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CALIBRATION OF A COMPRESSIVE GRAVITY THICKENING MODEL FROM A SINGLE BATCH SETTLING CURVE

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ABSTRACT

A compressive gravity thickening model (Kos and Adrian, 1974; Kos, 1978) was calibrated from a single batch settling curve. This model was originally formulated in terms of the total suspended solids concentration, C, the dynamic pressure gradient, $\partial P/\partial z$, and the gradient corresponding to the compressive yield strength, $\partial \sigma_y/\partial z$. Fitch (1975) demonstrated that the model could be formulated in terms of C and the solids concentration gradient, $\partial C/\partial z$. Utilizing this formulation the model was calibrated with data generated from elementary quantitative analysis of the steady-state conditions attained in continuous thickening experiments (Vaccari, 1984; Vaccari and Uchrin, 1989). In this investigation the model was calibrated from a single batch settling curve. This was done using the down-hill simplex optimization method proposed by Nelder and Mead (1965) in a curve-fitting capacity. The optimization routine adjusted the model coefficients in order to reduce the discrepancy between the simulated results of the model and the corresponding experimental batch settling data. When calibrated by this method the model was found to accurately predict experimental continuous thickening behavior observed for the same sludge.

KEYWORDS

Activated sludge; batch; calibration; compression; modeling; sedimentation; simulation; thickening.

INTRODUCTION

The Coe and Clevenger (1916) gravity thickening model, or equivalently the flux theory, has essentially been utilized as the basis for the design of the continuous gravity thickening process for over 75 years. A significant reason for the continued use of this theory is the fact that the model can be "calibrated" from a series of simple batch settling curves. However, both theoretically and experimentally identified inadequacies associated with this theory have become apparent. For example, the theory cannot predict the existence of a sludge blanket in an underloaded thickener; it does not account for the final distribution of solids concentrations in a completely settled sludge. These behaviors may be accounted for by including the interparticle compressive stress (Fitch, 1966; Dixon, 1977a, b; Vaccari 1984). In this investigation the Kos compressive gravity thickening model was calibrated with a single batch settling curve.

Thickening models have been useful for design of sedimentation basins. If a model capable of predicting dynamic behavior of thickening were available, and if it could be calibrated from relatively simple laboratory tests, it might be possible to extend the application of models to real time process control.

DEVELOPMENT OF THE COMPRESSIVE THICKENING MODEL

A variety of gravity thickening models can be derived from the mass and force balances pertaining to the floc structures. Fitch (1979) showed that a general differential equation, which defines the force balance on the flocs, can be utilized in the derivation of most thickening theories presented in the literature. To obtain solutions to this equation various terms are neglected and/or various assumptions are made concerning the functionalities of the constitutive relations (Fitch 1979).

The Continuity Equation

The continuity equation defines the mass balance pertaining to the solids phase of a flocculant slurry in a continuously operated gravity thickener. In this work the equation was based on the following assumptions.

- The system is one-dimensional and is oriented in the vertical direction.
- Both the solids and the water are conservative.
- Despite the fact that the solids actually consist of discrete particles, the concentration of the solid particles in the slurry was represented as continuous in space.
- In accordance with the results obtained by Kos (1978), the ratio of bound water to corresponding floc solids was assumed to remain constant.
- Both the water and the solids are incompressible.
- The transport of solids due to dispersion was considered negligible.

A hypothetical column of settling flocs was divided along the vertical, or z-axis, into individual layers with thickness Δz. Formulating the mass balance around the layer located between the levels z and $z+\Delta z$, resulted in the following form of the continuity equation:

$$-\partial C/\partial t = \partial G_s/\partial z + v_u \, \partial C/\partial z \tag{1}$$

where t is time; G_s is the solids settling flux; z = space coordinate; and v_u is the underflow velocity.

