The Binding Mechanism and a Low-Lying Resonance in $^{11}$Li

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Using a new $^9$Li-$n$ interaction taking into account the $^9$Li-core excitation, it is shown that the $^9$Li+$n$ model reproduces the observed binding energy of $^{11}$Li. This $^9$Li-$n$ interaction naturally explains the degeneracy of single particle $s_{1/2}$- and $p_{1/2}$-orbital states above the $^9$Li+$n$ threshold. Furthermore, by using the $^9$Li-$n$ interaction, a low-lying excited resonant state is obtained in $^{11}$Li together with the ground state, and they are found to be partners in mixed configurations of $(0p_{1/2})^2$ and $(1s_{1/2})^2$.

§1. Introduction

Recently, much attention has been concentrated on the study of neutron-rich nuclei. The first motivation for studying neutron-rich nuclei was provided by the observation of the large interaction cross-section of $^{11}$Li. It has been discussed in many works that the large interaction cross-section can be explained in terms of the so-called halo structure, which is a cloud of valence neutrons around the core nucleus.

Early in the study of the exotic structure of $^{11}$Li, calculations based on the $^{9}$Li+$n+n$ three-body model were carried out. In these models, the $^{9}$Li-$n$ interaction is chosen so as to reproduce the experimental $p$-wave resonance energy of $^{10}$Li at $E_r \sim 0.5$ MeV. However, despite the success of this model in explaining the halo structure of $^{11}$Li, the binding energy of $^{11}$Li is not reproduced. For example, in the cluster orbital shell model (COSM), which is one of the pioneering $^{9}$Li+$n+n$ models, the binding energy is too weak at least 1–2 MeV. Later, Thompson and Zhukov proposed a possibility of a virtual state, which is a low-lying $s$-state, in the $^{9}$Li+$n$ subsystem. Using a $^9$Li-$n$ potential with a strong state dependence, which produces the virtual state, they explained the experimental binding energy of $^{11}$Li and also the $^{9}$Li momentum distributions. The essential point of their model is that the $s$- and $p$-states are almost energetically degenerate near the $^9$Li+$n$ threshold. This model is different from the previous model which assumes only the presence of low-lying $p$-states. When there is such a virtual state, since the state of two neutrons in the $s$-orbit can strongly couple to those in the $p$-state, the ground state should have the mixed configuration $(s_{1/2})^2+(p_{1/2})^2$. However, in the analyses of Ref. 6), the mechanism of such a strong state dependence of the $^9$Li-$n$ interaction, which causes
the shell gap between the $p$-shell and the $sd$-shell to disappear, was introduced empirically, but was not explained. Experimentally, this mixed configuration of the large $s$-wave weight in the ground state of $^{11}$Li has been observed.\textsuperscript{7)}

The $^{9}$Li+$n$ subsystem of $^{11}$Li so far has been investigated through many experimental\textsuperscript{8)}-\textsuperscript{14)} and theoretical\textsuperscript{6)}-\textsuperscript{18)} studies, but the conclusion is still ambiguous even for the ground state.\textsuperscript{19)} Several experiments suggest an $s_{1/2}$-orbital configuration as the ground state of $^{10}$Li at $E \sim 0.0$ MeV.\textsuperscript{8)}-\textsuperscript{10)} These experiments support the existence of the virtual state in the $^{9}$Li+$n$ subsystem. On the other hand, several experiments suggest presence of a $p_{1/2}$-orbital configuration as the ground state of $^{10}$Li at $E \sim 0.5$ MeV.\textsuperscript{11)}-\textsuperscript{13)} Very recently, it was observed that both the $s$- and $p$-states are very low lying around the $^{9}$Li+$n$ threshold: $E^{s\text{-state}}$ is less than 0.05 MeV and $E^{p\text{-state}}$ is around 0.5 MeV.\textsuperscript{14)} This situation is very similar to that proposed by Thompson and Zhukov in order to explain the binding mechanism of the $^{9}$Li+$n$+$n$ system.\textsuperscript{6)}

Among $N = 7$ isotones, $^{11}$Be is well known as a nucleus that has an intruder $s$-orbital ground state. For $^{11}$Be, it may be possible to explain the parity inversion of $1s$- and $0p$-orbits by taking into account the coupling between $1s$- and $0d$-orbits, together with the deformation of the core nucleus.\textsuperscript{20)}-\textsuperscript{23)} However, we cannot use such an explanation for parity inversion in the $^{10}$Li nucleus, where the core deformation is considered to be not large, if it exists at all, contrary to the $^{10}$Be case. Then, the problem is to understand why the $s$-orbital state is low lying near the $^{9}$Li+$n$ threshold, if it is a physically meaningful situation. In a recent experiment for another $N = 7$ isotope, $^{9}$He, it was also shown that the $s$-orbital energy is lower than the $p$-orbital energy.\textsuperscript{14)} In the shell model picture, by considering the so-called pairing blocking effect, this systematic behavior of the parity inversion appearing in the $N = 7$ isotones was predicted by Sagawa et al.\textsuperscript{24)}

Recently, Katō, Yamada and Ikeda constructed a new $^{9}$Li-$n$ potential with a coupled channel model\textsuperscript{25)} in order to take into account the pairing blocking effect proposed by Sagawa et al.\textsuperscript{24)} In this model, they took into account the effect of the pairing-correlation of neutrons and the excited configuration from the $p_{3/2}$-orbit to the $p_{1/2}$-orbit in the $^{9}$Li-core. When a valence neutron is added to the $^{9}$Li-core in $^{10}$Li, the valence neutron is partly blocked from entering the $p_{1/2}$-orbit due to the Pauli principle, and in this sense, the $p$-wave potential of the $^{9}$Li-$n$ interaction receives a repulsive effect. However, the neutron’s $(s_{1/2})^{2}$ component in the $^{9}$Li-core is believed to be very small. Therefore, the repulsive effect from the Pauli blocking on the $s$-wave potential is weaker than in the case of the $p$-wave. As a result, the energy of the $s$-orbit in the simple shell model picture comes down to the same region as that of the $p$-orbit in $^{10}$Li.\textsuperscript{24), 25)}

