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Discussion

Two-Dimensional Transient Solutions for Crossflow Heat Exchangers With Neither Gas Mixed[1] and Transient Temperature Fields in Crossflow Heat Exchangers With Finite Wall Capacitance[2]

F. E. Romie.[3] Spiga and Spiga use exponential functions and the modified Bessel functions I_0 and I_1 to express the Green's functions used to find the exchanger responses to variations of fluid inlet temperatures. The purpose of this discussion is to show that the closely related Anzelius–Schumann functions, $F_0(u, v)$ and $G_0(u, v)$ and their n-times successive integrals, $F_n(u, v)$, $G_n(u, v)$ with respect to the second argument (Romie, 1987) are well suited to expression of the Green's functions.

The three basic functions are

$$F_{-1}(u, v)=\frac{\partial F_0(u, v)}{\partial v}=\exp(-u-v)(u/v)^{1/2}I_1(2\sqrt{uv})$$

$$G_{-1}(u, v)=\frac{\partial G_0(u, v)}{\partial v}=\exp(-u-v)I_0(2\sqrt{uv})$$

$$G_0(u, v)=\int_0^v G_{-1}(u, v')dv'$$

Two recurrence equations permit finding $F_n(u, v)$ $(n \geq 0)$ and $G_n(u, v)$ $(n > 0)$ in terms of these three functions. In particular, $F_0(u, v) = G_0(u, v) + G_{-1}(u, v)$ and $F_1(u, v) = (v - u)\, G_0(u, v) + v(F_{-1} + G_{-1})$.

For gas-to-gas ($V_a = V_b = 0$) crossflow exchangers the Green's functions derived by Spiga and Spiga (1987) can be written in an especially simple form using the $F_n - G_n$ functions:

$$T_a^G(x, y, t)=\delta(t)e^{-x}+F_{-1}(x, t)F_0(Rt, y)$$

$$T_b^G(x, y, t)=G_{-1}(x, t)G_0(Rt, y)$$

$$T_w^G(x, y, t)=G_{-1}(x, t)F_0(Rt, y)$$

$$\overline{T}_a^G(t)=\delta(t)e^{-N_a}+F_{-1}(N_a, t)F_1(Rt, N_b)/N_b$$

$$\overline{T}_b^G(t)=(1-F_0(N_a, t))G_0(Rt, N_b)/N_a$$

In this case the only integration required is that for G_0 for which rapid algorithms are indicated in the reference cited.

For liquid-to-liquid exchangers ($V_a \neq 0$ and $V_b \neq 0$) the Green's functions derived by Spiga and Spiga (1988) can be written in the following form. (The variables θ and η, $\theta \equiv t - V_a x$ and $\eta \equiv \theta - V_b y'/R$, are introduced here for brevity of notation.)

$$T_a^G(x, y, t)=\delta(\theta)e^{-x}+U(\theta)\left\{e^{-R\theta}F_{-1}(x, \theta) + \int_0^{y^*} F_{-1}(x, \eta)F_{-1}(R\eta, y')dy'\right\}$$

$$T_b^G(x, y, t)=U(\theta)\int_0^{y^*} G_{-1}(x, \eta)G_{-1}(R\eta, y')dy',$$

$$y^*=\min(y, R\theta/V_b)$$

$$T_w^G(x, y, t)=U(\theta)\left\{e^{-R\theta}G_{-1}(x, \theta) + \int_0^{y^*} G_{-1}(x, \eta)F_{-1}(R\eta, y')dy'\right\}$$

$$\overline{T}_b^G(t)=\frac{1}{N_a}\int_0^{N_a} T_b^G(x, N_b, t)dx \quad \text{and}$$

$$\overline{T}_a^G(t)=\frac{1}{N_b}\int_0^{N_b} T_a^G(N_a, y, t)dy$$

In this case the integrals must be evaluated numerically because η contains the variable of integration, y'. Thus the advantage of the $F_n - G_n$ functions is limited to conciseness of expression of the Green's functions when $V_a \neq 0$ and $V_b \neq 0$. Use of the polynomial approximations for I_0 and I_1, given by Abramowitz and Stegun (1964), gives rapid evaluation of F_{-1} and G_{-1}.

References

Abramowitz, M., and Stegun, I. A., 1964, *Handbook of Mathematical Functions*, National Bureau of Standards, p. 378.

Romie, F. E., 1987, "Two Functions Used in the Analysis of Crossflow Exchangers, Regenerators, and Related Equipment," ASME Journal of Heat Transfer, Vol. 109, pp. 519–521.

Authors' Closure

The comments of Dr. Romie do not modify our work, but rather put the results in a formally more compact and elegant form. We acknowledge with pleasure the ingenuity and care of Dr. Romie in properly establishing at an explicit level recurrence properties that were only implicit in our equations.

We feel grateful to him for the achieved improvement, and are happy to see that our articles have stimulated some interest.

[1]By G. Spiga and M. Spiga, published in the May 1987 issue of the ASME Journal of Heat Transfer, Vol. 109, pp. 281–286.

[2]By M. Spiga and G. Spiga, published in the February 1988 issue of the ASME Journal of Heat Transfer, Vol. 110, pp. 49–53.

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