

to the surface co-ordinates, and they contain some terms which logically should be deleted.

It has been pointed out that one part of the energy of bending of a shell is due to rotations of elements of the middle surface about their normal lines. In order to reconcile Love's theory with that of the author, this effect must be neglected, since Love's equation cannot correctly account for the rotation, inasmuch as the derivative u_y is lacking. Dr. Koiter indicates that this effect is always negligible if the tangential displacements u and v are small, and the radii of curvature are large compared to the thickness. It then follows that all of the u and v -terms may be discarded from C_2 and C_2' (Equation [8] of the paper), since these terms account for rotations of elements of the middle surface about their normal lines. This conclusion is practically useful, since the u and v -terms in C_2 and C_2' are a source of difficulty in the integration of the differential equations of shells. If the u and v -terms are discarded from C_2 and C_2' , and quadratic terms in w are neglected, then $C_2 = C_2'$. This relationship eliminates an anomaly from the theory, since the shearing strain γ_{xy} (Equation [11]) is now given by a linear equation in z , i.e.

$$\gamma_{xy} = 2 (EG)^{-1/2} (C_1 + C_2 z)$$

The approximate expression for C_2 and C_2' is

$$C_2 = C_2' = \frac{E_y w_x}{2E} + \frac{G_x w_y}{2G} - w_{xy}$$

Strictly speaking, the strain energy of a shell is not uniquely determined by the displacements of the middle surface. Analogously, the strain energy of a beam is not uniquely determined by the curvature of the neutral axis. For example, equal and opposite forces, applied laterally to a beam, change the strain energy without changing the curvature of the neutral axis. In order to obtain a unique strain-energy expression for shells, the author has used the usual assumption that the stresses σ_x , τ_{xz} , and τ_{yz} are negligible. Also, quadratic z -terms in the strain tensor have been neglected. Within the limitations of these assumptions, the author's derivation of the linear terms in u , v , and w in the strain-energy expression is rigorous, since the analysis is purely formal mathematics. The fact that some of these terms, although of the first degree, are still negligible, was not contemplated. Dr. Koiter has raised an important point in dis-

cussing the relative magnitudes of these terms, since it is essential to discard, from the differential equations of shells, all terms which do not have primary significance, if solutions of special problems are to be achieved.

The Dynamic Response of a Simple Elastic System to Antisymmetric Forcing Functions Characteristic of Airplanes in Unsymmetric Landing Impact¹

R. L. BISPLINGHOFF.² The author has demonstrated the severity of unsymmetrical landings in producing dynamic stresses. Since landings involving side drift occur frequently in service, and are often unsymmetrical, the critical features demonstrated in the present paper are of considerable importance in dynamic-landing analyses. It should be observed that the increased dynamic response factor in unsymmetrical landings may not be the only reason why dynamic stresses in unsymmetrical landings are more severe. In many cases, the wing configuration and the landing gear location are such that the landing force is applied somewhat closer to the node line in the lowest symmetric mode than in the lowest antisymmetric mode.

The dynamic-response-factor curves for antisymmetric forcing functions given by the author make a valuable addition to those already in the literature for other kinds of disturbances.

AUTHOR'S CLOSURE

The author wishes to thank Professor Bisplinghoff for his comments. The author agrees that the location of the landing-gear station may be closer to the node line of the lowest symmetric mode than to that of the lowest antisymmetric mode, and that this is an important reason why unsymmetric landings may produce severe dynamic stresses in such a case.

¹ By J. B. Woodson, published in the September, 1949, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 71, pp. 310-316.

² Associate Professor of Aeronautical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.