Coupled-Channels Study of the Nuclear Rainbow Phenomena for the $^{16}$O+$^{16}$O System

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(Received September 5, 2001)

We investigate refractive phenomena of the $^{16}$O+$^{16}$O system at $E_{\text{lab}} = 124, 145, 250$ and 350 MeV using the microscopic coupled-channels method. The coupling potentials are constructed by the double folding model using the three effective nucleon-nucleon interactions, M3Y, DDM3Y-ZR and DDM3Y-FR (DDM3Y1). With respect to the transition density matrix elements of the $^{16}$O nucleus, we employ the orthogonal condition model based on the microscopic $\alpha+^{12}$C cluster picture. We perform coupled-channels calculations that include both the $3^{-} (E_{\text{ex}} = 6.13 \text{ MeV})$ and the $1^{-} (E_{\text{ex}} = 7.12 \text{ MeV})$ excitations of the $^{16}$O nucleus. We show that the characteristic features of the nuclear rainbow phenomena can be reproduced successfully by our coupled-channels calculations with the DDM3Y-FR folding potential. The absolute values of the experimental cross sections for inelastic scattering at $E_{\text{lab}} = 350 \text{ MeV}$ can also be reproduced by our calculation. The coupling effects from inelastic channels are mainly absorptive and weakly repulsive in the refractive phenomena at these energies in the $^{16}$O+$^{16}$O system.

§1. Introduction

Scattering between heavy ions is usually governed by a strong absorption. In such situations, the cross sections are only sensitive to the surface region of the nuclear interaction and, therefore, the optical potential required to describe the measurements cannot be determined uniquely, especially at small internuclear distances. The last decade, however, has seen significant progress in our understanding of the optical potential that describes elastic scattering of two light heavy-ions.¹ This has mostly been a consequence of the precise and extensive measurements of elastic differential cross sections. The observation of the refractive features in elastic scattering is crucial for the unambiguous determination of optical potentials at small distances.²

The nuclear rainbow for light heavy-ion systems, which is the most remarkable example of refractive scattering, was first identified in the $^{16}$O+$^{16}$O system at $E_{\text{lab}} = 350 \text{ MeV}$.³ At backward angles in the elastic-scattering angular distribution, a pronounced minimum appears at about $\theta_{\text{cm}} = 43^\circ$ followed by a maximum at $\theta_{\text{cm}}$
= 50° and a smooth fall-off. The precise and complete measurements of elastic scattering have also been performed for the $^{16}\text{O}+^{16}\text{O}$ system at $E_{\text{lab}} = 124$, $145$, and $250$ MeV. They have shown the persistence of strong refractive features in the angular distributions.

The optical-model analyses of refractive scattering for the $^{16}\text{O}+^{16}\text{O}$ system in terms of phenomenological potentials have consistently resulted in potentials featuring a deep real part associated with a rather weak absorption. In the phenomenological studies of $^{16}\text{O}+^{16}\text{O}$ elastic scattering, a unique set of real optical potentials over a wide energy range has been determined by the observation of nuclear rainbow phenomena. The rainbow phenomena are sensitive to the depth of the real potential at small distances. At $E_{\text{lab}} = 350$ MeV, a pronounced Airy minimum, followed by a hump and an exponential rainbow tail, was clearly observed. This rainbow structure played an essential role for the unique determination of the real potential. In the optical-model analyses of the subsequent measurements for lower-energy elastic scattering at $E_{\text{lab}} = 124$, $145$ and $250$ MeV, discrete ambiguities of depth for the real potential were obtained. However, the discrete ambiguities were completely resolved by the requirement of consistency with the unique potential determined at $E_{\text{lab}} = 350$ MeV, namely a smooth continuation of the real volume integral per nucleon pair, $J_R$, from the lower-energy range to $E_{\text{lab}} = 350$ MeV.

Conventionally, folding models based on the M3Y and the density-dependent M3Y (DDM3Y) interactions have been used widely in studies of heavy-ion scattering. The folding model using the M3Y interaction had been successful in the description of low-energy heavy-ion scattering, which is dominated by peripheral collisions due to strong absorption. In this paper, we refer to this folding model as the “M3Y folding model”. The rainbow-like features seen at higher energies and larger angles, however, cannot be reproduced even qualitatively by the folding model based on density-independent interactions like M3Y. The folding potential with the M3Y interaction is too deep to reproduce the large-angle data. This is a clear indication that the effective nucleon-nucleon ($N-N$) interaction used in folding models should depend on the density of the surrounding medium. The DDM3Y interaction was proposed and first applied to the nuclear rainbow phenomena in $\alpha$-particle scattering and later also to heavy-ion scattering. In the folding model in its early stage, the exchange term was approximated by a zero-range (ZR) pseudo-potential in order to simplify the folding calculations of the exchange term. In this paper, we refer to this folding model using the DDM3Y interaction with the zero-range pseudo-potential for the exchange term as the “DDM3Y-ZR folding model”.

Recently, in microscopic folding model analyses of $^{16}\text{O}+^{16}\text{O}$ elastic scattering, successful descriptions of the refractive phenomena have been obtained by Khoa and his collaborators. This folding model is called the “generalised folding model,” because of the more exact [i.e., finite-range (FR)] treatment of the exchange term, which leads to a consistent prescription of the energy and density dependence of the folding potentials. The volume integral of the folding potentials per nucleon pair and its dependence on the incident energy are similar to those obtained in the phenomenological studies. In this paper, we refer to this folding model as the “DDM3Y-FR folding model”.
Since the first report of the DDM3Y-FR folding model, the DDM3Y-ZR folding model has not been used in analyses of refractive phenomena in heavy-ion scattering and it has been largely abandoned. No systematic study has been reported on the detailed comparison between the DDM3Y-ZR and DDM3Y-FR folding models in the description of refractive phenomena in heavy-ion scattering. In this paper, we also consider the DDM3Y-ZR folding model and apply it to analyses of the refractive phenomena in the $^{16}$O+$^{16}$O system and compare the results with those obtained by the DDM3Y-FR folding model.

In the folding-model studies of the refractive phenomena in the $^{16}$O+$^{16}$O system, all the analyses have been performed in the one-channel framework, and the coupling effects from inelastic channels have not explicitly been taken into account in these analyses. No systematic study of the coupling effect has been made to this time on the refractive phenomena in the $^{16}$O+$^{16}$O system. In the present paper, we perform a systematic study of the coupling effect from inelastic channels employing coupled-channels calculations based on the DDM3Y-ZR and DDM3Y-FR folding models in order to draw a decisive conclusion about the validity of these folding models.

In the present paper, we analyse refractive scattering of the $^{16}$O+$^{16}$O system at $E_{\text{lab}} = 124, 145, 250$ and $350$ MeV using a microscopic coupled-channels method based on the double folding model. The general wave function for scattering of two colliding nuclei with spin can be written in channel-spin representation, as

$$
\Psi^{JM}_{\alpha L S}(I_A I_a) \equiv \sum_{\gamma L' S'} X^{J \gamma L' S'}(I_A I_a, \alpha L S)(k_{\gamma}, r) \Phi^{JM}_{\gamma L' S'}(I_A I_a, J)(\hat{r}, x_A, x_a),
$$

where $\Phi^{JM}_{\gamma L' S'}(I_A I_a, J)(\hat{r}, x_A, x_a)$ is the spin-angle function. The quantity $X^{J \gamma L' S'}(I_A I_a, \alpha L S)(k_{\gamma}, r)$ is the radial wave function specified by the channel labels $\alpha, \beta, \gamma$, the total angular momentum $J$, the relative angular momentum $L$ and the channel spin $S$. The spin-angle functions are constructed through the vector coupling of the spherical harmonics, $y_{LM}(\hat{r}) = i^L Y_{LM}(\hat{r})$, and the nuclear internal wave
functions, denoted by \( \phi_{I_A \mu_A}(x_A) \) and \( \phi_{I_a \mu_a}(x_a) \). In the present case of scattering between two identical nuclei with an integer spin, the spin-angle functions require Boson symmetry \(^{18}\) with respect to the interchange of the projectile and the target as follows:

\[
\Phi_{\alpha LS(I_A I_a),J}^{M_J}(\hat{r}, x_A, x_a) = N_{I_A I_a} \left[ \begin{array}{c} [ \phi_{I_A}(x_A) \otimes \phi_{I_a}(x_a) ] S \otimes y_L(\hat{r}) \end{array} \right]_{JM_J} + \left[ [ \phi_{I_a}(x_a) \otimes \phi_{I_A}(x_A) ] S \otimes y_L(-\hat{r}) \right]_{JM_J},
\]

\[
N_{I_A I_a} = \left[ 2 \left( 1 + \delta_{I_A I_a} \delta_{\epsilon A \epsilon a} \right) \right]^{-\frac{1}{2}}.
\]

