

stress is

$$S_{ij}(M_B) = \sqrt{(S_{ij}(M_C))^2 + (S_{ij}(M_L))^2} \quad (3)$$

The foregoing maximum stress located somewhere inside the 0–90-deg segment. There is another maximum stress, with a reversed sign, located within the 180–270-deg segment.

In a cylindrical vessel, the situation is complicated by the irregular distribution of the stress. The stress due to  $M_C$  is distributed in a shape close to a shifted cosine curve, but the stress due to  $M_L$  is humped toward the neutral axis. Due to this off-axis peaking, it appears that an absolute sum may have to be taken to calculate the combined maximum stress. Nevertheless, in considering the fact that the stress field due to  $M_C$  is considerably narrower than a cosine distribution, equation (3) can still be used for cylindrical shells with good representation. In fact, this equation has been used by the piping code [3] since the 1950's.

Since the purpose of the calculation is to find the maximum stress intensity, the relative signs between the radial stress and the circumferential stress is important. Fortunately, this sign reversal only occurs at some of the circumferential membrane forces in WRC-297. One way of maintaining the sign is to take the  $S_{ij}(M_B)$  in equation (3) the same sign as that of the greater  $S_{ij}(M_C)$  and  $S_{ij}(M_L)$ . Even with this sign-preserving arrangement, the maximum membrane stress intensity calculated may still be smaller than the ones calculated at the four major axis points. However, the difference is insignificant. The stresses calculated at the four major axis points still need to be considered.

**Combined Normal Stress.** The combined maximum normal stress is determined by  $P$ ,  $M_C$ , and  $M_L$ . Since the stress due to  $P$  is uniform all around the attachment circumference, we can simply write

$$S_{ij} = S_{ij}(P) + S_{ij}(M_B) \quad (4a)$$

$$S_{ij} = S_{ij}(P) - S_{ij}(M_B) \quad (4b)$$

Equations (4a) and (4b) represent the maximum normal stresses at the two maximum points located on opposite sides of the attachment. Each equation further represents two stresses one at the outer, and the other the inner surface of the shell. These four locations are to be checked for the maximum stress intensity.

**Shear Stress due to  $M_T$ .** The shear stress due to torsional moment is uniform all around the attachment circumference. This stress can be expressed as  $SS(M_T)$ .

**Shear Stress due to  $V_C$  and  $V_L$ .** The shear stress due to  $V_C$  and  $V_L$  can be combined by

$$SS(V) = \sqrt{(SS(V_C))^2 + (SS(V_L))^2} \quad (5)$$

**Total Shear Stress.** The total maximum shear stress is the absolute sum of the shear stress due to torsion and the shear stress due to combined shear force. That is,

$$SS = SS(M_T) + SS(V) \quad (6)$$

This maximum shear stress generally does not occur at the same location as the maximum normal stress. However, since the shear stress is insignificant in most of the cases, it can be conservatively considered as occurring at the same location where the maximum normal stress occurs.

**Maximum Stress Intensity.** The stress intensity can be calculated by the maximum shear stress theory using the normal stress and shear stress calculated by equations (4) and (6), respectively. The WRC bulletins have given detailed formulas for this calculation. A total of four stress intensities representing the maximum and minimum stress points and both outside and inside surfaces should be calculated. The maximum value is then used for the design. To satisfy certain Code

[4] requirements, the maximum membrane stress intensity and the total stress intensity may also need to be separated.

## Conclusions

Regardless of the warning given by the WRC Bulletin 107 that there is no assurance that the absolute maximum stress intensity in the shell will be located at one of the eight points (four major axis points each having outside and inside surfaces) considered in the example calculations, many designers still use only the stresses calculated there for design. This practice creates inconsistencies in designs and may introduce as much as a 40-percent nonconservatism. The present article outlined the procedures for calculating the maximum stress intensities both at and off the major axis points. This maximum stress intensity should be used in the design evaluations.

## References

- 1 Wichman, K. R., Hooper, A. G., and Mershon, J. L., "Local Stresses in Spherical and Cylindrical Shells Due to External Loadings," WRC Bulletin No. 107, Aug. 1965, revised Mar. 1979.
- 2 Mershon, J. L., Mokhtarian, K., Ranjan, G. V., and Rodabaugh, E. C., "Local Stresses in Cylindrical Shells Due to External Loadings on Nozzles—Supplement to WRC Bulletin No. 107," WRC Bulletin No. 297, Aug. 1984.
- 3 ANSI Code for Pressure Piping, ANSI/ASME B31.3 Chemical Plant and Petroleum Refinery Piping, ASME, New York, 1984.
- 4 ASME Boiler and Pressure Vessel Code, Section VIII, Pressure Vessels, Div. 2, Alternative Rules, ASME, New York, 1983.