The Force Balance Equation

Similar to the derivation of the continuity equation, the general form of the force balance equation was developed for the hypothetical layer of slurry located between the levels z and $z+\Delta z$. This included the gravitational force due to the unbuoyed weight of the solids, the interparticle compressive force, and the force resulting from the dynamic pressure gradient due to the drag force generated by the relative movement of the flocs and water. By taking the limit as Δz goes to zero, the following form of the force balance was ultimately obtained:

$$gC(1-\rho_w/\rho_s) = \partial P/\partial z + \partial\psi/\partial z \tag{2}$$

where g is acceleration due to gravity; ρ_w is the density of water; ρ_s is the density of solids; P = dynamic pressure; and ψ is the interparticle compressive stress. In Equation (2) the left-hand side depicts the gravitational force, while the forces due to dynamic pressure and the interparticle compressive stress gradients, respectively, are represented on the right-hand side.

The Michaels and Bolger Model

Michaels and Bolger (1962) developed mathematical relations which described how the initial zone settling velocity (ISV) in batch thickening experiments will vary with initial sludge column height. To account for the compressive force, the compressive yield strength, or the yield stress, σ_y, associated with a slurry was defined. The compressive yield strength defines the maximum compressive load that the original floc network can support without compressing to a higher concentration. The Michaels and Bolger model was mathematically extended by Fitch (1966) to the continuous thickening process. As a result, a general form of the model which was applicable to continuous gravity thickening was established. Since the force balance represented by Equation (2) is applied in the region where compression occurs, the compressive stress, ψ, is equivalent to the compressive yield strength, σ_y. Furthermore, Equation (3) describes the system since the compressive yield strength is assumed to be a function of solids concentration.

$$\partial\sigma_y/\partial z = (d\sigma_y/dz)\cdot(\partial C/\partial z) \tag{3}$$

For the Michaels and Bolger model the dynamic pressure gradient is defined by Equation (4), or Darcy's law, where the resistivity, k, is assumed to be a function exclusively of solids concentration.

$$\partial P/\partial z = kv_s \tag{4}$$

Substituting Equations (3) and (4) into Equation (2) and solving for solids settling velocity, v_s, results in:

$$v_s = (gC(1-\rho_w/\rho_s)/k) - ((d\sigma_y/dC)\cdot(\partial C/\partial z)/k);\quad C > C_c \tag{5}$$

The compression point concentration, C_c, defines the lowest solids concentration at which interparticle compressive stress exists. As Equation (5) indicates, compression reduces the settling velocity.

Two extreme cases can be examined using equation (5). One is the case where $\partial C/\partial z = 0$, which, from equation (3), implies an absence of a compressive stress gradient. This is the situation arrived at experimentally when the ISV is measured using tall columns (greater than 1.2 m). The second case is where $v_s = 0$. This corresponds to the situation where settling is complete; the solids have compacted into a layer at the bottom of the thickener. For a given solids concentration the maximum settling velocity, v_m, is defined for the first case by setting the solids stress gradient, $\partial C/\partial z$, in Equation (5), equal to zero:

$$v_s = v_m = gC(1-\rho_w/\rho_s)/k \tag{6}$$

This velocity, which is a function exclusively of solids concentration, is equivalent to the settling velocity predicted by the Coe and Clevenger (1916) gravity thickening model. For the second extreme case the settling velocity defined by Equation (5) is set equal to zero. An expression for the concentration gradient associated with the final settled volume of sludge in a batch settling test, dC^*/dz, can thus be obtained. The compressibility function, K, is defined as dC^*/dz expressed in terms of solids concentration.

$$K = dC^*/dz = gC(1-\rho_w/\rho_s)/(d\sigma_y/dC) \tag{7}$$

Substituting Equation (6), and then Equation (7), into Equation (5) results in the general form of the Michaels and Bolger (1962) model which is applicable to continuous gravity thickening.

$$v_s = v_m(1-(\partial C/\partial z)/K);\quad C > C_c \tag{8}$$

As opposed to the Coe and Clevenger (1916) gravity thickening model, the settling velocity, v_s, is defined as a function of the solids concentration gradient, $\partial C/\partial z$, as well as the solids concentration. In Equation (8) the compressibility function, K, defines the compressive thickening properties of the sludge while the remaining properties are defined by the settling velocity in the absence of a solids compressive stress gradient, v_m.