As another typical core+$n$+$n$ system having a halo structure, $^{6}$He has been studied in detail with the Hybrid-$TV$ model.\textsuperscript{5)} The Hybrid-$TV$ model is one of core+$n$+$n$ models and may be the best method to treat the shell model picture.\textsuperscript{5)} Applying the $^{4}$He+$n$+$n$ model for $^{6}$He is expected to be more valid than applying the $^{9}$Li+$n$+$n$ model for $^{11}$Li, because the $^{4}$He-core is much stabler than the $^{9}$Li-core. The ground state properties of $^{6}$He are explained very well with the simple $^{4}$He+$n$+$n$ model.\textsuperscript{26)}
Furthermore, the excited resonant state of $^{6}\text{He}$ can be reproduced by applying the complex scaling method (CSM)\(^ {27}\) to the $^{4}\text{He}+n+n$ model either without or with the effective three-body interaction.\(^ {26,28}\)

For excited resonant states in $^{11}\text{Li}$, several low-lying states have been observed\(^ {29-33}\). In the normal shell model picture, for $^{11}\text{Li}$ of the $N=8$ system, since the neutron shell is closed, only a proton’s excitation to the $p_{1/2}$-orbital state ($E_x \sim 3\text{ MeV}$) is expected to exist in the low energy region. The observation of several low-lying excited states should be considered as further evidence for the disappearance of the shell gap between the $p$-shell and the $sd$-shell of the neutrons in neutron-rich nuclei. With regard to the observed $E_x \sim 1\text{ MeV}$ state, there are several investigations of the excitation mechanism: the soft dipole resonance,\(^ {34-36}\) the Efimov state,\(^ {6,32,37,38}\) the pairing excitation.\(^ {24,39}\) However, no concrete conclusion has yet been obtained.

Within the three-body model, as pointed out in Ref. 6), if there is a low-lying $s$-state in the $^{9}\text{Li}+n$ subsystem in addition to a virtual state in the $n+n$ subsystem, many low-lying excited $3/2^-$ states might appear. Since the Efimov condition, which is the zero energy limit for all the $s$-states of the subsystem, is almost satisfied, many discrete states might appear in the low energy region.\(^ {6,32,37}\) Cobis et al. obtained such Efimov-like solutions with a $^{9}\text{Li}+n+n$ model. However, as they pointed out, many of these states might not be true three-body resonant solutions due to the nature of treatment of the boundary condition for the three-body system.\(^ {37,40}\)

In the shell model, using the WBP interaction,\(^ {41}\) which reproduces the intruder $s$-states in the $N=7$ isotones partly due to the pairing blocking effect,\(^ {42}\) a low-lying excited $3/2^-$ state in $^{11}\text{Li}$ is predicted at $E_x = 1.49\text{ MeV}$.\(^ {39}\) However, as they mentioned, the theoretical error is as large as about 1 MeV because of a buried state in the continuum states (no boundary condition for the unbound states).

At the present stage, as suggested in Ref. 6), we know that the $^{9}\text{Li}+n+n$ three-body model assuming a low-lying $s$-state in the $^{9}\text{Li}+n$ subsystem works well in explaining the ground state ($3/2^-$) properties of $^{11}\text{Li}$. Also, using a $^{9}\text{Li}-n$ interaction with a low-lying $s$-state, many excited $3/2^-$ states (Efimov-like states) might be obtained.\(^ {6,32,37}\) Furthermore, the assumption regarding the $^{9}\text{Li}-n$ interaction that has a low-lying $s$-state is supported by the shell model picture of the pairing blocking effect.\(^ {24}\) A low-lying excited $3/2^-$ state in $^{11}\text{Li}$ is also predicted by the shell model.\(^ {39,33}\) With these points in mind, we think that the study of $^{11}\text{Li}$ with the $^{9}\text{Li}+n+n$ three-body model based on the shell model picture with an accurate treatment of the three-body resonant problem for excited $3/2^-$ states is important. Now, we have the three-body model with the shell model picture: the Hybrid-TV model\(^ {5,26}\) with the new $^{9}\text{Li}-n$ interaction.\(^ {25}\) And we have the accurate treatment of three-body resonant problem: a recently developed method of the CSM.\(^ {26,27}\) It is the time to study the binding mechanism and the low-lying excited $3/2^-$ states on the same footing.

In this paper, we demonstrate that the problem of the small binding energy predicted by a previous simple $^{9}\text{Li}+n+n$ model\(^ {4,43}\) can also be solved by employing the new $^{9}\text{Li}-n$ interaction. Essential properties of the ground state is considered to be similar as the previous work by Thompson and Zhukov,\(^ {6}\) because the new $^{9}\text{Li}$-
interaction has an effectively strong state dependence, which makes the $s$-state a virtual state. The physical reason of the virtual state may be the effect of the $^9$Li-core excitation, like the pairing blocking effect proposed by Sagawa et al.\textsuperscript{24}) Due to the degeneracy of the $s_1/2$- and $p_1/2$-orbits, the ground state of $^{11}$Li has a large mixing of $(p_{1/2})^2$- and $(s_{1/2})^2$-configurations, and we obtain an excited resonant state with a mixed configuration similar to that of the ground state in the low energy region.

In the next section, we briefly explain the $^9$Li+$n$+$n$ model and the $^9$Li-$n$ interaction used here. In §3, the calculated results are given, and in §4, a summary and conclusion are given.

§2. Model and method

The experimental observation of the halo structure in $^{11}$Li suggests a double density structure consisting of two valence neutrons and the $^9$Li-core. Two valence neutrons are extremely weakly bound to the $^9$Li-core, which has a normal density. To describe such a halo structure of $^{11}$Li, we consider the Hamiltonian

$$H^{(11}\text{Li}) = H_c^{(9}\text{Li}) + H_N(2n) + V_{\text{coup}},$$  \tag{2.1}

where $H_c^{(9}\text{Li})$, $H_N(2n)$ and $V_{\text{coup}}$ are the intrinsic Hamiltonian of the $^9$Li-core, the relative motion of the valence neutrons, and the coupling potential between the degrees of freedom of the valence neutrons and the $^9$Li-core, respectively.

