Comparing the impact of time displaced and biased precipitation estimates for online updated urban runoff models

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ABSTRACT

When an online runoff model is updated from system measurements, the requirements of the precipitation input change. Using rain gauge data as precipitation input there will be a displacement between the time when the rain hits the gauge and the time where the rain hits the actual catchment, due to the time it takes for the rain cell to travel from the rain gauge to the catchment. Since this time displacement is not present for system measurements the data assimilation scheme might already have updated the model to include the impact from the particular rain cell when the rain data is forced upon the model, which therefore will end up including the same rain twice in the model run. This paper compares forecast accuracy of updated models when using time displaced rain input to that of rain input with constant biases. This is done using a simple time–area model and historic rain series that are either displaced in time or affected with a bias. The results show that for a 10 minute forecast, time displacements of 5 and 10 minutes compare to biases of 60 and 100%, respectively, independent of the catchments time of concentration.

Key words | data assimilation, radar, rainfall runoff, storm movement, uncertainty

INTRODUCTION

Time displaced input data and updated models

When using rain gauge data as input to urban drainage models there will usually be a time-offset error in the modelled runoff because the rainfall detected by the gauge and the measured runoff are not synchronized due to the time it takes for the rain to travel the distance between the rain gauge and the catchment. Multiple studies from England (Marshall 1980; Graham 2002) and Sweden (Niemczynowicz 1987) have found mean storm cell velocities close to 10 m/s, but also frequent events with much lower velocities. For illustration, if a gauge is situated 5 km from a catchment and a storm travels at a velocity of 5 m/s from the catchment towards the rain gauge, or vice versa, the rain data will be displaced 10 minutes in time compared to the actual rain hitting the catchment. The time-offset still exists when the storm direction is not parallel with the axis between the gauge and the catchment, but it is then harder to quantify since it depends upon the shape and spatial correlations within the rain cell. The existence of this time-offset error is common knowledge among modellers and can in offline simulations be corrected for by identifying the time-offset and displacing the modelled runoff in time before evaluating against measured runoff. For online models it is, however, not possible to identify the time-offset before well into an event, since a lot of data is needed to distinguish the time-offset from other error sources. It is therefore not possible, in online situations, to consequently displace the modelled runoff in time to take account of the time-offset, meaning that the time-offset will result in reduced model performance, as investigated by McCuen et al. (2006).

When a model is run online, it will preferably be updated by assimilating measured system data into the model to keep the model from drifting away from reality. In this article the term forecast is used when the updated model is run beyond the newest available system measurements based on rain data only (without considering whether the rain data are from measurements or forecasts). When a model has been updated based on system measurements, the state of the model is a result of the precipitation that has hit the ground up to the time of update. If the model forcing (rain input) used in the following forecast is not
completely synchronized with reality, the same part of the rain might be included in the model twice in case the rain forcing is delayed: first due to the update and then again due to the time displaced model forcing. If, on the contrary, the rain forcing is detected too early the update will eliminate the impact from part of the rain cell instead. Therefore the importance of correct timing of the precipitation input increases when the models are updated to reality.

Figure 1 illustrates an idealized example where the rain gauge data are delayed compared to the actual rain. Due to the time displacement of the model forcing, the forecast will end up exaggerating the peak runoff and the updating actually ends up worsening the model performance. Note that delayed forcing on an updated model will always result in larger peak values in the forecast, but that the opposite cannot be assumed when the rain data arrive too early. In the latter case the forecast will miss the rain that should enter the model right after the time of update, but will use rain from the immediate future instead. This will reduce the quality of the forecast but not make a noticeable difference to the statistical properties of the forecast.

The forecast in Figure 1 shows an example of an actual hydrological forecast way beyond the last update, as would be the case if the forecast were to be used in, e.g. model predictive warning and control. The forecasts that are part of a sequential data assimilation scheme will only last until the next update, and they will therefore not drift as far away from the observations as indicated on the figure.

The frequency of updates can depend upon the frequency of which system measurements can be acquired, but also upon the time it takes to compute the update, since many data assimilation techniques become very computationally burdensome for large and complex systems. If it takes 10 minutes to compute a system wide update and the model is required to produce the most recent state estimates at all times, then the model has to be propagated 20 minutes forward in time based on rain input only, in order to catch up with the 10 minutes spent in updating the model and the 10 minutes it takes to produce the following update. Therefore even an update procedure that could update a model to match reality perfectly would not remove the need for high quality rain data. Besides having a direct negative effect on the forecast quality, time displaced input data also reduce the effect of most data assimilation schemes since these are usually designed to analyse and correct amplitude errors, not synchronization errors in time (Ravela et al. 2007).