The functions v_m and K can be measured independently of each other. The velocity v_m can be measured by the same techniques utilized to determine the ISV for the classical flux model (APHA, 1989). The compressibility function, K, can be measured using a static settling test conducted in small columns (Vaccari, 1984). However, flocculant sludges such as activated sludge do not conform to the Michaels and Bolger (1962) model. As Kos (1978) showed, these materials are non-Darcian in the sense that compression affects the resistivity, k. Kos developed models which are more general than Equation (8):

$$v_s = v_s(C, \partial C/\partial z);\quad C > C_c \tag{9}$$

Nevertheless, Vaccari (1984) also showed that Equation (8) can still be used to model activated sludge if the empirical expressions for v_m and K are fitted to realistic experimental conditions, rather than extreme cases. Vaccari showed that this could be done utilizing nonlinear least-squares regression analysis with data obtained from laboratory steady-state continuous thickening experiments. This work will improve on the current situation by showing that the model can be fitted using a single batch settling curve, greatly simplifying the experimental requirements.

Empirical Expressions for v_m and K

The empirical equation proposed by Tracy (1973), which expresses the settling velocity, v_m, as a function of solids concentration, was utilized in this investigation. The Tracy equation is represented by Equation (10) where the parameters B_1 and B_2 define thickening properties of the sludge.

$$v_m = B_1/(C - B_2);\ \ C > C_c \tag{10}$$

At concentrations less than or equal to C_c, a linear extension of the Tracy equation was employed (Vaccari 1984). The empirical expression represented by Equation (11) was developed in order to define the compressibility function in the gravity thickening model.

$$K = K_m e^{(-C/S)};\ \ C > C_c \tag{11}$$

The parameters K_m and S define the compressive thickening properties of the sludge. However, when the model was calibrated from continuous thickening experiments utilizing the procedure demonstrated by Vaccari (1984), statistics indicated that the estimated value for K_m was not significantly different from unity. As a result, the parameter K_m was eventually dropped from the model. As evident from Equation (11), when the parameter K_m is equal to unity, the compressibility function, K, will be limited to values which are less than 1. Although this was acceptable for the sludge utilized in this particular investigation, this may not be appropriate with all flocculant slurries.

BEHAVIOR OF THE COMPRESSIVE THICKENING MODEL

The behavior of the compressive gravity thickening model is illustrated in Figure 1 which depicts the settling flux curves predicted by the model. The Coe and Clevenger (1916) thickening model allows a single value for the solids settling flux, G_s, at each value of solids concentration. In contrast, the compressive model is not limited to a single value. As indicated by Equation (8), above C_c the settling velocity, v_s, and therefore the settling flux, will vary linearly from a maximum which occurs when $\partial C/\partial z = 0$, to zero when $\partial C/\partial z = K$. Thus the flux curve is "filled-in" below the maximum flux. Even though the operating line shown falls below the maximum flux, it represents the area in the sludge blanket having a solids flux corresponding to the values of C and $\partial C/\partial z$ which can be found from the Figure. Figure 1 also shows how the concentration distribution in the blanket of a thickener can have a sigmoidal shape. This can be seen by following the operating line from its lower end, where it intersects the concentration axis at the underflow concentration, upward. The operating line passes from below the line marking a concentration gradient of 20×10^{-3} g/L/cm, to above it, and then below again until the operating line meets C_c at the top of the sludge blanket. This indicates that the concentration drops rapidly, then more slowly, then rapidly again, from the bottom of the thickener up through the sludge blanket.

As the operating line approaches the maximum flux line, the height of the compression sludge blanket increases, possibly until it exceeds the depth of the thickener. Thus, it is possible for a thickener to be overloaded in terms of height of compression layer, even though it is underloaded in terms of the classical flux theory.

FORMULATION OF THE SIMULATION PROGRAMS

The general unsteady-state gravity thickening model was defined by the force balance obtained when Equations (10) and (11) were substituted into Equation (8), or the linear extension of the Tracy equation when $C \leq C_c$, together with the mass balance as expressed by Equation (1). The FORTRAN programs formulated to simulate the unsteady-state gravity thickening process correspond to the numerical solution of these model equations. The solution was obtained by solving the force and mass balances as simultaneous, coupled, partial differential equations. This technique was previously utilized by Vaccari (1984) to successfully simulate the unsteady-state thickening process. Simulation programs were developed for both the batch and continuous gravity thickening processes by incorporating the appropriate boundary conditions into the numerical solution. Simulation of the batch thickening process was required in order to calibrate the compressive gravity thickening model with data obtained from batch thickening experiments. The calibration was then assessed by simulating the continuous thickening process and comparing the results with data obtained from the continuous experiments.