In many calculations employing the $^9$Li+$n$+$n$ model, the simple shell model configuration of $(0s)^4(0p_{3/2})^1(0p_{3/2})^4$ is assumed for the $^9$Li-core. (This is the so-called frozen core assumption.) However, there is neither an experimental nor theoretical basis for this assumption. Here, we direct our attention to the important correlation of neutrons in the $^9$Li-core in addition to the importance of the pairing correlation of the valence neutrons.

We write the $^9$Li-core wave function as

$$\Phi_{\text{gr}}^{(9}\text{Li}) = \sum_{i=0}^\infty c_i \Phi_i^{(9}\text{Li}),$$  \tag{2.2}

where $\Phi_0$ corresponds to the $(0s)^4(0p_{3/2})^1(0p_{3/2})^4$ configuration, and $\Phi_i (i > 0)$ represent the core excited configurations.

When the $^9$Li system exists in isolation, the coefficients $c_i$ are determined by solving the following Schrödinger equation:

$$H_c^{(9}\text{Li}) \Phi_{\text{gr}}^{(9}\text{Li}) = E_{\text{gr}} \Phi_{\text{gr}}^{(9}\text{Li}).$$  \tag{2.3}

However, this must be dynamically solved for the $^9$Li-core, which is coupled with the valence neutrons in $^{11}$Li. When we express the wave function of the valence neutrons coupled with the $i$-th core configuration $\chi_i^{2n} (\xi_1, \xi_2)$, the wave function of $^{11}$Li is written

$$\Phi_{\text{gr}}^{(11}\text{Li}) = \sum_{i=0}^\infty A[\chi_i^{2n} (\xi_1, \xi_2) \Phi_i^{(9}\text{Li})],$$  \tag{2.4}
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where $\xi_1$ and $\xi_2$ are appropriate coordinates of the two valence neutrons. This expression describes dynamical changes of the $^9$Li-core configuration as the valence neutrons approach the $^9$Li-core. The wave function $\chi_{i}^{2n}(\xi_1, \xi_2)$ is determined by solving the Schrödinger equation

$$H(^{11}\text{Li})\psi_{\text{gr}}(^{11}\text{Li}) = E\psi_{\text{gr}}(^{11}\text{Li}).$$

We adopt a single-channel approach with an effective $^9$Li-$n$ potential based on the picture provided by Feshbach’s projection method. Here, the projection operators $P$ and $Q$ are defined as

$$P\psi_{\text{gr}}(^{11}\text{Li}) = A[\chi_{0}^{2n}(\xi_1, \xi_2)\phi_{0}(^{9}\text{Li})],$$

$$Q\psi_{\text{gr}}(^{11}\text{Li}) = (1 - P)\psi_{\text{gr}}(^{11}\text{Li}),$$

where $P + Q = 1$ and $PQ = 0$. Then, we have the couple-channel equation:

$$\begin{align*}
(E - H_{PP})P\psi_{\text{gr}}(^{11}\text{Li}) &= H_{PQ}Q\psi_{\text{gr}}(^{11}\text{Li}), \\
(E - H_{QQ})Q\psi_{\text{gr}}(^{11}\text{Li}) &= H_{QP}P\psi_{\text{gr}}(^{11}\text{Li}),
\end{align*}$$

where $H_{PP} = PH(^{11}\text{Li})P$, $H_{QQ} = QH(^{11}\text{Li})Q$, $H_{PQ} = PH(^{11}\text{Li})Q$, $H_{QP} = QH(^{11}\text{Li})P$. Eliminating $Q\psi_{\text{gr}}(^{11}\text{Li})$, we obtain the effective single channel equation,

$$(E - H_{\text{eff}})P\psi_{\text{gr}}(^{11}\text{Li}) = 0,$$

where

$$H_{\text{eff}} = H_{PP} + H_{PQ} \frac{1}{E - H_{QQ}} H_{QP}.$$

The $P$-space corresponds to the usual $^9\text{Li}+n+n$ model, where the closed neutron configuration $(0s)^4(0p_{3/2})^1_\pi (0p_{3/2})^4_\nu$ is assumed for $^9\text{Li}$.

In this paper, for $H_{PP}$, we employ the following three-body cluster Hamiltonian of the orthogonality condition model (OCM): $^{43}$

$$H_{PP} \approx \sum_{i=0}^{2} T_i - T_{\text{cm}} + \sum_{k=1}^{2} V_{k}^{\text{fold}}(^9\text{Li}-n) + v_{12}(2n)$$

$$+ \lambda \sum_{k=1}^{2} \sum_{n,l,j} |nlj;k\rangle \langle nlj;k|. \quad (2.12)$$

Here, $T_i$ is the kinetic energy operator of the valence neutrons and the core, $T_{\text{cm}}$ is the kinetic energy operator of the center of mass motion, $V^{\text{fold}}$ is the $^9\text{Li}$-$n$ folding potential, $v_{12}$ is the nucleon-nucleon interaction between the valence neutrons. Also, $|nlj\rangle \langle nlj|$ is the pseudo-potential $^{44}$ necessary to project out the Pauli forbidden states to a highly excited energy state with a large $\lambda$ value ($10^4$–$10^8$ MeV). The additional term coming from the elimination of the $Q$-space configurations of the $^9\text{Li}$-core has a complex non-locality and a state dependence. This additional interaction originating from the dynamic nature of the $^9\text{Li}$-core is expressed as

$$H_{PQ} \frac{1}{E - H_{QQ}} H_{QP} = \Delta V. \quad (2.13)$$
Then, the effective interaction, excluding $v_{12}$, in the $^9$Li+$n+n$ system can be written

$$V^{\text{eff}}(^9\text{Li}-n-n) = \sum_{k=1}^{2} V_k^{\text{fold}}(^9\text{Li}-n) + \Delta V. \quad (2.14)$$