Here, \( N_{I_A I_a} \) denotes the normalisation factor which depends on the spin \( I \) and on \( \epsilon \), representing any other quantum numbers of the target and the projectile. The radial wave functions \( \chi \) satisfy the coupled-channels equations:

\[
\left\{ \frac{\hbar^2}{2\mu} \left[ \frac{d^2}{dr^2} + k_\beta^2 - \frac{L'(L' + 1)}{r^2} \right] - F_{\beta L'S'(I_A' I_a')},\beta L'S'I_A' I_a' \right\}(r) \chi_{\beta LS(I_A I_a),\alpha LS(I_A I_a)}^{J}(k_\gamma, r)
\]

\[
= \sum_{\gamma L''S'' \neq \beta L'S'} F_{\beta L'S'(I_A' I_a'),\gamma L''S''(I_A'' I_a'')}^{J}(r) \chi_{\gamma L''S''(I_A'' I_a''),\alpha LS(I_A I_a)}^{J}(k_\gamma, r).
\]

Here, \( \mu \) is the reduced mass and \( k_\beta \) is the asymptotic wave number in the \( \beta \) channel. The quantities \( F_{\beta L'S'(I_A' I_a'),\alpha LS(I_A I_a)}^{J}(r) \) represent the diagonal (\( \beta = \alpha \)) and the coupling (\( \beta \neq \alpha \)) potentials.

The coupling potentials in Eq. (4) are generated from the double folding model. Their nuclear part can be written in the form

\[
F_{\beta L'S'(I_A' I_a'),\alpha LS(I_A I_a)}^{J}(r)
\]

\[
= \int \Phi_{\beta LS'(I_A' I_a'),J}^{M_J}(\hat{r}, x_A, x_a) V(r, x_A, x_a) \Phi_{\alpha LS(I_A I_a),J}^{M_J}(\hat{r}, x_A, x_a) \, d\hat{r} \, dxdx.
\]

\[
= \sum_{\lambda} \int L' + L - \lambda \right) S' + L - J - \lambda \hat{L} \hat{L} W(S'L'SL : J \lambda) \langle L'0L0 | \lambda0 \rangle
\]

\[
\cdot 2 N_{I_A I_a} N_{I_A' I_a'} \left[ V_{\beta LS'(I_A' I_a'),\alpha S(I_A I_a)}^{\nucl}(r) + (-)^{S'} V_{\beta LS'(I_A' I_a'),\alpha S(I_A I_a)}^{\nucl}(r) \right],
\]

\[
\text{where } \hat{L} = (2L + 1)^{1/2}. \text{ Here } W(S'L'SL : J \lambda) \text{ and } \langle L'0L0 | \lambda0 \rangle \text{ are the Racah and the Clebsch-Gordan coefficients. The quantities } V_{\beta LS'(I_A' I_a'),\alpha S(I_A I_a)}^{\nucl}(r) \text{ represent the}
\]
reduced matrix elements of a tensor operator of rank $\lambda$. The reduced matrix elements in our model are given by the following explicit expression:

\[
V_{\beta S'(I'_A I'_a), \alpha S(I_A I_a)}(r) = \frac{1}{\sqrt{4\pi}} \hat{S}' \hat{S} \hat{I}'_A \hat{I}'_a \sum_{\lambda_A \lambda_a} \left\{ \begin{array}{ccc} I'_A & I'_a & S' \\ I_A & I_a & S \\ \lambda_A & \lambda_a & \lambda \end{array} \right\} \cdot \left\{ \int d\hat{r} d\hat{r}_A d\hat{r}_a v_D(\rho, s) \rho^{\lambda_A, I_A}_{ I'_A, I'_a}(r_A) \rho^{\lambda_a, I_a}_{ I'_a, I_a}(r_a) \left[ [y_{\lambda_A}(\hat{r}_A) \otimes y_{\lambda_a}(\hat{r}_a)]_\lambda \otimes y_{\lambda}(\hat{r}) \right]_00 \right. \\
+ \int d\hat{r} d\hat{t} d\hat{s} v_{EX}(\rho, s) \exp \left[ i \frac{K(r)}{\mu} \cdot s \right] \rho^{\lambda_A, I_A}_{ I'_A, I_A}(p) \hat{j}_1(k_F(p)s) \rho^{\lambda_a, I_a}_{ I'_a, I_a}(t) \hat{j}_1(k_F(t)s) \\
\left. \cdot \left[ [y_{\lambda_A}(\hat{p}) \otimes y_{\lambda_a}(\hat{t})]_\lambda \otimes y_{\lambda}(\hat{r}) \right]_00 \right\}.
\]  

(6)

In this expression we have introduced the Wigner 9-$j$ symbol. The coordinates used in the folding integrals are defined in Fig. 1. The first term in Eq. (6) is the direct term in the double folding model. The second term is the single nucleon knock-on exchange term, in which the two nucleons, which interact via $v_{EX}$, are interchanged. The single-nucleon exchange between the two nuclei is expected to be the leading correction arising from antisymmetrisation. The expression of the exchange term in Eq. (6) is that of the generalised folding model, which takes account of a more exact prescription for the single-nucleon knock-on exchange within a local-potential approximation. The quantity $K(r)$ is the local momentum of relative motion in the entrance channel. Because of the existence of the local momentum in Eq. (6), the folding calculation in the elastic channel is a self-consistent problem. The function $\hat{j}_1(x)$ represents $3(\sin x - x \cos x)/x^3$. The quantity $k_F(r)$ is the average local Fermi momentum for the density-matrix expansion. The quantities $\rho^{\lambda_A, I_A}_{ I'_A, I_A}(r_A)$ and $\rho^{\lambda_a, I_a}_{ I'_a, I_a}(r_a)$ represent the multipole components of the densities. To treat the density matrix element, we carry out a multipole expansion of the density:

\[
\rho_{I'_m', I_m}(r) \equiv \sum_{\lambda \nu} \langle I_m \lambda \nu | I'_m' \rangle \rho^{\lambda A, I_A}_{ I'_A, I_A}(r) y^{*}_{\lambda \nu}(\hat{r}).
\]  

(7)

We employ the M3Y-Reid potential for the effective $N-N$ interaction. This potential is given as follows:

\[
v_D(s) = 7999 \frac{e^{-4s}}{4s} - 2134 \frac{e^{-2.5s}}{2.5s},
\]  

(8)

\[
v_{EX}(s) = 4631 \frac{e^{-4s}}{4s} - 1787 \frac{e^{-2.5s}}{2.5s} - 7.847 \frac{e^{-0.7072s}}{0.7072s}.
\]  

(9)

In the M3Y and DDM3Y-ZR folding models, the exchange part of the effective $N-N$ interaction has been evaluated by a zero-range pseudo-potential whose strength is chosen to reproduce experimental data and has a weak energy dependence, \(^8\)

\[
v_{EX}(s) \approx -276 [1 - 0.005 (E/A)] \delta(s).
\]  