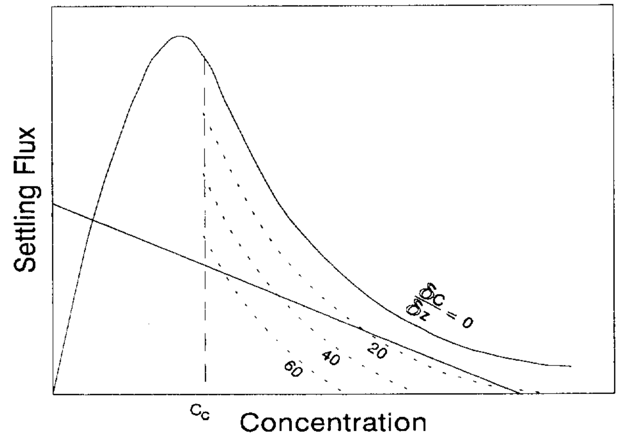

Fig. 1. Settling flux curve for the compressive gravity thickening model (Vaccari and Uchrin 1989).

MODEL CALIBRATION UTILIZING BATCH THICKENING EXPERIMENTS

The calibration procedure based on batch thickening experiments consisted of determining the parameter values C_c, B_l, B_2, and S which minimized the discrepancy between the simulated batch settling curve and the corresponding experimental settling curve. Calibration was achieved by employing the down-hill simplex method proposed by Nelder and Mead (1965). As presented by Nelder and Mead, the down-hill simplex technique is actually utilized to determine values for the independent variables which minimize the corresponding function. However, this optimization scheme can also be applied to the problem of curve-fitting and is particularly well suited for use with nonlinear models similar to the unsteady-state gravity thickening model. Divergence is impossible and determination of function derivatives is not required.

When applied in a curve-fitting capacity, the down-hill simplex method determines values for the model parameters which minimize the sum of the squares of the errors, *SSE*. In this investigation, the errors were defined by the discrepancy between the simulated batch settling curve and the corresponding experimental curve. The effect of flocculation on the initial accelerating portion of the settling curve was not modeled. The algorithm only compared the model to data subsequent to the beginning of the linear portion of the settling curve.

To calibrate the thickening model a modified version of the batch simulation program was utilized as a subroutine within the down-hill simplex algorithm. An initial set of parameters was assumed based on the values obtained when the model was calibrated from the continuous thickening experiments. For this initial set of parameter values the subroutine simulated the batch settling curve, performed the comparison with the experimental curve, and computed the value for the *SSE*. Based on the value obtained for the *SSE*, the down-hill simplex algorithm selected a revised set of parameter values. The algorithm then sent these revised values to the subroutine in order to compute the new value for the *SSE*. Consequently, calibration of the model proceeded in an iterative fashion until the value of the *SSE* could not significantly improve the fit in terms of a specified stopping criterion.

CONTINUOUS THICKENING EXPERIMENTS

The continuous thickening apparatus consisted of four simultaneously operating gravity thickeners operating in a total recycle mode. Each thickener was constructed from clear acrylic tubing measuring 180 cm in length with an inside diameter of 10 cm. An underflow outlet was provided at the bottom of each thickener and an overflow outlet was located at a height of 160 cm. In order to measure the solids concentration profile, sampling ports were provided every 10 cm from the bottom of the thickener to a height of 120 cm. Each thickener was equipped with a stirring mechanism which was intended to eliminate floc bridging and reduce the effects of shear stress at the wall. The stirrer was designed in accordance with the American Public Health Association (1989) specifications pertaining to the stirring mechanism utilized in both the settled sludge volume (SSV) test and the sludge volume index (SVI) test. In addition, the stirrer was equipped with a system of sludge rakes which transported the sludge on the bottom of the thickener toward the centrally located underflow. Further experimental details are provided by Cacossa (1993).

