in determining the neutron star composition.\(^{29}\) We are developing our formalism for such investigations.\(^{19}\) At this stage, we can conclude that in general it is not possible to reach the critical abundance of 15% needed for the occurrence of direct Urca processes, which are believed to be responsible for fast neutron star cooling.\(^{29}\)

We have restricted ourselves to the $s$-wave $NN \rightarrow NN\Delta$ transition potential. The contribution from other channels, as well as the $NN \rightarrow \Delta\Delta$ transition, are small, according to Refs. 2), 3) and 30), but it may affect our results at very high densities.\(^{30}\) According to Ref. 32), the $V_{28}$ potential\(^{31}\) used in Ref. 30) does not describe the low-energy data well. In addition, it is well known that the BHF approach fails at high densities. Therefore the claims regarding the effect of $\Delta$ due to other partial
waves as well as \( \Delta \Delta \) states are not reliable. We hope in future works to look into this matter in detail.

MM would like to thank Tehran University for supporting him under the grants provided by the Research Council.

References

21) G. M. Huber et al., nucl-exp/9912001.
22) E. A. Pasyuk et al., nucl-exp/9912004.
28) K. Heyde, Basic Ideas and Concepts in Nuclear Physics (IOP publishing, Bristol, 1994).