(10)
For the density dependence of the DDM3Y-ZR and DDM3Y-FR folding models, we adopt the conventional density-dependent form,\(^9\)

\[ v_{D,EX}(\rho, s) = f(\rho) \, v_{D,EX}(s), \tag{11} \]

\[ f(\rho) = C \left( 1 + a e^{-b \rho} \right), \tag{12} \]

where \(a\), \(b\) and \(C\) are parameters characterising the density dependence. In generating the folding potential, the overlapping densities \(\rho\) in Eq. (12) are assumed as

\[ \rho = \frac{1}{2} \left[ (\rho_{I'm_A'm_A} + \rho_{I'A'm_A}) + (\rho_{I'm_a'm_a} + \rho_{I'a'm_a}) \right], \tag{13} \]

where we take the diagonal densities at the position of each nucleon for the direct term and those at the midpoint between two nucleons for the exchange term. In the case of the zero-range pseudo-potential expressed in Eq. (10) is used as the exchange term, the parameter values of the density-dependent factor are taken from Fig. 1 of Ref. 9), while, in the case of the generalised folding calculation including the finite-range form of Eq. (9), the parameter values are taken from Ref. 6), where this type of density dependence is called DDM3Y1. The weak energy-dependent factor \(1 - 0.002 E_{\text{lab}}/A\) is additionally introduced into the generalised folding model. The DDM3Y1 reproduces the equilibrium density and the binding energy of normal nuclear matter within the Hartree-Fock scheme. In our paper, we refer to the density dependent M3Y interaction with the zero-range pseudo-potential as “DDM3Y-ZR” and to that with the finite-range exchange potential (DDM3Y1) as “DDM3Y-FR”.

The diagonal and transition densities of \(^{16}\)O are calculated from the internal wave functions generated by the orthogonal condition model (OCM)\(^{20}\) based on the microscopic \(\alpha^{+12}\)C cluster picture. The fundamental properties of \(^{16}\)O (i.e. binding energies, electromagnetic features and so on) have been extensively studied by OCM. In our calculations, we have included the \(0^+\) (ground state), \(3^- (E_{\text{ex}} = 6.13 \text{ MeV})\) and \(1^- (E_{\text{ex}} = 7.12 \text{ MeV})\) states of \(^{16}\)O. The \(3^-\) and \(1^-\) states are well represented by the collective excitations and they have a structure similar to that of the ground state. The existing data for electron scattering between these states can be accurately reproduced by calculations using the OCM transition densities. The calculated electric transition probability from the \(3^-\) state to the ground state in \(^{16}\)O \[i.e. B(E3; 3^- \rightarrow 0^+)\] is about 1.5 times larger than that found from the experimental data. We have modified the transition density between the \(3^-\) and \(0^+\) states to reproduce the experimental electric transition probability as well as the electron scattering data.\(^{21}\) We also assume that the transition density between the \(1^-\) and \(3^-\) states can be represented by a differential form of average between the diagonal densities of the \(3^-\) and \(1^-\) states, whose strength is adjusted to reproduce the electric transition probability, since transition densities of \(^{16}\)O are not available.\(^{20}\) We do not include the \(0^+_2 (E_{\text{ex}} = 6.05 \text{ MeV})\) and \(2^+ (E_{\text{ex}} = 6.92 \text{ MeV})\) states in our coupled-channels calculations, though these states are bound states of \(^{16}\)O in the same excitation energy region (below the \(\alpha^{+12}\)C threshold). Coupling effects from the \(0^+_2\) and \(2^+\) excitations to the elastic channel are not expected to be prominent,
since these states have an $\alpha+^{12}\text{C}$ cluster structure which differs from the structure of the ground state.

We define the full coupling potentials of Eq. (4) as follows:

$$F_{J}^{\beta L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}(r) = N_{R} F_{\beta L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}^{\nucl}(r) + F_{\beta L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}^{\text{coul}}(r) - i \left\{ \delta_{\beta\alpha} W \left[ 1 + \exp \left( \frac{r - r_I}{a_I} \right) \right]^{-1} N_I (1 - \delta_{\beta\alpha}) F_{\beta L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}^{\nucl}(r) \right\} \right. \tag{14}$$

The quantities $F_{\beta L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}^{\nucl}(r)$ represent the double folding potentials constructed by the M3Y, DDM3Y-ZR and DDM3Y-FR folding models. We refer to these potentials as M3Y, DDM3Y-ZR and DDM3Y-FR folding potentials, respectively. In Eq. (14), $N_{R}$ is a renormalisation factor of the real folding potential. The diagonal and coupling potentials obtained from the folding model consist of real parts only, since we use the M3Y-Reid potential as the effective $N-N$ interaction.

In order to express the effects of absorption, we introduce an imaginary part into the diagonal and coupling potentials. We use the conventional Woods-Saxon shape for the diagonal part. With respect to the coupling part, we assume that the imaginary potential has the same shape as the real folding potential, which is multiplied by $N_I$ as an adjustable strength factor. In first attempts, we employ simple four-parameter fits, employing $N_{R}$, $W$, $r_I$ and $a_I$, but setting $N_I = 0.0$, to simplify the optimum searches for the potential parameters. Then, we discuss the dependence of our calculation on $N_I$ in §4.4. For the search process, we use an automatic local-potential search code (ALPS)\textsuperscript{22}) to minimise the conventional $\chi^2$ values per data point in one-channel calculations. In the case of the coupled-channels calculations, we determine the “best-fit” parameters for optimum fits to the data by subjective judgement (in fact, by eye). The Coulomb part of coupling potentials, denoted by $F_{\lambda L'S'\langle I'A \rangle_{\alpha LS}(I_A I_a)}^{\text{coul}}(r)$, is calculated by a folding process similar to that used for the nuclear part, by replacing the effective $N-N$ interaction and the nucleon matter density by the Coulomb potential between two protons and the charge density. We do not include the Coulomb excitation effects, since they are not expected to play an important role in scattering between light heavy-ions in the present energy range.

§3. One-channel calculations

In this section, we investigate to what extent the refractive phenomena in the $^{16}\text{O}+^{16}\text{O}$ system can be reproduced by the one-channel calculations using the M3Y, DDM3Y-ZR and DDM3Y-FR folding potentials.

3.1. Analyses with the M3Y and DDM3Y-ZR folding potentials at $E_{\text{lab}} = 350$ MeV

In this subsection, we analyse the nuclear rainbow phenomena in $^{16}\text{O}+^{16}\text{O}$ scattering at $E_{\text{lab}} = 350$ MeV using the one-channel calculations with the M3Y and DDM3Y-ZR folding potentials. We also discuss how and why the M3Y folding po-
tential results in failure.

The angular distributions for elastic scattering at $E_{\text{lab}} = 350$ MeV in the $^{16}\text{O} + ^{16}\text{O}$ system obtained from the one-channel calculations are shown in Fig. 2. The dashed and solid curves are the results of the optical model fits to the data obtained from the one-channel calculations with the M3Y and DDM3Y-ZR folding potentials, respectively. In both cases, the renormalisation factor $N_R$ is taken to be close to unity. The parameter values corresponding to the calculations are given in Table I. The calculation with the DDM3Y-ZR folding potential reproduces the observed angular distribution fairly well in the entire angular range, while the calculation with the M3Y folding potential fails to reproduce the backward-angle data for $\theta_{\text{cm}} > 50^\circ$ as long as the $N_R$ value is kept close to unity. These results agree with those of Ref. 10.

The fit to the backward-angle data by the M3Y folding model can be improved if

Table I. The parameter values obtained by the optimum fits at the listed energies with the one-channel calculations using the M3Y, DDM3Y-ZR and DDM3Y-FR folding potentials.