Batch thickening experiments were utilized to monitor the settling properties of the sludge. During the calibration experiments the ISV and the SVI of the sludge were monitored as a function of time in order to detect any possible changes in settling properties. Each continuous thickening column was operated at conditions representing a different operating line on the flux plot. After steady-state was established the solids concentration distributions were determined by obtaining samples every 10 cm. and then measuring the total suspended solids concentrations gravimetrically. A new set of conditions was established and allowed to come to steady-state for eight hours before a similar measurement was performed. Thus two sets of four column experiments were carried out with the same sludge within an eight hour period.

The model was fitted to the eight columns of continuous data using the method of Vaccari (1984). A statistical comparison of the residuals indicated that there was no difference between the two sets of four columns. SVI and ISV data also indicated no changes in sludge properties over the eight hour period.

BATCH THICKENING EXPERIMENTS

The stirred columns constructed for the continuous thickening apparatus were also utilized to perform the batch thickening experiments. At the outset of each batch test the sludge was fed into the settling column to a height of 120 cm utilizing the bottom fill procedure developed by Dick (1965) to minimize the apparent flocculation period. The subsidence of the solids-liquid interface was monitored for 60 minutes in order to provide the data utilized to construct the batch settling curve.

A series of batch thickening experiments was also conducted in one liter graduate cylinders in order to provide the information required to monitor the settling properties of the sludge. With the exception of the stirring mechanism, which was omitted from these tests, both the ISV and the SVI were determined in accordance with the specifications provided by the American Public Health Association (1989). In addition, the batch settling curves generated from these tests were also utilized to successfully calibrate the compressive gravity thickening model.

RESULTS AND DISCUSSION

Calibration from Continuous Thickening Data

As a first step, the method of Vaccari (1984) was used to calibrate the model from the continuous thickening data. The resulting model was then used to predict the batch settling curves. Figure 2 shows two of the settling curves, together with simulation results, for two of the one-liter cylinder batch columns, and including one of the highest initial concentrations used and one of the lowest. Both show close agreement with the linear portion of the curve, indicating that the ISV is accurately predicted. The interface velocity in the compression portion of the curve is also reasonably well predicted.

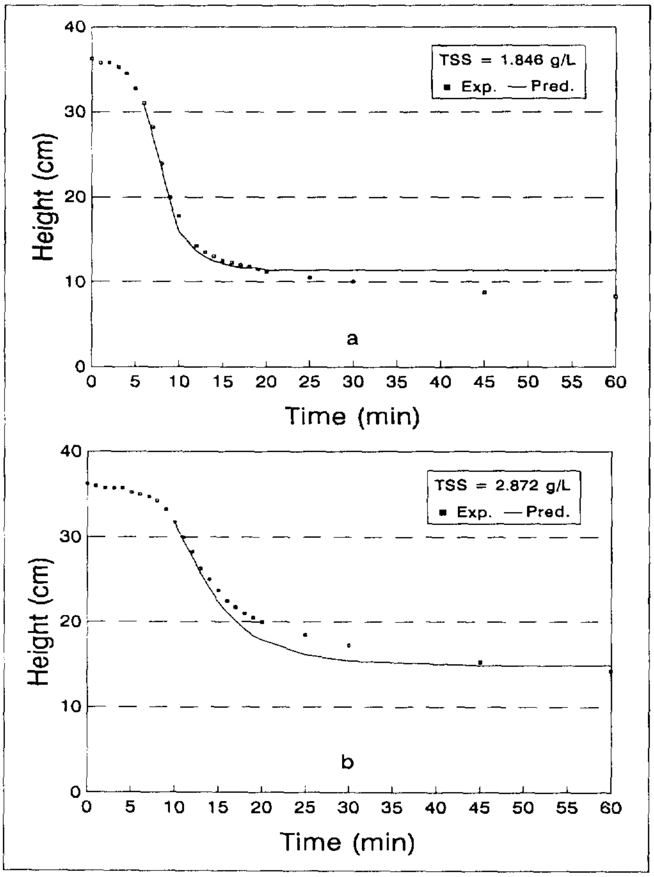

Fig. 2. Simulation of batch settling tests with calibration from continuous thickening experiments.