## DISCUSSION

### R. Natarajan<sup>2</sup>

At the outset, I would like to congratulate the author for bringing out certain important points which a designer sometimes forgets while using design charts. However, there are some points which are worth mentioning about this paper:

1 While discussing the inconsistency about the location of the maximum stress in a nozzle-spherical sheet intersection, it is expected that the designer will define the geometry and the loading using the same coordinate system. The location of the maximum stress, and hence the inconsistency in defining the maximum stress location, is due to the misunderstanding by the designer and not due to the examples given in WRC-107 or WRC-297.

2 While calculating the combined stress due to bending moments, mention should be made that the flexibility of the nozzle has not been completely considered. Further, the boundary conditions at the nozzle and cylinder ends also affect the value and location of these maximum values.

### K. Mokhtarian<sup>3</sup>

I have the following general comments to make on Peng's paper:

1 We have found that generally the maximum stress due to a longitudinal moment occurs at the 0-deg azimuth. We do not agree with the shape of the stress curve due to  $M_L$  in Fig. 3(b).

2 The last three sentences of the last paragraph in the subsection "Stresses due to  $M_C$  and  $M_L$ " are not clear and appear to contain conflicting statements.

3 Normally, the designer has to face the question of combining the stresses due to pressure with those due to

<sup>2</sup>Mankato State University, Mechanical Engineering Department

<sup>3</sup>CBI Na-Con, Inc., Oak Brook Engineering

mechanical loads. I do not know of any simple way of providing those guidelines now, but eventually this question will have to be addressed.

#### Z. F. Sang<sup>4</sup>

As stated in the paper by L. C. Peng, WRC-107 and WRC-297 published by PVRC are excellent references for calculating local stresses in nozzles and attachments. Indeed, they are widely used in the design of pressure vessels and have become indispensable tools.

The author summarizes inconsistencies occurring in some designs due to the designers misapplying the data presented in the aforementioned two documents. He also presents a method and procedure for calculating the maximum stress intensity. This is of importance and needs to be understood by designers. It should prove to be an aid in applying the two documents correctly.

I am in agreement with Dr. Peng's opinion about the inconsistency and nonconservation, which will be created in the design procedure if a designer cannot determine the maximum stress intensity. In the paper, the formula which is developed for calculating intensity seems to hold only for round radial nozzles and attachments on spherical shells. Only in this case are the stresses due to radial load  $P$  and torsional moment  $M_T$  uniform. For other shapes, particularly in the case of a rectangular attachment on a cylinder, the stresses are not uniform along the perimeter of the attachment.

In the section "Location of Maximum Stress" the author states that "the calculations involve only the secondary stress." From a stress classification point of view, stresses due to external load on an attachment include not only secondary stresses, but also primary ones. This is important, because there are different allowable stresses associated with different stress categories.

<sup>4</sup>University of Illinois at Chicago, Mechanical Engineering Department

With reference to the calculation of the maximum stress intensity, it is noted that the maximum shear stress generally is not located at the same point where the maximum normal stress occurs. But the author assumes that they do occur at the same locations. Is this a conservative assumption?

#### AUTHOR'S CLOSURE

In thanking Messrs. R. Natarajan, K. Mokhtarian, and Z. F. Sang for the valuable discussions, the author would like to make a brief closure.

This paper's main concern is the misapplication of the bulletins, not the validity of the bulletins which are excellent works. The nozzle flexibility and the vessel end condition, just as other geometrical parameters, have definite effects on the stress shape. The main point is if the interaction exists between the two moment components.

The off-axis peak stress due to  $M_L$  may not exist on small  $d/D$  vessels, but it does exist on other vessels, as demonstrated by Prof. Steel, and various pipe branch tests. There is indeed some confusion in the last three sentences concerning the stresses due to  $M_C$  and  $M_L$ . Because of the combination method proposed, the stress loses the orientation after the calculation. With the proposed sign tracking method, the maximum calculated membrane stress intensity may be occasionally smaller than the stress calculated at the four major corners. One way to correct the problem is to reverse one of the stress signs when the situation is detected. The author agrees that there is no simple way to combine the pressure and the mechanical load effects. Publication of some of the NRC approved methods, for instance, should be encouraged.

The secondary stress mentioned by Dr. Sang should have been more accurately stated as local stress. The inclusion of higher shear stress is always conservative in the calculation of the stress intensity when it is taken as twice the maximum shear stress.