Rain data for urban runoff models

Rain gauges are the most widely used tool we have for quantifying precipitation intensity. There are, however, numerous commonly acknowledged conceptual problems associated with the use of rain gauge data for runoff modelling, since the rain gauge cannot measure the spatial variability or movement of the rain across a catchment, as investigated by Andersen et al. (1991), de Lima et al. (2009) and Seo & Schmidt (2012). Furthermore the shape and intensity of a rain cell is always changing so the rain cell that is detected by a gauge located outside a catchment will not be identical to the rain cell when it passes the catchment, making the time-offset error only one of numerous problems with the representativeness of rain gauge estimates. Despite these problems and despite the last decades' technological progress within the field of weather radars, the use of radar data for hydrological applications is still limited. Besides the fact that most hydrologists are accustomed to rain gauge data this can be because many radar rain estimates

Figure 1 | (Left) Situation where a rain cell passes over a catchment before moving towards a rain gauge situated outside the catchment. (Right) The corresponding observed runoff from the catchment and the modelled runoff based on the rain gauge data. The dashed line shows a single model forecast made from the time of update using a model that has been updated to match reality but using the (time displaced) rain gauge data as model forcing.
are still affected by significant errors that are difficult to quantify (Berne & Krajewski 2003). A meta study from McMillan et al. (2012) shows that the standard deviation of the error of radar rain estimates as a proportion of the rain rate typically lies in the range 0.5–0.5 for hourly data. In a recent thorough study of the uncertainties of radar rainfall estimates produced using the Hydro-NEXRAD algorithm it was found that ‘radar-rainfall uncertainty is characterized by an almost three times greater standard error at higher resolutions (15-minute and 0.5 km scale) than at lower resolutions (1-hour and 8 km)’ (Seo 2010). This means that it can be extra challenging to use radar data for urban catchments where the spatial and temporal scales are relatively small. Fortunately, the quality of weather radar precipitation estimates are continuously improved (e.g. Krämer & Verworn 2009), and in the recent years several research projects have shown some success in using radar data for urban runoff models (e.g. Thorndahl et al. 2013). Since radar data can be treated in many different ways in order to produce rain estimates the associated spatial and temporal uncertainties cannot be quantified in a general way. In the current study the radar uncertainty is simply represented as a constant bias.

The rather large uncertainty in short term rain estimates from weather radars is to some extent counterbalanced by the radars ability to estimate the spatial extent of the rain storm and to estimate where it is raining at a specific time. An investigation of the importance of the spatial distribution and extent of rainstorms can be found in Kitchen & Blackall (1992), Berne et al. (2004) and Vaes et al. (2005). The literature is, however, scarce on the interplay between updated models and time displaced input data.

Study aims

The aim of the current study is to compare the impact of temporal and quantitative errors in rain input for online runoff models that are being updated from system measurements. The quantitative error is chosen to be a multiplicative bias. This gives an idea about how good radar data should be before being equally good as data from a rain gauge situated outside the catchment – when disregarding all other rain gauge error sources than the time-offset. A lumped, single catchment, time-area model is used. This has the benefit, compared to a distributed model, of being insensitive to the speed, direction and shape of a rain cell passing the catchment. Furthermore it is shown that the time-area model makes it possible to obtain results that are independent of the area and time of concentration of the catchment. Thereby the choice of model makes it possible to isolate the effect of the time displacement when also assuming that the model is perfect, the rain gauge measures rain intensity 100% correctly – but time displaced, and the update procedure updates the model exactly to the true state of the system.

METHODOLOGY

Setup

The purpose is to investigate how big a bias rain forcing for a runoff model can be affected with before it corresponds to a given time displacement of the same data in terms of model forecast accuracy, when the model is updated from system measurements. This is done by selecting a rain series as being the true actual rainfall which is propagated through a simple time–area model to produce the true model states at any time. The true model states are used to update the model 100%, meaning that the updating simply replaces the model states with the true model states (without changing the model parameters). For all time steps the updated model is used as starting point for forecasts during which the only input to the model is perturbed rainfall data created by either displacing the true rain series in time or affecting it with an error. For all forecasts the forecast error is quantified and by interpolating between these results the effect of time displacing input data to an updated model can be quantified in terms of equivalent error on the input data. The quality of the various forecasts is held up against each other to find out how big a bias a given time displacement corresponds to. When using a squared error measure (such as RMSE or Nash–Sutcliffe Efficiency Index) to evaluate forecast quality and using a simple time–area runoff model, the results become independent of the catchment’s time of concentration as well as the direction of the time displacement and bias.