The model does not incorporate flocculation and so no attempt was made to simulate the portion before the linear segment. The prediction curve was shifted in time to make the first point in the linear segment match the data. It is significant that the model accurately predicts interface settling velocity until well into the compression phase. These results prove that the same set of model coefficients are capable of describing both batch and continuous thickening of a sludge. This is strong evidence validating the model for activated sludge thickening.

Calibration from Batch Thickening Data

However, it is of greater practical significance to be able to calibrate the model from batch experiments, which are much easier to conduct. A more severe test of the model would then be to predict continuous thickening experimental results from batch calibration. The down-hill simplex method was utilized for this purpose.

The parameters found using the algorithm were somewhat sensitive to the initial guesses used. In any case, all sets of parameters gave similar predictions for both batch and continuous systems. The parameters selected for further study below were the ones which gave the lowest *SSE*. Further research is needed to ensure that the algorithm will always find a global optimum.

The algorithm was successful for eight of the nine experiments conducted in one liter graduated cylinders as well as for the 120 cm high column. The single one liter batch thickening experiment which could not satisfactorily calibrate the model corresponded to an initial solids concentration of 4.205 g/L. This concentration was substantially higher than the initial concentrations associated with the other one liter tests. In this test the sludge interface height remained essentially constant throughout most of the 60 minute batch test. As a result of these and other experiments, it was concluded that reasonable results could only be assured with batch data if the 60 minute settled volume was no more than half the initial sludge volume.

The eight sets of model parameters were then utilized to simulate each of the eight steady-state concentration distributions obtained from the continuous thickening experiments. In each case the simulated concentration distributions were able to adequately predict the experimental distribution. The results were more favorable, however, when the batch calibration was performed with data obtained from the 120 cm batch thickening experiment. The best set of model parameters obtained when the down-hill simplex method was applied to the single batch settling curve from a 120 cm batch thickening test are:

B_1 = 3.4947 (cm-g/min-L)
B_2 = 1.4788 (g/L)
S = 30.0243 (g/L)
C_c = 2.723 (g/L)

Figure 3 shows the simulated batch settling curve, as well as the corresponding experimental settling data, using the above parameters. The high degree of fit was to be expected, since the fitting procedure is equivalent to a curve-fitting operation.

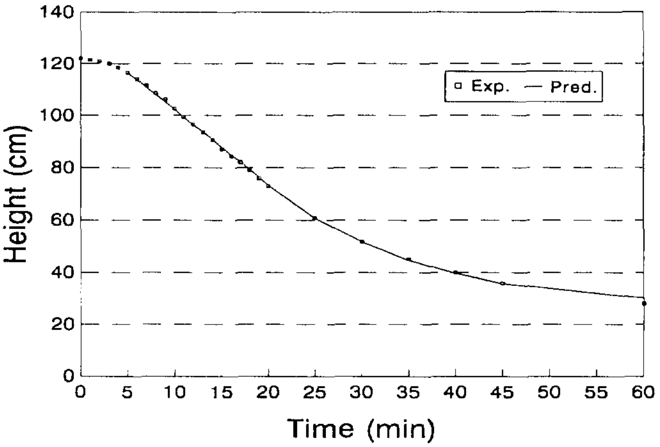

Fig. 3. Results of down-hill simplex optimization.

Of greater interest is how well the fitted model predicts the continuous thickening data. The eight steady-state continuous thickening experiments were simulated using the coefficients given above and the respective experimental solids loading rates and underflow velocities. The FORTRAN program which simulates the unsteady-state continuous thickening process was run until steady-state conditions were attained. The experimental and simulated concentration distributions for four of the continuous thickeners, selected to cover the range of experimental underflow concentrations, are depicted in Figure 4. The other