<table>
<thead>
<tr>
<th>Model</th>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$N_R$</th>
<th>$W$ (MeV)</th>
<th>$r_1$ (fm)</th>
<th>$a_1$ (fm)</th>
<th>$J_R$ (MeV fm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M3Y</td>
<td>145</td>
<td>0.984</td>
<td>20.61</td>
<td>5.666</td>
<td>0.772</td>
<td>403</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>1.026</td>
<td>28.86</td>
<td>5.884</td>
<td>0.574</td>
<td>402</td>
</tr>
<tr>
<td>DDM3Y-ZR</td>
<td>145</td>
<td>1.150</td>
<td>20.21</td>
<td>5.345</td>
<td>1.172</td>
<td>344</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>1.139</td>
<td>30.30</td>
<td>5.481</td>
<td>0.886</td>
<td>332</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>1.085</td>
<td>31.38</td>
<td>5.485</td>
<td>0.747</td>
<td>301</td>
</tr>
<tr>
<td>DDM3Y-FR</td>
<td>145</td>
<td>0.916</td>
<td>14.96</td>
<td>6.222</td>
<td>0.632</td>
<td>339</td>
</tr>
<tr>
<td></td>
<td>250</td>
<td>0.963</td>
<td>28.39</td>
<td>5.710</td>
<td>0.742</td>
<td>334</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>0.923</td>
<td>30.93</td>
<td>5.631</td>
<td>0.654</td>
<td>304</td>
</tr>
</tbody>
</table>

Fig. 2. The calculated cross section for elastic scattering at $E_{\text{lab}} = 350$ MeV in the $^{16}\text{O} + ^{16}\text{O}$ system. The dashed and solid curves are the results of the optimum fits to the data obtained from the one-channel calculations with the M3Y and the DDM3Y-ZR folding potentials, respectively.

Fig. 3. The comparison between the M3Y and DDM3Y-ZR folding potentials for the $^{16}\text{O} + ^{16}\text{O}$ system at $E_{\text{lab}} = 350$ MeV. The dashed and solid curves represent the M3Y and the DDM3Y-ZR folding potentials, respectively. The dotted curve represents the unique potential of Ref. 7).
we reduce the value of $N_R$ down to about 0.6, but this results in a forward-angle
diffraction pattern completely out of phase with the experimental data (not shown in
the figure). This implies that the M3Y folding potential (with $N_R \approx 1.0$) is too deep
at short distances, although it gives a good account of the potential depth around
the nuclear surface region.

Figure 3 compares these folding potentials for this energy multiplied by the
renormalisation factor $N_R$ of Table I. The phenomenological “unique potential”
(called “A-potential” in Ref. 7)) is also shown for comparison. It is clearly seen that
the DDM3Y-ZR folding potential bears a close resemblance to the unique potential
in the entire radial range, while the M3Y folding potential is too deep at short and
medium distances. This is clear evidence that the density dependence of the effective
$N$-$N$ interaction plays an essential role in the folding procedure.

The experimental data shown in Fig. 2 have a dip at $\theta_{cm} = 43^\circ$ followed by a
maximum at $\theta_{cm} = 50^\circ$ and a smooth fall-off. The dip at $\theta_{cm} = 43^\circ$ was identified
as the first Airy minimum in the phenomenological optical-model analyses.\textsuperscript{7,23)}
The dip reproduced by the DDM3Y-ZR folding potential at the same angle is also
identified to be the first Airy minimum, while that obtained by the M3Y folding
potential is found to be the third Airy minimum. These results can be understood
from the large difference in the potential depth at short distances as is seen from
Fig. 3.

3.2. Analyses with the DDM3Y-ZR folding potentials at $E_{lab} = 124$, 145 and 250
MeV

Next, we analyse refractive scattering of the $^{16}\text{O} + ^{16}\text{O}$ system at lower energies
($E_{lab} = 124, 145$ and 250 MeV) using the one-channel calculations with the DDM3Y-
ZR folding potentials. The refractive feature also persists at these energies in the
cross sections observed at backward angles. Since the DDM3Y-ZR folding potential
with the renormalisation factor $N_R \approx 1.1$ well reproduces the nuclear rainbow at
$E_{lab} = 350$ MeV, we first impose the constraint that the $N_R$ value is taken to be
close to the value for $E_{lab} = 350$ MeV ($N_R \approx 1.1$) in the parameter-search process
at these energies. More explicitly, we allow the value of $N_R$ to vary in the range
1.1±0.05. This is to examine whether the intrinsic energy dependence carried by the
folding model itself can be consistent with the observed rainbow phenomena.

The calculated cross sections obtained by the DDM3Y-ZR folding potential are
represented by the solid curves in Fig. 4. The parameter values obtained from
the optimum fits are listed in Table I. The calculated cross sections successfully
reproduce the refractive features at the backward angles. On the other hand, the
dip positions of the forward-angle oscillatory cross sections deviate systematically
to smaller angles from the experimental ones. We have made an extensive search
for the parameter set but, as long as we impose the above stated constraint on $N_R$,
it was not possible to obtain a satisfactory fit to the forward-angle data with any
choice of parameter set for the imaginary potential ($W$, $r_I$ and $a_I$).

When we remove the constraint on $N_R$ and search for the optimum values for $N_R$
as well as the other three parameters for the imaginary potential so as to reproduce
the angular distribution in the entire angular range, we find parameter sets with
$N_R \approx 0.9$, which give good quality of fits to the data in the entire angular range for $E_{lab} = 124$ and 145 MeV (not shown in the figure). However, the smaller value of $N_R \approx 0.9$ is found to give an unacceptable result at $E_{lab} = 350$ MeV. Specifically, it fails to reproduce the characteristic rainbow feature of the cross sections at backward angles ($\theta_{cm} > 30^\circ$). Therefore, we cannot accept this family of parameter sets with $N_R \approx 0.9$.

The above results are quite consistent with the phenomenological optical-model study of elastic scattering in this system. In the phenomenological study,\(^4\),\(^5\),\(^7\) different families of potentials that reproduced the angular distributions equally well over the entire angular range were obtained. It is found that one of the families has an energy dependence of $J_R$ similar to that obtained from the present folding potential with $N_R \approx 1.1$ (given in Table I). Another family has an energy dependence similar to that of the folding potential with $N_R \approx 0.9$. This family was ruled out in phenomenological studies for the same reason, namely the failure to reproduce the nuclear rainbow at $E_{lab} = 350$ MeV. The value of the renormalisation factor $N_R \approx$

Fig. 4. The angular distributions for the $^{16}$O+$^{16}$O system at $E_{lab} = 124$, 145, 250 and 350 MeV obtained from one-channel calculations. The solid and dotted curves represent the results obtained by the DDM3Y-ZR and DDM3Y-FR folding potentials, respectively.
1.1 for the present $^{16}\text{O}+^{16}\text{O}$ system is also consistent with that for the folding-model study $^{12)}$ of the $^{12}\text{C}+^{12}\text{C}$ system with the DDM3Y-ZR folding potential.

Thus, the discrepancies between the experimental data and calculational results at forward angles shown in Fig. 4 imply that the DDM3Y-ZR folding model with $N_R \approx 1.1$ does not provide correct magnitudes of the nuclear potential around the nuclear surface region, to which the cross sections at forward angles are particularly sensitive. On the other hand, it gives a correct account of the potential depth at short distances, to which the refractive cross sections are sensitive. In other words, the radial shape of the DDM3Y-ZR folding potential is not capable of reproducing both diffractive features at forward angles and refractive features at backward angles simultaneously, although it is much better than that of the M3Y folding potential.

3.3. **Analyses with the DDM3Y-FR folding potentials**

In this subsection, we demonstrate the difference between the DDM3Y-ZR and the DDM3Y-FR folding potentials. In the one-channel calculations using the DDM3Y-FR folding potential, successful descriptions of the refractive phenomena have been reported in Ref. 6).

The calculated angular distributions at $E_{\text{lab}} = 124, 145, 250$ and 350 MeV with the DDM3Y-FR folding potential are represented by the dotted curves in Fig. 4. The parameter values for the optimum fits are listed in Table I. The refractive phenomena at the backward angles are well reproduced by the DDM3Y-FR folding potentials. In contrast with the case of the DDM3Y-ZR folding potential, the Fraunhofer-type oscillations at the forward angles are also reproduced well by this potential. In particular, the calculated dip positions for the DDM3Y-FR folding potential agree with the experimental ones much better than those for the DDM3Y-ZR folding potential.