Rain data perturbations

A time series of rain gauge measurements is defined as being the actual precipitation \( P_{\text{true}} \) which is used to create the perturbed rain data \( P_t \) that are used as model forcing in the forecast period. The two perturbations \( P_{\text{disp}} \) and \( P_{\text{bias}} \) are used where \( P_{\text{disp}} \) is the result of time displacing \( P_{\text{true}} \) while \( P_{\text{bias}} \) is a result of scaling \( P_{\text{true}} \) with a constant bias. This means that the rain data used for the forecasts are perfect.
except for either a bias or a time displacement. The perturbations are constant for all events.

The biased rain data are produced by multiplying \( P_{\text{true}} \) with a constant:

\[
P_{\text{bias}}(t) = c \cdot P_{\text{true}}(t), \quad c = 1 \pm \frac{\text{bias}}{100}
\]  

(1)

where \( c \) is a scaling factor based on the bias that is a constant bias in percentage.

\( P_{\text{bias}} \) can be seen as a rough emulation of radar data or simply as biased rain gauge data. This perturbation possesses the desirable quality that the effect on the runoff is independent of the catchment characteristics of a linear runoff model; if the bias is 50% the runoff is likewise affected with a factor of 50%, so the results are generally interpretable.

The time displaced rain data are calculated as:

\[
P_{\text{disp}}(t) = P_{\text{true}}(t - a)
\]  

(2)

where \( a \) is the time displacement in minutes.

The model

The runoff model used is a simple linear time–area model where the runoff is calculated as a moving average of the recent precipitation (Chow et al. 1964):

\[
Q(t) = \frac{A}{T_c} \sum_{i=t-T_c}^{t} P(i)
\]  

(3)

where \( A \) is the effective area of the catchment, \( T_c \) is the time of concentration and \( P(t) \) is the precipitation depth for a time step.

The forecast

The observed runoff \( Q_{\text{obs}} \) is produced by propagating the true precipitation \( P_{\text{true}} \) through the model:

\[
Q_{\text{obs}}(t) = \frac{A}{T_c} \sum_{i=t-T_c}^{t} P_{\text{true}}(i)
\]  

(4)

When an \( x \) minute forecast is made with an updated model, the forecasted runoff \( Q_x \) at time \( t \) is a result of both \( P_{\text{true}} \) and the forecast model forcing \( P_l \) (which equals either \( P_{\text{disp}} \) or \( P_{\text{bias}} \)). At the time of update \((t - x)\) the equation for \( Q_x \) equals that of \( Q_{\text{obs}} \). When a forecast is made \( x \) time steps ahead of the time of update, the moving average of the runoff calculation includes \( x \) elements of \( P_l \) instead of \( x \) elements of \( P_{\text{true}} \). This can be formulated as:

\[
Q_x(t) = \frac{A}{T_c} \left( \sum_{i=t-T_c}^{t-x} P_{\text{true}}(i) + \sum_{i=t-x}^{t} P_l(i) \right), \quad \text{for} \quad x < T_c
\]  

(5)

where the summation of elements of \( P_{\text{true}} \) is what is left of the update at the time of forecast and the sum of \( P_l \) is the precipitation forcing used in the forecast period. As \( x \) approaches \( T_c \) the impact of the update diminishes and \( Q_x \) converges against a normal model simulation based on \( P_l \).

Forecast error

The forecast error \( e \) can be formulated from (4) and (5):

\[
e(t) = Q(t) - Q_{\text{obs}}(t)
\]  

\[
e(t) = \frac{A}{T_c} \left( \sum_{i=t-T_c}^{t-x} P_{\text{true}}(i) - \sum_{i=t-x}^{t} P_{\text{true}}(i) + \sum_{i=t-x}^{t} P_l(i) \right) = \frac{A}{T_c} \sum_{i=t-x}^{t} (P_{\text{true}}(i) - P_l(i)) = \frac{A}{T_c} \sum_{i=t-x}^{t} e_{PF}(i), \quad \text{for} \quad x < T_c
\]  

(6)

where \( e_{PF}(t) \) is the difference between the true rainfall and the perturbed rain data used as precipitation forcing during the forecasting period.

When comparing the effect of different model forcings on the forecasts using an error measure based on the sum of errors, squared or not, both \( A \) and \( T_c \) disappear. This means that if a 10 minute displacement of the forecast rain data results in the same sum of errors as a bias of 50% on the same data for a given forecast horizon, then this applies for all \( T_c \). Thus the results become independent of catchment characteristics as long as the forecast horizon is not above \( T_c \).