In the more simple case of $^{10}\text{Li} (= ^9\text{Li}+n)$, the corresponding couple-channel equation was solved by applying a coupled-channel orthogonality condition model (CCOCM).\textsuperscript{25} Here, for the $^9\text{Li}+n$ system, we assume the $^9\text{Li}$-core configuration defined in Eq. (2.2) as

$$\Phi_0 : \left[ (0s)^4 (0p_{3/2})^4 \pi (0p_{3/2})^4, J=0 \right]_{3/2},$$

$$\Phi_1 : \left[ (0s)^4 (0p_{3/2})^4 \pi (0p_{3/2})^2, J_1=0 \right]_{3/2},$$

$$\Phi_2 : \left[ (0s)^4 (0p_{3/2})^4 \pi (0p_{3/2})^2, J_1=0 \right]_{3/2},$$

$$\ldots$$

$$\Phi_i : \left[ (0s)^4 (0p_{3/2})^4 \pi (0p_{3/2})^2, J_1=0 \right]_{3/2},$$

$$\ldots$$

These configurations contain only the two-neutron pairing excitations within the usual shell model picture. As discussed in Ref. 25), for the $^9\text{Li}+n$ system, we can consider the pairing blocking effect with the above $^9\text{Li}$-core wave function.

In Fig. 1, we show the radial dependences of the additional interaction $\Delta V^{\text{Pauli}}$ (dotted curve) and $V^{\text{fold}}$ (dashed curve) for the $p$-wave, which were investigated in the explicit couple-channel calculation of the $^9\text{Li}+n$ system.\textsuperscript{25} The solid curve represents the effective $^9\text{Li}$-n potential, which is expressed as

$$V^{\text{eff}}(^9\text{Li}-n) = V^{\text{fold}}(^9\text{Li}-n) + \Delta V^{\text{Pauli}}. \quad (2.16)$$

The folding potential $V^{\text{fold}}(^9\text{Li}-n)$ is parameterized by $\delta$ parameter for the intermediate range term of the effective nuclear interaction. The $\Delta V^{\text{Pauli}}$ is determined by using CCOCM, so as to reproduce the binding energy of the $p_{1/2}$-orbital neutron in the $^9\text{Li}+n$ system. The details of the $^9\text{Li}-n$ folding potential and the other terms of the $^9\text{Li}-n$ interaction are explained in Refs. 25) and 16).

Here, we should mention that the strength of the additional Pauli blocking potential ($\Delta V^{\text{Pauli}}$) has an ambiguity due to the assumed $^9\text{Li}$-core wave function, the applied effective nucleon-nucleon interaction, and the strength of the spin-orbit interaction. Furthermore, there might also exist an effect originating from other core excited configurations for the core nucleus, which cannot be expressed with two-neutron pairing excitations, and the weight of two-neutron pairing configurations in Eq. (2.15) for $^{11}\text{Li}$ might change. It may be possible to solve these problems using a more microscopic model for $^{11}\text{Li}$ such as a broken $^9\text{Li}$-core cluster model. However, it is not clear what sort of nucleon-nucleon interaction should be employed in order to describe the change of the $^9\text{Li}$-core quantitatively in $^{10}\text{Li}$ and $^{11}\text{Li}$. At this time, we believe that there is no known nucleon-nucleon interaction that can reproduce
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Fig. 1. The radial dependence of the $^{9}$Li-$n$ potential.\textsuperscript{25)} The solid curve shows the effective $^{9}$Li-$n$ potential for the $p$-wave, $V^{\text{eff}}$. The dotted curve shows the Pauli forbidden potential, $\Delta V^{\text{Pauli}}$, and the dashed curve shows the folding potential part, $V^{\text{fold}}$.

quantitatively the change of the $^{9}$Li-core. For this reason, we simply assume that the effective interaction of the $^{9}$Li+$n+n$ system, $V^{\text{eff}}(^{9}$Li-$n-n$), can be written as a superposition of the effective $^{9}$Li-$n$ interaction, $V^{\text{eff}}(^{9}$Li-$n$), and that the change of the effective $^{9}$Li-$n$ interaction from that for $^{10}$Li to $^{11}$Li can be expressed as follows with the parameter $\alpha$:

$$V^{\text{eff}}(^{9}$Li-$n-n) \approx \sum_{k=1}^{2} V^{\text{eff}}_{k}(^{9}$Li-$n),$$  \hspace{1cm} (2.17)

$$V^{\text{eff}}_{k}(^{9}$Li-$n) \approx V^{\text{fold}}_{k}(^{9}$Li-$n) + \alpha \Delta V^{\text{Pauli}}_{k}. \hspace{1cm} (2.18)$$

As mentioned above, $\Delta V$ of $^{11}$Li cannot be expressed only in terms of $\Delta V^{\text{Pauli}}$ of $^{10}$Li, and $\Delta V^{\text{Pauli}}$ itself may be reduced in $^{11}$Li (e.g., $\alpha \sim 0.5$, due to the double counting of the Pauli forbidden potential as discussed in \S3). Furthermore, very recently, another new mechanism causing the state-dependence of the core-$n$ interaction was proposed by Otsuka et al. considering the spin-isospin dependent term of the nucleon-nucleon interaction,\textsuperscript{45)} and this might be a complementary effect with the pairing blocking effect.\textsuperscript{24)} The state dependence resulting from such a complementary effect is also effectively included as $\Delta V^{\text{Pauli}}$ in the present procedure of determining the potential strength from the $^{10}$Li spectrum. As a first step in the study with the state-dependent $^{9}$Li-$n$ interaction, we investigate the case $\alpha = 1$.

In order to solve the $^{9}$Li+$n+n$ Hamiltonian, we use the variation method with the so-called Hybrid-$TV$ model.\textsuperscript{5)} Using this model, it is expected that we can solve accurately both core-$n$ and $n$-$n$ correlations using a small number of basis function
as discussed in Refs. 5) and 26). The variation function for the Hybrid-TV model can be expressed by a superposition of a V-type function and a T-type function as

$$\Psi_{JM} = \Phi_{JM}(V) + \Phi_{JM}(T).$$  \hspace{1cm} (2·19)

The details of the Hybrid-TV model are given in Ref. 26). As far as a practical method for obtaining the three-body resonances is concerned, we use the CSM, as explained in detail in Refs. 26) and 43).