We now investigate the cause of this difference in fits between the DDM3Y-ZR and DDM3Y-FR folding potentials. Figure 5 compares the M3Y, DDM3Y-ZR and DDM3Y-FR folding potentials at $E_{\text{lab}} = 145$ MeV. These folding potentials have been multiplied by the renormalisation factor $N_R$, given in Table I, which

![Fig. 5. The M3Y, DDM3Y-ZR and DDM3Y-FR folding potentials at $E_{\text{lab}} = 145$ MeV for the $^{16}\text{O}+^{16}\text{O}$ system. These folding potentials have been multiplied by the renormalisation factor obtained from the one-channel fit. The solid and dotted curves are the DDM3Y-ZR and DDM3Y-FR folding potentials, respectively. The M3Y folding potential is represented by the dashed curve.](https://academic.oup.com/partICLE/article-abstract/107/2/377/1879847)
is used to fit the data in the one-channel calculation. The DDM3Y-FR folding potential (represented by the dotted curve) is the same as the folding potential called “DDM3Y1” in Ref. 6), except for a difference in the nucleon density distribution of $^{16}\text{O}$ adopted. There is a slight difference in the radial shape between the DDM3Y-FR and DDM3Y-ZR (solid curve) folding potentials. This difference is much smaller than that between the M3Y (dashed curve) and these two folding potentials. The difference originates in the finite-range treatment of the exchange term [see Eq. (9)] of the effective $N$-$N$ interaction in the folding procedure. The small but non-negligible difference in the potential shape makes an appreciable improvement on the fits to the data at forward angles.

Here, we should note the difference in the $N_R$ values between the DDM3Y-ZR and DDM3Y-FR folding potentials (see Table I). As discussed above, the $N_R$ values for the DDM3Y-ZR folding model ($N_R \approx 1.1$) have been fixed by the criterion that the calculation reproduces backward-angle rainbow scattering, at the expense of poor fits to the forward-angle data. This gives $J_R$ values of 301–344 MeV fm$^3$ for $E_{\text{lab}} = 124$–350 MeV. On the other hand, similar values of $J_R$ are obtained from the optimum fits to the data by the DDM3Y-FR folding potentials with the renormalisation factor of $N_R \approx 0.94$. The DDM3Y-FR folding potentials with $N_R = 1.0$ have $J_R$ values larger than those for the DDM3Y-ZR potentials with $N_R = 1.0$ by about 15–20%, which originates from the different treatments of the exchange term. It should also be noted that the renormalised values of $J_R$ and their energy dependence for these folding potentials are quite similar to those for the phenomenological optical potentials. $^4,^5$ The DDM3Y-ZR and DDM3Y-FR folding potentials are qualitatively equivalent from the viewpoint of the proper depth of potentials for reproducing refractive scattering at backward angles. However, the more proper (i.e. finite-range) treatment of the exchange term of the $N$-$N$ interaction gives a more appropriate shape of the potential, which makes it possible to give a very precise and consistent description of elastic scattering in the entire angular range.

§4. Coupled-channels calculations

The remarkable “success” of the DDM3Y-FR folding potential in the one-channel framework should be re-examined in the coupled-channels framework, which takes account of the dynamical effects originating from non-elastic channels. In this section, we study the dynamical effects of the inelastic channels on the angular distribution of elastic scattering. We investigate whether the “success” of the one-channel description by the DDM3Y-FR folding potential remains valid in the coupled-channels framework. We also investigate whether the coupling effects of inelastic channels could improve the fits to the scattering data at forward angles in the case of the DDM3Y-ZR folding potential. For these purposes, we perform coupled-channels calculations in which the single and mutual excitations of $^{16}\text{O}$ to the $3^- (E_{\text{ex}} = 6.13 \text{MeV})$ and $1^- (E_{\text{ex}} = 7.12 \text{MeV})$ excited states are taken into account. To save computational time, the mutual excitation to the $1^-$ state is excluded, since the coupling of this channel to elastic scattering is expected to be small in the present energy region.
4.1. **Coupled-channels calculations with the DDM3Y-FR folding potentials**

We first investigate the effects of channel coupling on the refractive phenomena at $E_{\text{lab}} = 124, 145, 250$ and $350$ MeV using coupled-channels calculations with the DDM3Y-FR folding potentials. Here, we take the imaginary strength factor of the coupling potentials $N_I$ of Eq. (14) to zero; that is, only the diagonal potentials have an imaginary part of the Woods-Saxon form. The role of the imaginary part of the coupling potentials will be discussed in §4.4. We perform coupled-channels calculations and search for optimum fits (by eye) to the data by adjusting the three parameters of the imaginary potential together with the renormalisation factor $N_R$ for the folding potential.

The “best-fit” angular distributions obtained from the coupled-channels calculations with the DDM3Y-FR folding potentials are represented by the solid curves in Fig. 6. The calculated cross sections reproduce the experimental data successfully over the entire angular range, as in the case of the one-channel calculations discussed in §3.3, which are represented by the dotted curves in Fig. 6. The descriptions of refractive scattering remain good in the presence of the coupling effects from inelastic channels and the fit to the data is even improved in some cases. The parameter values for the optimum fits by the coupled-channels calculations are listed in the lower part of Table II. It should be noted that the parameter values are different from those for the one-channel calculations listed in Table I. The small difference in the calculated angular distributions between the one-channel and coupled-channels results shown in Fig. 6 does not necessarily mean that the coupling effect is small. The main difference is seen in the imaginary part, which suggests that the coupling effect is mainly an absorption. It should be also noted that the renormalisation factor $N_R$ for the real folding potential has been changed slightly from the values for the one-channel calculations. The amount of this change is less than 2% at all energies, which, however, is essential in reproducing the right positions of the Airy minima in the observed angular distributions. This is a clear example demonstrating how the refractive features of the cross section are sensitive to small changes in the real potential depth. The small changes (less than 2%) of the $N_R$ values suggest that the channel coupling has only a minor effect on the real part of the potential. The coupling effects in terms of the dynamic polarisation potential are discussed in

### Table II. The parameter values obtained by the optimum fits at the listed energies with the coupled-channels calculations using the DDM3Y-ZR and DDM3Y-FR folding potentials.

<table>
<thead>
<tr>
<th>Model</th>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>$N_R$</th>
<th>$W$ (MeV)</th>
<th>$r_1$ (fm)</th>
<th>$a_1$ (fm)</th>
<th>$J_R$ (MeV fm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC with 124</td>
<td>1.135</td>
<td>16.0</td>
<td>5.65</td>
<td>0.711</td>
<td></td>
<td>342</td>
</tr>
<tr>
<td>DDM3Y-ZR 250</td>
<td>1.160</td>
<td>21.5</td>
<td>5.00</td>
<td>1.000</td>
<td></td>
<td>349</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>1.105</td>
<td>29.5</td>
<td>5.40</td>
<td>0.730</td>
<td>307</td>
</tr>
<tr>
<td>CC with 124</td>
<td>0.930</td>
<td>15.5</td>
<td>5.67</td>
<td>0.710</td>
<td></td>
<td>344</td>
</tr>
<tr>
<td>DDM3Y-FR 250</td>
<td>0.980</td>
<td>26.0</td>
<td>5.58</td>
<td>0.740</td>
<td></td>
<td>340</td>
</tr>
<tr>
<td></td>
<td>350</td>
<td>0.945</td>
<td>28.8</td>
<td>5.54</td>
<td>0.654</td>
<td>311</td>
</tr>
</tbody>
</table>
detail in the next subsection.