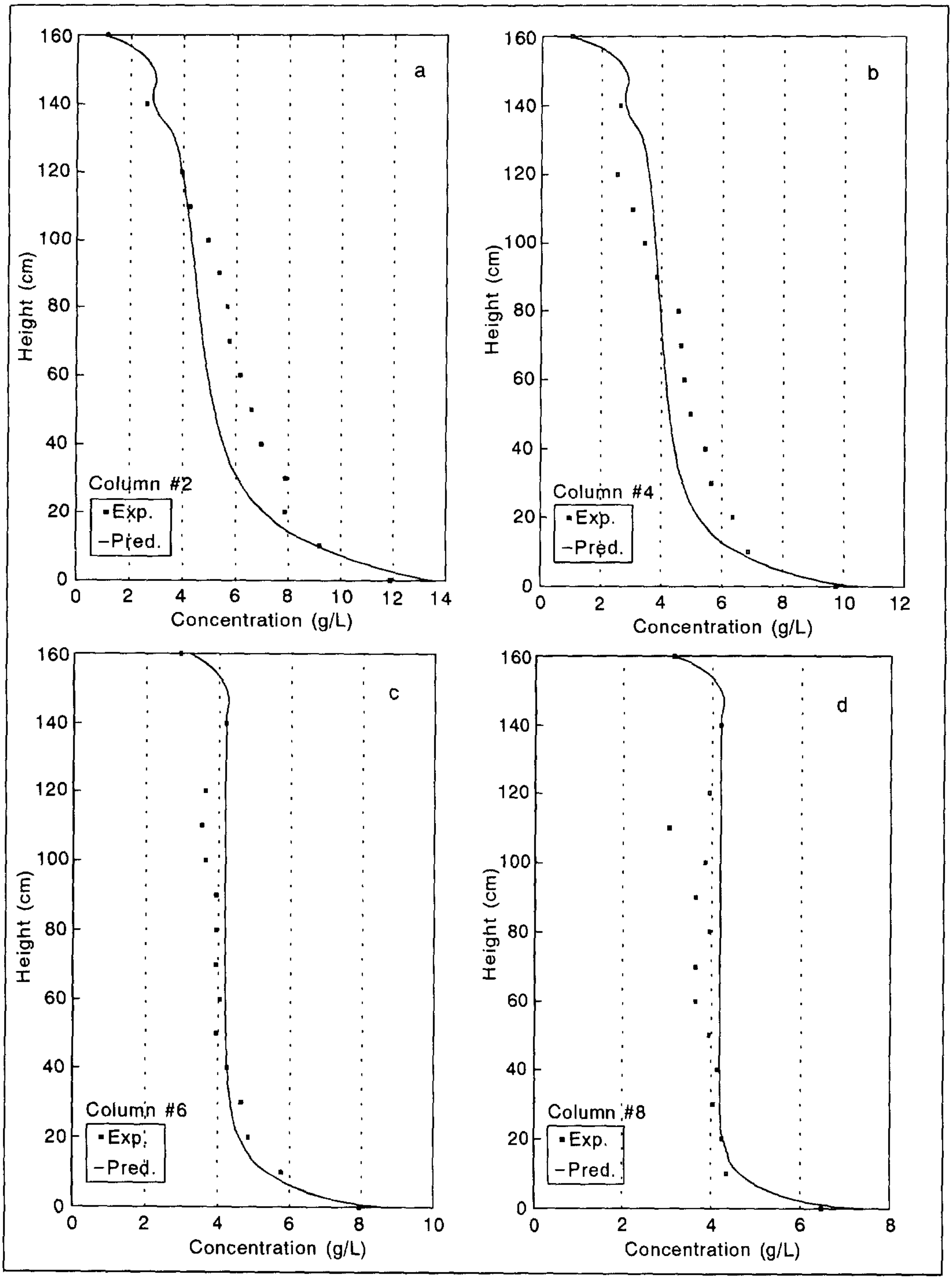

Fig. 4. Simulation of steady-state concentration distributions using batch calibration

four showed similar goodness-of-fit. Figure 4 shows that the predicted concentration distribution provided a reasonably good representation of the experimental results obtained in the continuous thickeners. The most significant trend evident was the over-prediction of the underflow solids concentrations. In the majority of the thickeners, however, the extent of this discrepancy was relatively insignificant.

CONCLUSIONS AND RECOMMENDATIONS

- Batch thickening experiments for fitting the compressive thickening model should be conducted in columns with a minimum depth of 120 cm and minimum diameter of 10 cm.
- Only settling curves which show a 60 minutes sludge volume, SSV_{60}, less than 50% of the initial volume, should be used for calibration purposes.
- A compressive gravity thickening model based on force and mass balances was capable of describing both batch and continuous thickening.
- The compressive thickening model could be accurately calibrated from a single batch settling test in order to predict continuous thickening concentration distributions.

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