§3. Results

Firstly, we investigate the ground state of $^{11}\text{Li}(3/2^-)$ using the $^9\text{Li}$-$n$ potential in Eq. (2·16), taking into account the $^9\text{Li}$-core excitation. In Fig. 2, we plot the energy convergence of the lowest $3/2^-$ state of $^{11}\text{Li}$. The folded nucleon-nucleon interaction in the $^9\text{Li}$-$n$ potential is the Hasegawa-Nagata No. 1 (HN No. 1) potential. The potential parameters used here are those given in Ref. 25): $\delta = 0.245$ and $V_{ls} = 51.52$ MeV·fm$^3$. The nucleon-nucleon interaction between the valence neutrons is the Minnesota potential with the exchange parameter value $u = 0.95$. In Fig. 2, the calculated energy of $^{11}\text{Li}$ measured from the $^9\text{Li}$+$n$+$n$ threshold is presented as a function of the number of basis states represented by the channel number. The upper solid curve with squares represents the energies calculated with COSM (V-type). The channel configuration of COSM is as follows: channel No. 1 is $(p_1/2)^2$, channel No. 2 is $(p_1/2)^2+(s_1/2)^2$, channel No. 3 is $(p_1/2)^2+(s_1/2)^2+(d_5/2)^2$, and so on. The maximal configuration (in channel No. 28) for two valence neutrons is $(l = 14, j = 27/2)^2$. Here, $(l, j)^2$ means $[\frac{3}{2}^- \otimes \frac{1}{2}]_{j} \otimes [\frac{3}{2}]_{j} 0]_{3/2^-}$. The energy convergence with COSM is very slow, and it does not converge even with a large model space including a higher angular momenta, which corresponds to higher shell orbits in the shell model picture. The lower solid curve with circles, whose values

![Fig. 2. The calculated energies of the ground state for the hybrid-TV model and COSM. The $l = L = S = 0$ configuration is employed in the $T$-type bases of the hybrid-TV model. \hspace{1cm} (26), \hspace{1cm} (43) The strength parameters of the $^9\text{Li}$-$n$ folding potential with HN No. 1, \hspace{1cm} (46) $\delta = 0.245$, is employed. \hspace{1cm} (25)\hspace{1cm}](https://academic.oup.com/ptp/article-abstract/107/3/543/1809777)
are calculated with the hybrid-TV model, exhibits rapid convergence.\(^{26,43}\) The channel configuration of the hybrid-TV model is as follows: channel No. 1 is \((T\text{-type base}) + (p_{1/2})^2\), channel No. 2 is \((T\text{-type base}) + (p_{1/2})^2 + (s_{1/2})^2\), channel No. 3 is \((T\text{-type base}) + (p_{1/2})^2 + (s_{1/2})^2 + (d_{5/2})^2\), and so on. Here, we only include a \(l = L = S = 0\) channel for the \(T\text{-type bases}\) which mostly describe the \(0^+\) pairing correlation between valence neutrons. It is discussed that the slow convergence for COSM originates from the insufficient description of the \(0^+\) pairing correlation between valence neutrons.\(^{5}\) The value to which the energy converges with the hybrid-TV model is \(-0.51\) MeV, whose value is obtained only within the channel of \((T\text{-type base}) + (p_{1/2})^2 + (s_{1/2})^2 + (d_{5/2})^2 + (d_{3/2})^2 + (f_{7/2})^2 + (f_{5/2})^2\) (Channel No. 6). In other words, we can treat it within three major shell configurations (V-type) of valence neutrons and the pairing base (T-type) using the hybrid-TV model.

In Table I, we give the energies calculated using the conventional \(^9\)Li-\(n\) potential in which the \(^9\)Li-core excitation was not taken into account.\(^{43}\) The present energy of \(E = -0.51\) MeV shown in Fig. 2 found using the new \(V_{\text{eff}}(9\text{Li}-n)\) potential is much more stronger binding than the previous value, \(E = 0.35\) MeV, found using the conventional \(V_{\text{fold}}(9\text{Li}-n)\) potential. Since the experimental energy is \(\sim -0.34\) MeV, the binding energy is nearly reproduced using the new \(^9\)Li-\(n\) potential with HN No. 1. There are two reasons why the calculated binding energy of \(^{11}\)Li is slightly larger than the experimental value. One reason is that the potential strength of the \(s\)-state is slightly too large, because it gives a very loosely bound state (0.03 MeV), even in the \(^9\)Li+\(n\) system. Experimentally, we know that the \(^9\)Li+\(n\) system does not have any bound state, and it is believed to be a virtual (unbound) state. Another reason for the overbinding is that the \(^9\)Li-\(n\) potential using HN No. 1 is stronger than those using more realistic effective nucleon-nucleon interactions such as the Hasegawa-Nagata No. 2 potential (HN No. 2)\(^{46}\) or the Modified Hasegawa-Nagata Potential (MHN).\(^{48}\) Because HN No. 1 has no odd-state component that is repulsive property for the \(3/2^-\) state of \(^{11}\)Li, the binding energy becomes large. This is also reflected by Table I, where the resonance energy of \(^{11}\)Li for HN No. 1 (0.35 MeV) is seen to be smaller than that for HN No. 2 (0.72 MeV) and MHN (0.73 MeV).