In order to examine the validity of the present coupled-channels calculations with the DDM3Y-FR folding potential, we also analyse the cross sections for inelastic scattering. Figure 7 shows the cross sections for inelastic scattering to the 3− state at $E_{\text{lab}} = 350$ MeV obtained from the same coupled-channels calculation as that used to obtain Fig. 6. It should be noted that the data plotted in Fig. 7 are not pure 3− inelastic cross sections. They contain both the 3− ($E_{\text{ex}} = 6.13$ MeV) and

![Diagram showing angular distributions for the 16O+16O system at various energies.](https://example.com/diagram.png)

Fig. 6. The angular distributions for the 16O+16O system at $E_{\text{lab}} = 124, 145, 250$ and 350 MeV obtained from the one-channel and coupled-channels calculations with the DDM3Y-FR folding potentials (Tables I and II). The solid and dashed curves represent the results of the coupled-channels and one-channel calculations, respectively.
Fig. 7. The differential cross section for inelastic scattering of the $^{16}\text{O}+^{16}\text{O}$ system at $E_{\text{lab}} = 350$ MeV. The experimental data\textsuperscript{24)} are the summed intensities of the $3^-_1$ and $2^+_1$ excitations. The solid curve represents the calculated inelastic cross section for the $3^-_1$ excitation with the DDM3Y-FR folding potential.

$2^+_1(E_{\text{ex}} = 6.92$ MeV) excitations.\textsuperscript{24)} The transition between the $0^+_1$ (ground) and $2^+_1$ states is expected to be weak because of the large difference in nuclear structure between the two states, as mentioned in §2. Thus, we have not included the $2^+_1$ state in the present coupled-channels calculations and in Fig. 7 we simply compare the experimental data with the calculated cross section for inelastic scattering to the $3^-_1$ state. The calculation reproduces the magnitude of the observed cross section at forward angles that carries the dominant fraction of the total inelastic cross section to the $3^-_1$ state. It should be noted that the observed cross section for inelastic scattering has less distinct diffraction minima in the angular distribution at forward angles than the calculated one. This may partly be due to contributions from the $2^+_1$ state ignored in the calculation, since the diffractive pattern of the cross section for the $2^+_1$ state should be out of phase with that for the $3^-_1$ state. It is seen that the calculated cross section is greater than the experimental values at large angles. Similar deviation from the data at large angles was also reported in a previous DWBA analysis\textsuperscript{24)} of the same data. The origin of the discrepancy in this angular range was not clarified in Ref. 24), and it has not been found in the present work either.

4.2. Effects of channel coupling

We have found the successful description of the refractive phenomena in the $^{16}\text{O}+^{16}\text{O}$ system with the coupled-channels calculation using the DDM3Y-FR folding potential. In this subsection, we discuss the effects of channel coupling in terms of the dynamic polarisation potential.
In order to see the net coupling effects explicitly, we perform one-channel calculations using the same potential parameters ($W$, $r_I$, $a_I$ and $N_R$) as those used in the coupled-channels calculations (solid curves in Fig. 6) discussed in the previous subsection, namely the parameter values labelled “CC with DDM3Y-FR” in Table II. The results of the one-channel and coupled-channels calculations using the same parameter values are compared in Fig. 8 for $E_{\text{lab}} = 145$ MeV. In both calculations, we have switched off the Boson symmetry between the two identical $^{16}$O nuclei in order to clarify the characteristic angular distribution of the nuclear rainbow, i.e. the Airy minima. Note that the introduction of the Boson symmetry gives rise to additional oscillations around $\theta_{\text{cm}} = 90^\circ$, which makes it difficult to clearly identify the true Airy minima.

The large difference between the dotted curve (one-channel) and the solid curve (coupled-channels) in Fig. 8 shows the effects of coupling to the inelastic channels. The cross section obtained from the coupled-channels calculation is smaller than that obtained from the one-channel calculation for $\theta_{\text{cm}} > 25^\circ$. This may imply that the coupling to the inelastic channels induces an additional absorption in the elastic channel. It is also seen that the positions of the Airy minima at $\theta \sim 53^\circ$ and $\sim 73^\circ$ are shifted forward by $\Delta \theta_{\text{cm}} \sim 2-4^\circ$ under the influence of the channel coupling. The angular location of an Airy minimum is very sensitive to the real potential depth and it shifts forward when the real potential becomes shallower. Thus, the forward shift of the minima seen in Fig. 8 suggests that the coupling from the inelastic channels has an effect of reducing the real potential depth by a small amount, that is, a weakly repulsive effect. The coupling effects at other energies, $E_{\text{lab}} = 124, 250$ and $350$ MeV, are found to be quite similar to those at $E_{\text{lab}} = 145$ MeV.

In order to investigate the above-mentioned nature of the coupling effects in a quantitative manner, we evaluate the dynamic polarisation potential which simulates the coupling effect from the inelastic channels in terms of an effective local potential. To this end, we search for the “optical potential”, $U_{\text{opt}}(r)$, which precisely reproduces the coupled-channels result of the elastic cross sections (i.e. the solid curve in Fig. 8) in the one-channel framework. More precisely, we regard the elastic cross sections obtained from the coupled-channels calculation as virtual experimental data and search for the optical potential $U_{\text{opt}}(r)$ which gives an optimum fit to the virtual data.

Here, we assume that the form factors for $U_{\text{opt}}(r)$ are exactly the same as those used in the one-channel study in §3, i.e. the real part given by the DDM3Y-FR folding potential multiplied by a renormalisation factor $N_R$ and the imaginary part with the three-parameter Woods-Saxon form. We then search for the set of parameter values which give an optimum fit to the virtual data (i.e. the coupled-channels result). The resulting parameter values of the optical potential $U_{\text{opt}}(r)$ are found to be very close to those given in Table I (labelled by DDM3Y-FR) obtained from the one-channel fit to the real experimental data. This is quite reasonable, because the coupled-channels calculation (regarded as the virtual data) itself reproduces the real experimental data very well, as shown in Fig. 6.
Now, we define the dynamic polarisation potential as follows:

$$\Delta U(r) = U_{\text{opt}}(r) - U_{\text{bare}}(r).$$  (15)

The quantity $U_{\text{bare}}(r)$ represents the “bare potential”, which gives the dotted curve in Fig. 8 in the one-channel calculation and is the same potential as that used in the coupled-channels calculation, whose parameters are given in Table II, as mentioned above. Since the optical potential $U_{\text{opt}}(r)$ reproduces well the coupled-channels result (the solid curve in Fig. 8), the dynamic polarisation potential $\Delta U(r)$ simulates the difference between the dotted curve and the solid curve in Fig. 8, which represents the coupling effects of the inelastic channels.

Figure 9 shows the dynamic polarisation potential $\Delta U(r)$ for scattering at $E_{\text{lab}} = 145$ MeV. The bold-solid and bold-dotted curves represent the imaginary and real parts of the dynamic polarisation potential, respectively. For comparison, the imaginary and real parts of the bare potential $U_{\text{bare}}(r)$ are represented by the thin-solid and thin-dotted curves in the same figure. It should be noted that the real part of $\Delta U(r)$ has the same radial form as that of the folding potential, since the real parts of $U_{\text{opt}}(r)$ and $U_{\text{bare}}(r)$ are assumed to have the same form factor as that of the folding potential in the present analysis.

The imaginary part of $\Delta U(r)$ has a surface-peaked shape with a negative sign.

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Fig. 8. The unsymmetrised cross section at $E_{\text{lab}} = 145$ MeV in the $^{16}\text{O}+^{16}\text{O}$ system. The solid curve is the result obtained from the coupled-channels calculation with the DDM3Y-FR folding potential. The dotted curve represents the schematic one-channel calculation with the same parameters as those used in obtaining the solid curve.

Fig. 9. The comparison between the dynamic polarisation potential and the bare potential at $E_{\text{lab}} = 145$ MeV in the $^{16}\text{O}+^{16}\text{O}$ system. The bold-solid and bold-dotted curves are the imaginary and real parts of the dynamic polarisation potential, respectively. The imaginary and real parts of the bare potential are represented by the thin-solid and thin-dotted curves.
At the peak position around $R = 5$ fm, its magnitude is greater than one third of the imaginary part of the bare potential. This indicates that the coupling to the inelastic channels induces a substantial amount of additional absorption in the surface region of the potential. Contrastingly, the real part of $\Delta U(r)$ has a positive sign with a strength of about 2% of the real part of the bare potential, indicating a weakly repulsive nature of the channel coupling effect. The small repulsive effect, however, gives rise to a forward shift of Airy minima by $\Delta \theta_{cm} \sim 2–4^\circ$, as is mentioned above.