In a similar way, we solve for the lowest \(3/2^-\) energy of \(^{11}\)Li by using the \(^9\)Li-\(n\) potential with MHN. For this potential, the \(s\)-state of the \(^9\)Li+\(n\) system is a virtual state, and it has an odd-state component. With this potentials, two problems of the \(^9\)Li-\(n\) potential with HN No. 1 are avoided. Since the \(^9\)Li-\(n\) interaction with MHN is weaker than that of HN No. 1, we can expect that the ground state of \(^{11}\)Li becomes a more loosely bound state than that of HN No. 1. However, it has no bound state, and the obtained resonant energy of the lowest \(3/2^-\) state is \(E_r = 0.59\) MeV, which does not differ greatly from the previous value, \(E_r = 0.73\) MeV, given

| Table I. The \(3/2^-\) state energies of \(^{11}\)Li with the conventional \(^9\)Li-\(n\) folding potential.\(^{43}\) |
|-----------------|-------------|---------|-------------|
| \(E_{\text{3}^-/2^-}^{(11\text{Li})}(\text{MeV})\) | \(\delta\) | \(V_0^{1/2}\) |
| HN No. 1        | 0.35–0.01  | 0.0866  | 51.52       |
| HN No. 2        | 0.72–0.08  | 0.1200  | 44.62       |
| MHN             | 0.73–0.07  | 0.0442  | 44.20       |
in Table I. Here, we infer that the obtained 3/2$^-$ state may not be the ground state. The reason why we reach such a conclusion is that we cannot obtain the dominant s-wave solution for unbound states with the present method (CSM). In the CSM, the calculating limit of the S-matrix pole for the Gaussian potential is on the fourth quadrant of the second-Riemann energy-sheet. In order to check this conclusion, we strengthen the $\delta$ parameter. Doing so, we easily find that a very slight modification of $\delta$ (0.157$\rightarrow$0.162) creates another 3/2$^-$ solution, which is very loosely bound, in addition to the above resonant solution. In other words, there exist two low-lying 3/2$^-$ states in $^{11}\text{Li}$ with the new $^9\text{Li}$-$n$ potential, taking into account the $^9\text{Li}$-core excitation.

In Table II, we list the calculated energies of the 3/2$^-$ states of $^{11}\text{Li}$. The second column lists the bound state solutions. The third column lists resonant solutions. The fourth and fifth columns list solutions obtained within only the $l = L = S = 0$ (T-type) + $(s_{1/2})^2$ (V-type) channel and the $l = L = S = 0$ (T-type) + $(p_{1/2})^2$ (V-type) channel, respectively. (The potential parameters used here are the same as those given in Ref. 25.) As seen in the second and the third columns of Table II, for HN No. 1, two 3/2$^-$ solutions are obtained, a bound state solution ($-0.51 \text{ MeV}$) and a resonant solution ($0.21-0.05 \text{ MeV}$). This suggests the new possibility of a low-lying resonance in $^{11}\text{Li}$. For MHN, we obtain only one solution, though we can easily obtain two solutions as mentioned above, if we slightly strengthen the $^9\text{Li}$-$n$ potential.

This mechanism responsible for the existence of two low-lying energy solutions is explained as follows. The ground state of $^{11}\text{Li}$ is expressed in terms of the superposition of many configurations in the shell model picture, where the configurations of the two valence neutrons in ($nlj$)-orbits are mixed due to the pairing correlation. In the present potential model, the p-orbital and s-orbital energies for the valence neutron are very near each other. For this reasons, the s-wave component and the p-wave component of valence neutrons in $^{11}\text{Li}$ are strongly coupled with each other. Therefore, two solutions, one with an s-wave dominant and one with a p-wave dominant, are expected with a energy gap between them. In other words, when the solution with the s-wave dominant is obtained as the ground state, the solution with the p-wave is obtained as an excited state. The obtained excited state is considered as the pairing excitation by the energy gap (0.72 MeV for HN No. 1).

As seen from the fourth column in Table II, if we drop the p-wave channel, we cannot obtain any solutions, because we cannot derive the virtual-like state solution or the broad resonant solution in the present method. Contrastingly, as seen from

<table>
<thead>
<tr>
<th></th>
<th>bound state</th>
<th>resonance</th>
<th>($s_{1/2})^2$</th>
<th>($p_{1/2})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HN No. 1</td>
<td>$-0.51$</td>
<td>$0.21-0.05$</td>
<td>not found</td>
<td>$0.30-0.00$</td>
</tr>
<tr>
<td>MHN</td>
<td>not found</td>
<td>$0.60-0.09$</td>
<td>not found</td>
<td>$0.68-0.05$</td>
</tr>
</tbody>
</table>
The Binding Mechanism and a Low-Lying Resonance in $^{11}\text{Li}$

Fig. 3. The dependence of the $s$-wave energy ($2^-$) of $^{10}\text{Li}$ on the $\delta$ parameter for MHN (solid curve) and HN No. 1 (dashed curve).

the fifth column, even if we drop the $s$-wave channel, we get resonant solutions. The resonant energies of these solutions are very near those of the resonant solutions in the third column. Therefore, we can expect that the excited resonant solutions have large $p$-wave components.

Since the $(p_{1/2})^2$ component of the pairing neutron in the $^9\text{Li}$-core is somewhat ambiguous, we can change the value of $\delta$ slightly. Microscopically, this corresponds to changing the weight of the core-excitation to the $p_{1/2}$-orbit in $^9\text{Li}$. The present weight of the $p_{1/2}$ configuration is $\sim 25\%$, which is a reasonable value from the shell model picture, but it has an ambiguity. In fact, as mentioned above, the $^9\text{Li}+n$ potential of HN No. 1 given in Ref. 25 causes the $s$-state of $^{10}\text{Li}$ to be bound. This is inconsistent with the observation of the virtual state. In order to find a reasonable $\delta$ of the virtual state, we investigate the energy of the $2^-$ state ($s$-state) in $^{10}\text{Li}$. In Fig. 3, the solid curve represents the energy for MHN, and the dashed curve represents that for HN No. 1. As seen from Fig. 3, the $^9\text{Li}+n$ system is bound from $\delta = 0.234$ for HN No. 1, and from $\delta = 0.164$ for MHN. Thus, we should use a value smaller than $\delta = 0.163$ for HN No. 1 and $\delta = 0.233$ for MHN at least, because the $s$-state is observed slightly above the $^9\text{Li}+n$ threshold (e.g. $E < 50$ keV).

By using parameter values for the virtual states $\delta = 0.233$ for HN No. 1 and $\delta = 0.163$ for MHN, the ground state energy of $^{11}\text{Li}$ is obtained as $E = -0.32$ MeV for HN No. 1 and $E = -0.01$ MeV for MHN. This calculated energy of HN No. 1 is very near the experimental energy of $E = -0.34$ MeV, and that of MHN represents a very loosely bound state. The present $p$-state ($1^+$) energy of $^{10}\text{Li}$ is fitted to the typical energy of 0.42 MeV. In a recent experiment on the unnatural parity state ($1^+$), Bohlen et al. obtained results suggesting a smaller value of 0.24 MeV ($\Gamma = 0.10$ MeV) than their previous value of 0.42 MeV ($\Gamma = 0.15$ MeV). In the original study, in which the virtual state in the $^9\text{Li}+n$ subsystem is assumed, Thompson and Zhukov employed a $p$-state energy of 0.175 MeV–0.35 MeV. By fitting the $p$-state
energy of $^{10}$Li to 0.24 MeV ($\Gamma = 0.13$ MeV for HN No. 1, $\Gamma = 0.12$ MeV for MHN), we obtained the ground state energies of $^{11}$Li as $E = -0.52$ MeV for HN No. 1 and $E = -0.12$ MeV for MHN. In other words, the binding energy of $^{11}$Li is almost reproduced within the experimental uncertainty for the $p$-state in $^{10}$Li. Here, the $s$-state of $^{10}$Li is also reproduced as a virtual state.

Furthermore, in order to see the dependence on $\delta$ of the $3/2^-$ states in $^{11}$Li, in Fig. 4, we plot the energies of the $3/2^-$ solutions. The solid curves represent the calculated energies ($E_r$), and the dashed curve represents the calculated decay widths ($E_r \pm \Gamma/2$). For small values of $\delta$, which correspond to the situation in which the $^9$Li-core excitation is small, only a resonant state is obtained. When $\delta$ is somewhat large, the resonance energy is lower, and the decay width is larger. However, as $\delta$ becomes much large, another $3/2^-$ solution is obtained as a bound state below the resonant solution. The reason why this bound state suddenly appears can be understood by considering the solution of the large $s$-wave component, which cannot...
be obtained as an unbound state in the present method.

In order to confirm the large s-wave component in the calculated ground state, we determined the weights of the partial waves. For the results listed in Table III, the value of $\delta$ was chosen to reproduce the experimental binding energy of $^{11}\text{Li}$. The weight of the s-wave is 63.6% for HN No. 1, while the weight of the s-wave is 79% for MHN. The main reason for this difference comes from the p-wave resonant pole position. Since the resonance energy of the p-wave channel (fifth column in Table II) for HN No. 1 is nearer the s-wave energy (threshold energy) than that for the MHN potential, the s-wave and the p-wave are strongly coupled. In the original paper in which the virtual state in the $^9\text{Li}+n$ subsystem is assumed, Thompson and Zhukov also calculated the weights of the partial waves. Among the given parameter sets, that of P4 gives results nearest to those of the present calculation. For the set of P4, the p-wave energy of $^{10}\text{Li}$ is fitted to $E = 0.35$ MeV (the value obtained presently is $E = 0.42$ MeV), and the virtual state position of $^{10}\text{Li}$ is determined to reproduce the ground state energy of $^{11}\text{Li}$, $E = -0.31$ MeV (the present one is $E = -0.34$ MeV). Their calculated weight of the s-wave is 64% and the values obtained for the p-wave is 30%, which are very similar to the values obtained for HN No. 1 (64% for the s-wave and 29% for the p-wave). The $^9\text{Li}-n$ potential given by Thompson and Zhukov is a simple Woods-Saxon type and does not have a spin doublet component reflecting the spin of the $^9\text{Li}$-core ($3/2^-$). Therefore, the present results for HN No. 1 correspond to their results, because the spin splitting between $1^+$ and $2^+$ in the $^9\text{Li}+n$ subsystem is very small for HN No. 1 due to the absence of odd state component in the microscopic nucleon-nucleon interaction. The calculated root-mean-squared radii, $R_m = 3.63$ fm for HN No. 1 and $R_m = 3.67$ fm for MHN, are larger than the experimental values. This large value of $R_m$ originates from the large s-wave component, as discussed by Thompson and Zhukov. They also obtained the large value as $R_m = 3.64$ fm with the P3 potential ($R_m = 3.73$ fm for P4), which is a recommended potential to reproduce the binding energy and the $^9\text{Li}$ momentum distribution. Hence it is a reasonable to conjecture that the present wave function for the ground state is similar to the previous ones obtained by Thompson and Zhukov.

We think that only the $^9\text{Li}+n+n$ model cannot explain the fact that the binding energy is too weak because of the channels beyond the three-body model. For $^6\text{He}$, it is known that the difference in binding energy is a few hundred keV. Then, the s-wave potential used in Table III might be slightly stronger, because the $\delta$ parameter is determined to reproduce the binding energy. If the p-wave potential is stronger, the p-wave component in the ground state becomes larger, which makes the root-mean-squared radius smaller, due to the centrifugal barrier. In order to investigate this, we change the potential strength ($\delta$) of MHN, making the p-wave potential slightly stronger ($\delta = 0.06$) and the s-wave potential slightly weaker ($\delta = 0.173$). Then, the p-wave component becomes 42.5% and the s-wave component becomes 51.6%, and the binding energy, 0.34 MeV, is reproduced. For this potential, the root-mean-squared radius has a smaller value of $R_m = 3.40$ fm, as expected.

In Fig. 5, we display typical calculated $3/2^-$ energy levels and the experimental low-lying states in the $^9\text{Li}+n+n$ threshold energy region. The potential parameters are the same as those employed in the calculation whose results are given in Table
Table III. Weights of the configuration in the ground state wave function of the loosely bound state.