4.3. Coupled-channels calculations with the DDM3Y-ZR folding potentials

Next, we consider the same kind of coupled-channels calculations using the DDM3Y-ZR folding potential, in place of DDM3Y-FR, in order to see whether the poor fits to the data at forward angles in the one-channel calculations with the DDM3Y-ZR folding potentials (the solid curves in Fig. 4) could be improved by including the channel coupling.

The “best-fit” results of the coupled-channels calculations using the DDM3Y-ZR folding potentials are represented by the solid curves in Fig. 10. For comparison, the one-channel results are represented by the dotted curves, which are the same as the solid curves in Fig. 4. The parameter values used in the calculations are listed in the upper part of Table II, labelled “CC with DDM3Y-ZR”. These parameters are obtained in the same manner as in the case of the coupled-channels calculations.

![Fig. 10. Same as Fig. 6, but with the DDM3Y-ZR folding potentials.](https://academic.oup.com/ptp/article-abstract/107/2/377/1879847)

Fig. 10. Same as Fig. 6, but with the DDM3Y-ZR folding potentials.
using the DDM3Y-FR folding potentials; that is, we have searched for the potential parameters which give optimum fits (by eye) to the data. Proper fits to the large-angle data are essential in order to fix the value of the renormalisation factor $N_R$ for the folding potential. The resultant values of $N_R$ are close to those of the one-channel calculations listed in Table I. It should be noted that an extra dip appears at $\theta_{\text{cm}} \sim 63^\circ$ in the case of the coupled-channels calculation at $E_{\text{lab}} = 350$ MeV. This dip disappears as the imaginary coupling strength $N_I$ increases (see Fig. 12). This dip does not correspond to the Airy minimum but originates from the effects of channel coupling. The position of the first Airy minimum remains the same as that for the one-channel case, i.e. at $\theta_{\text{cm}} = 43^\circ$.

As seen in the figure, the resultant angular distributions obtained from the coupled-channels calculations are close to those obtained from the one-channel calculations, except for minor differences at the middle and backward angles. No visible improvement of fits to the experimental data at forward angles has been obtained in the coupled-channels calculations. The major difference of the parameter values between the one-channel and coupled-channels calculations exists in the imaginary part, as seen in Tables I and II. This is similar to the situation seen in the case of the DDM3Y-FR folding potential, as discussed in the previous subsection. This is a reasonable result, because the characteristic features of the reaction dynamics, such as effects of coupling to the inelastic channels, should not depend strongly upon a small difference between the basic interactions, such as DDM3Y-FR and DDM3Y-ZR. Therefore, the failure of a consistent description of elastic scattering suggests that the DDM3Y-ZR folding model has a crucial problem in predicting the precise shape of the real potential between composite nuclei.

4.4. Effects of imaginary coupling on the cross sections

Before drawing a conclusion on the validity of these folding models, we investigate another aspect of the dynamical effect ignored in the above-mentioned coupled-channels analyses. This concerns the imaginary part of the coupling potentials, which is known to affect the basic properties of the coupling effect or, in other words, the nature of the dynamic polarisation potentials. In this subsection, we investigate the effects of the imaginary part of the coupling potentials, represented by the parameter $N_I$ in the present coupled-channels framework [see Eq. (14)], on the elastic and inelastic cross sections. All the calculations in this subsection are based on the DDM3Y-ZR folding model. We investigate whether a consistent description of the elastic and inelastic cross sections can be obtained from the coupled-channels calculation with the DDM3Y-ZR folding potential using a non-zero value of $N_I$. To clarify the effect of $N_I$ on the cross sections, we perform schematic coupled-channels calculations in which only the $N_I$ value is changed, with all other parameter values unchanged. Here, we adopt positive values for $N_I$, which correspond to an absorption.

First, we examine the $N_I$ dependence of the elastic cross section at $E_{\text{lab}} = 350$ MeV. The cross section at forward and backward angles calculated with different values of $N_I$ is shown in Figs. 11 and 12, respectively. The calculation with $N_I = 0.0$ (the bold-solid curve) is the same as that shown in Fig. 10.
In Fig. 11, it is seen that the peak and dip positions of the Fraunhofer-type oscillation in this forward-angle region shift to larger angles and that the peak heights decrease as the value of $N_I$ increases. The Fraunhofer-type oscillation at forward angles is known to be sensitive to the real potential in the nuclear surface region. This $N_I$ dependence of the cross section at forward angles suggests that the imaginary coupling affects the real part of the dynamic polarisation potential in the nuclear surface region. In fact, the shift of the peak and dip positions induced by the introduction of non-zero values of $N_I$ corresponds to a reduction of the real potential strength around the nuclear surface by a substantial amount. For example, in order to reproduce the angular shift induced by the change of $N_I$ from 0.0 to 0.8, the real potential strength in the surface region needs to be reduced by about 30%.

In comparison to the forward-angle case, the effect of $N_I$ on the backward-angle cross section looks rather complicated, as seen in Fig. 12. One should, however, note that the position of the Airy-minimum at $\theta_{cm} \simeq 43^\circ$ is almost unchanged, despite the drastic change of $N_I$ from 0.0 to 0.8. It shifts toward larger angles slightly (by $\Delta \theta_{cm} \approx 2^\circ$) as $N_I$ increases from 0.0 to 0.8. The position of the Airy minimum is known to be very sensitive to the real potential depth. In fact, the same amount of shift as above occurs if we change the value of $N_R$ by only a few percent. This is in striking contrast to the large change in the $N_I$ value. This result suggests that the real potential at short distances is not strongly affected by the introduction of an

![Fig. 11](https://example.com/fig11.png)

**Fig. 11.** The $N_I$ dependence of the elastic-scattering cross section for forward angles at $E_{lab} = 350$ MeV, using the coupled-channels calculation with the DDM3Y-ZR folding potential.

![Fig. 12](https://example.com/fig12.png)

**Fig. 12.** The $N_I$ dependence of the elastic-scattering cross section for backward angles at $E_{lab} = 350$ MeV, using the coupled-channels calculation with the DDM3Y-ZR folding potential.
imaginary part to the coupling potentials in terms of $N_I$.

In this way, the introduction of the imaginary part in the coupling potentials gives rise to an appreciable shift of the forward-angle diffraction pattern without changing the position of the Airy minimum at backward angles. This effect of $N_I$ on the elastic angular distributions seems favourable for improving the poor fits to the data at $E_{\text{lab}} = 145$ and 250 MeV in the forward-angle region, shown in Fig. 10, where the calculations with the DDM3Y-ZR folding potential deviate from the data in the forward-angle region. However, this is not necessarily the case for elastic scattering at $E_{\text{lab}} = 350$ MeV. The calculation with $N_I = 0.0$ already gives a reasonable fit to the data at this energy, as shown in Fig. 10. Furthermore, the coupled-channels calculations with the DDM3Y-FR folding potential give almost perfect fits to the data for elastic scattering at all energies, as seen in Fig. 6, and there seems no room for introducing any additional parameter for the imaginary coupling terms. In this way, the situation depends on the incident energy and the interaction adopted in the analysis. It is rather difficult to judge only from the analysis of elastic scattering whether we should introduce an imaginary part into the coupling potentials.