<table>
<thead>
<tr>
<th></th>
<th>((s_1/2)^2)</th>
<th>((p_1/2)^2)</th>
<th>others</th>
<th>Energy (MeV)</th>
<th>(R_m) (fm)</th>
<th>(\delta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HN No. 1</td>
<td>63.6%</td>
<td>28.9%</td>
<td>7.5%</td>
<td>-0.34</td>
<td>3.63</td>
<td>0.234</td>
</tr>
<tr>
<td>MHN</td>
<td>79.0%</td>
<td>15.6%</td>
<td>5.4%</td>
<td>-0.34</td>
<td>3.67</td>
<td>0.180</td>
</tr>
</tbody>
</table>

\(\text{in MeV}\)

\[
\begin{array}{c}
0.34 \quad \Gamma = 0.08 \\
0.56 \quad \Gamma = 0.18 \\
1.25 \pm 0.25 \\
1.20 \pm 0.1
\end{array}
\]

Finally, in Fig. 6, we plot the \(\alpha (= 0.5-1.0)\) dependence of the ground state energy in order to show the ambiguity of the present model. In many three-body calculations, we assume that the \(^9\text{Li}-n\) interaction which is determined in \(^{10}\text{Li}\) is not different for the \(^9\text{Li}+n+n\) system. This assumption might not be valid for the real \(^{11}\text{Li}\) system. In naive picture of the pairing blocking effect, when a \(p_{1/2}\) valence
The neutron is added to $^9\text{Li}$, the neutron becomes subject to a repulsive potential from the pairing two neutrons of the $p_{1/2}$-orbit in the $^9\text{Li}$-core due to the Pauli principle. However, if one more valence neutron is added to the $^{10}\text{Li}$ nucleus, the neutron does not experience such a repulsive potential, because the $^9\text{Li}$-core is already reconfigured from the free $^9\text{Li}$ nucleus. For this reason, the pairing blocking potential for each valence neutron in $^{11}\text{Li}$ is expected to be half ($\alpha = 0.5$) that for one valence neutron in $^{10}\text{Li}$. In the original paper on the pairing blocking effect, Sagawa et al. discussed the mechanism of the shell-gap between $0p$- and $1s$-orbits in $N = 7$ isotones disappears.$^{24}$ They mentioned that three comparable effects are important to explain the disappearance of the shell gap in $N = 7$ isotones: the monopole interaction, $s$-$d$ coupling, and pairing blocking. Other effects such as the dynamical change of the $^9\text{Li}$-core from that in $^{10}\text{Li}$ to that in $^{11}\text{Li}$ is also considered. Although only the $\Delta V_{\text{Pauli}}$ is assumed in the present model, as mentioned above, the mechanism by which $\Delta V$ is produced is not only that of the pairing blocking effect.$^{24,45}$ Therefore, we can consider $\Delta V$ due to other mechanisms as being effectively included in $\Delta V_{\text{Pauli}}$ in the present $^9\text{Li}$-$n$ potential. With this understanding, we should consider $\alpha$ to be one of the parameters of the present model, rather than simply considering $\alpha$ to take a value near $\sim 0.5$.

In Fig. 6, the lower solid curve corresponds to HN No. 1, the upper solid curve to the MHN. The dotted curve is the experimental energy. The value of $\alpha$ becomes smaller, the binding energy becomes stronger. As seen from Fig. 6, the calculated energy for small values of $\alpha$ gives the binding energy that is much stronger than the experimental one. Since the value of $\alpha$ exhibits the strength of the additional potential, we understand that we need the same order of the additional repulsive potential ($\alpha \sim 0.8$ for MHN). Though the mechanism responsible for the state dependence of $p$- and $s$-waves might not be only the pairing blocking effect, we infer that it is important to consider that the state dependence of the $^9\text{Li}$-$n$ interaction originates from the $^9\text{Li}$-core excitation.

![Fig. 6. The dependence of the ground state energy on $\alpha$.](https://academic.oup.com/ptp/article-abstract/107/3/543/1809777)
§4. Summary and conclusion

In summary, we analyzed the binding mechanism acting in $^{11}$Li with the $^9$Li+$n+n$ model. Using the new $^9$Li-$n$ potential constructed by taking into account the $^9$Li-core excitation, we found a reasonable solution for the $^{11}$Li ground state. The $s$-wave component in the ground state wave function is larger than 50%. This is very different from the ground state wave functions found in the previous calculations that assume only the low-lying $p$-state $^4, ^{43}$ (where the $p$-wave component is $\sim 90\%$). This result is very similar to that of a previous calculation carried out by Thompson and Zhukov, where the virtual state in the $^9$Li+$n$ subsystem was originally proposed. The essential point is that the degeneracy of the single particle energies causes the ground state of $^{11}$Li to be a strongly mixed configuration of $s$- and $p$-waves.

For the excited state, we studied the second $3/2^-$ state. This state is understood as a pairing excitation of the valence neutrons from the ground state, because the ground state has the large mixed configuration of $s$- and $p$-waves. However, we cannot obtained the $3/2^-$ state as the so-called Efimov state. The excited $3/2^-$ state is surely obtained in the very low energy region $E_x = 0.5$–1 MeV. Since the low-lying state in $^{11}$Li observed by Gornov et al., which is suggested as the second $3/2^-$ state, has $E_x = 1.02$ MeV, $^{33}$ the newly predicted state may have already been observed. However, the calculated energy has a small ambiguity, because the present model has parameters that should be determined from the experimental data of the subsystems ($^9$Li+$n$, $n+n$). Thus, the experimental determination of the $p$-wave resonant state and the $s$-wave virtual state in $^{10}$Li should make the binding mechanism of $^{11}$Li clearer. Furthermore, it is expected that a more realistic pairing interaction increases the energy gap between the $3/2_1^-$ and $3/2_2^-$ states. The present effective nucleon-nucleon interaction of the Minnesota potential $^{47}$ has a small off-diagonal energy between the $(s_{1/2})^2$ channel and the $(p_{1/2})^2$ channel, which causes the energy gap to be small.

In the near future, we would like to study the pairing interaction in detail with a more developed coupled-channel model of $^{11}$Li, including the $^9$Li-core excitation.

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References

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