In order to obtain information that can help to solve this problem, we investigate the effect of imaginary coupling on inelastic cross sections. The coupled-channels calculations are performed using the DDM3Y-ZR folding potential. Figure 13 compares the cross sections calculated with $N_I = 0.0$ and 0.8 for elastic and inelastic scattering at $E_{\text{lab}} = 350$ MeV. The solid and dashed curves are the results of the coupled-channels calculations with $N_I = 0.0$ and 0.8, respectively. The experimental data are also shown in the figure. Since the schematic calculation with $N_I = 0.8$ (shown in Figs. 11 and 12) apparently deviates from the elastic-scattering data, we have readjusted other potential parameters ($N_R$, $W$, $r_I$ and $a_I$) so that the calculation with $N_I = 0.8$ reproduces the elastic-scattering data with a quality similar to that in the case with $N_I = 0.0$. It should be noted that the upper part of Fig. 13 shows not the schematic result but the improved fit to the elastic data. Both calculations reproduce the angular location and the cross section of the Airy minimum observed at $\theta_{\text{cm}} = 43^\circ$. The parameter values thus obtained are $N_R = 1.055$, $W = 34.5$ MeV, $r_I = 5.15$ fm and $a_I = 0.670$ fm for the calculation with $N_I = 0.8$. In Fig. 13, the forward-angle parts of the inelastic cross section are magnified and are displayed in the inset. It is seen that the absolute value of the inelastic cross section obtained from the calculation with $N_I = 0.8$ is larger than that with $N_I = 0.0$ by a factor of about 2, which leads to values larger than the experimental cross sections.

This information suggests that we can put a constraint on the $N_I$ value from the simultaneous analysis of elastic and inelastic scattering in the coupled-channels calculation. In the present case of the $^{16}\text{O}+^{16}\text{O}$ system at $E_{\text{lab}} = 350$ MeV, large values of $N_I$ cannot be accepted for a consistent description of elastic and inelastic scattering by the coupled-channels calculation with the DDM3Y-ZR folding potential. This should also be the case for the coupled-channels calculation with the DDM3Y-FR folding potential, by which a consistent description of elastic and inelastic scattering has been obtained without invoking the imaginary coupling terms, as already seen in Fig. 6.

As we have seen, the analyses of $^{16}\text{O}+^{16}\text{O}$ scattering at $E_{\text{lab}} = 350$ MeV sup-
Fig. 13. The optimum fits obtained from the coupled-channels calculation with the DDM3Y-ZR folding potential for elastic and inelastic scattering at $E_{\text{lab}} = 350$ MeV. The solid curves are the results with $N_I = 0.0$. The dashed curves are the results with $N_I = 0.8$. The cross section for inelastic scattering at forward angles is magnified in the small panel.

port the use of a small or even vanishing value of $N_I$ for a consistent description of elastic and inelastic scattering by the coupled-channels calculations with either the DDM3Y-ZR or DDM3Y-FR folding potentials. It is a non-trivial problem to determine whether this is also the case for lower-energy scattering at $E_{\text{lab}} = 124$, 145 and 250 MeV, because no experimental data for inelastic scattering exist for these incident energies. In fact, as long as a vanishing value of $N_I$ is adopted, the coupled-channels calculation with the DDM3Y-ZR folding potential fails to reproduce the forward-angle data of elastic scattering at these energies, and the use of large values of $N_I$ seems to be favourable for improving the poor fits at the forward angles, as mentioned above. By contrast, the coupled-channels calculations with the DDM3Y-FR folding potentials reproduce the elastic-scattering data at these energies without introducing any imaginary coupling terms, i.e. with $N_I = 0.0$. The lack of inelastic-scattering data at these low energies prevents us from further testing the
optimum values of $N_I$ to be used at these energies.

These results may indicate the failure of the DDM3Y-ZR folding model and the success of the DDM3Y-FR folding model in predicting the proper shape and energy dependence of the nuclear interaction for the $^{16}$O+$^{16}$O system. A small or vanishing value of $N_I$ would be reasonable for the present $^{16}$O+$^{16}$O scattering system, because the $^{16}$O nucleus is one of the most stable double-closed-shell nuclei, and the observation of the refractive phenomena represented by the nuclear rainbow is the best indication of the weak-absorption nature of this scattering system. Hence, we have no positive result giving a reason to introduce any strong imaginary part artificially in the coupling potential in the $^{16}$O+$^{16}$O system.

§ 5. Summary

In the present paper, we have analysed refractive phenomena observed in $^{16}$O+$^{16}$O scattering at $E_{\text{lab}} = 124, 145, 250$ and 350 MeV by using the microscopic coupled-channels method based on the double folding model. In the coupled-channels calculation, we include coupling to the $3^-_1$ ($E_{\text{ex}} = 6.13$ MeV) and $1^-_1$ ($E_{\text{ex}} = 7.12$ MeV) excited states of the $^{16}$O nucleus. We have tested various folding models based on the M3Y, DDM3Y-ZR and DDM3Y-FR effective interactions through analyses of nuclear rainbow phenomena observed in elastic scattering. Inelastic scattering to the $3^-_1$ state was also analysed at the highest incident energy.

We first demonstrated the failure of the M3Y folding model in reproducing the nuclear rainbow at $E_{\text{lab}} = 350$ MeV. The M3Y folding potential is too deep to reproduce the data at backward angles. We have also shown that the DDM3Y-ZR folding model, which has been used widely in microscopic studies of heavy-ion scattering, fails to give a consistent description of elastic scattering over the entire range of scattering angles at lower energies, although it gives a good fit to the data at $E_{\text{lab}} = 350$ MeV. At the lower energies, the radial shape of the DDM3Y-ZR folding potential is not capable of providing a consistent description of the Fraunhofer-type oscillation at forward angles and the refractive features at backward angles, although the DDM3Y-ZR folding model is much better than the M3Y folding model which ignores the density dependence of the effective $N$-$N$ interaction.

We have reconfirmed the successful description of elastic scattering given by the one-channel calculation with the DDM3Y-FR folding potential, which was reported in Ref. 6). We have discussed the difference between the DDM3Y-FR and DDM3Y-ZR folding potentials in some detail. It was shown that there exists a slight difference in the radial shape between the folding potentials, which originates from the different treatments of the exchange part of the effective $N$-$N$ interaction in the folding procedure, namely the finite-range and the zero-range treatments. The slight but non-negligible difference in the radial shape leads to an appreciable difference between calculated cross sections over the entire range of scattering angles. The DDM3Y-FR folding model gives more-successful descriptions of refractive scattering than the DDM3Y-ZR one, especially at lower energies.

Next, we investigated the coupling effects from inelastic channels on the refractive phenomena. It was shown that the coupled-channels calculation with the
DDM3Y-FR folding potential consistently reproduces the cross section for inelastic scattering (Fig. 7), as well as the characteristic features of the nuclear rainbow phenomena for elastic scattering (Fig. 6). The successful descriptions of refractive scattering with the DDM3Y-FR folding model are maintained when the influence of the coupling effects from inelastic channels is included. It was shown that the coupling effect gives rise to an appreciable amount of surface absorption, while the effect on the real potential is rather small in the present scattering system.

By contrast, we found that the difference between the data and calculational results at forward angles encountered in the one-channel calculation with the DDM3Y-ZR folding potential cannot be removed by the coupling effects from inelastic channels. We have investigated the role of the imaginary part of the coupling potentials represented by the parameter value of $N_I$ in the present coupled-channels framework. The introduction of the imaginary coupling gives rise to an appreciable effect on the elastic scattering mainly at forward angles. This appears to be a favourable effect for improving the poor fits to the low-energy data in the calculations with the DDM3Y-ZR folding potential. However, the magnitude of the inelastic scattering cross section becomes larger as $N_I$ increases, which leads to values larger than the experimental cross sections. As a result, we cannot obtain a consistent description of elastic and inelastic scattering using the coupled-channels calculation with the DDM3Y-ZR folding potential for any value of $N_I$ for the imaginary coupling.

Recently, angular distributions of $^{16}\text{O} + ^{16}\text{O}$ elastic scattering have been measured and analysed with the phenomenological optical model by Nicoli et al.\textsuperscript{25) at nine energies between $E_{\text{lab}} = 75$ and 124 MeV. Below $E_{\text{lab}} = 75$ MeV, extensive resonance data from the Yale group\textsuperscript{26) exist for this system. It is of interest to study the resonant phenomena through an extension of the present analyses in refractive phenomena for the lower-energy region.

Acknowledgements

We thank Dr. F. C. Barker for a careful reading of the manuscript. We would like to express our sincere thanks to Dr. H. G. Bohlen and Dr. D. T. Khoa for providing us with the $^{16}\text{O} + ^{16}\text{O}$ experimental data in numerical form.

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