Nash–Sutcliffe

The Nash–Sutcliffe Efficiency Index (NSE) is used to quantify the accuracy of the forecasts. As can be seen from (7) NSE is deeply dependent on \( T_c \), but since \( T_c \) can be isolated to the denominator and since the forecast forcing is only represented in the nominator, the result of intercomparisons between performances of different input types will be the
same, regardless of catchment characteristics (Tc).

$$\text{NSE} = 1 - \frac{\text{SS}_\text{err}}{\text{SS}_\text{tot}} = 1 - \frac{\sum_{i=1}^{n} e(i)^2}{\sum_{i=1}^{n} (Q_{\text{obs}}(i) - Q_{\text{obs}})^2}$$

$$= 1 - \frac{\sum_{i=1}^{n} \left( \sum_{j=1}^{t} C_{i,j} P_{\text{true}}(t) - TcP_{\text{true}}(t) \right)^2}{\sum_{i=1}^{n} \left( \sum_{j=1}^{t} C_{i,j} P_{\text{true}}(t) \right)^2}$$

(7)

where $n$ is the total number of time steps.

**Symmetries**

The use of this idealized setup results in some desirable properties of symmetry in the results with respect to the perturbations of the rain data. When the performance is calculated from the sum of the squared errors it does not matter whether a time displacement is forward or backwards in time, or whether a bias is positive or negative. For the bias this comes directly from the squaring of the error term. In regards to the displaced data the symmetry can be acknowledged by considering the rules of infinite series saying that:

$$\sum_{n}^{\infty} |a_n - a_{n+c}| = \sum_{n}^{\infty} |a_n - a_{n+c}|$$

(8)

where $a$ is an arbitrary infinite time series.

In our case this means the sum of the squared errors are the same irrespective of the direction of the time displacement.

**Probabilistic interpretation**

A major part of the uncertainty related to radar rain estimates is relating to choosing the correct conversion between radar reflectivity and rain intensity. For a calibrated weather radar this is unlikely to result in rain estimates that are constantly biased throughout multiple events, as assumed in the previous section. In the following it is assumed that an unknown event-wise constant conversion factor exists, which can relate radar reflectivity to actual rain intensity, and that the relationship between the chosen conversion factor and the true conversion factor is log-normal distributed with a median of one and some realistic upper and lower boundaries. In the results section the impact of having such a log-normal distributed event-wise constant scaling of the rain data is investigated. By defining events as being separated by a period without rain of at least the time of concentration of the catchment in question, the runoffs from each event become independent of each other. This makes it possible to calculate the expected value of the sum of squared errors SS$_\text{err}$ (and thereby the expected value of NSE) for a given distribution of the scaling factor $c$ without doing Monte Carlo model runs. This follows from:

$$E(\text{SS}_\text{err}) = E\left( \sum_{n}^{\infty} \text{SS}_\text{err,n} \right) = \sum_{n}^{\infty} E(\text{SS}_\text{err,n})$$

$$= \sum_{n}^{\infty} \int_{-\infty}^{\infty} \text{SS}_\text{err,n}(c)f(c)dc$$

$$= \int_{-\infty}^{\infty} \sum_{n}^{\infty} \text{SS}_\text{err,n}(c)f(c)dc = \int_{-\infty}^{\infty} f(c) \sum_{n}^{\infty} \text{SS}_\text{err,n}(c)dc$$

(9)

where SS$_\text{err,n}(c)$ is the sum of squared errors from the $n$'th event and $f(c)$ is the probability density function for $C$. That is: the expected value of SS$_\text{err}$ when $c$ is varying from event to event can be estimated by probability weighing SS$_\text{err}$ from multiple model runs with constant $c$ for all events.

**RESULTS AND DISCUSSION**

The results shown in this section are produced using 10 years of data (1995–2005) from rain gauge 28184 of the national Danish tipping bucket network (Jørgensen et al. 1998).

Figure 2 (left) shows NSE as a function of the forecast time for an updated model with a Tc of 40 minutes for various time displacements and biases of the rain data. The NSE starts at one for a forecast of 0 minutes since this just compares two identical model runs. After Tc minutes the NSE becomes constant since there is no updated part left in the model and the NSE becomes a result of a model run with the perturbed rain input only. For other Tc’s the shape of the figures would be the same, except that they would be constant from the Tc used and the lower range of NSE would be different. It can be seen that the time displaced data perform better the shorter the forecast horizon is, measured in NSE. Figure 2 (right) shows the performance of time displaced data in equivalent performing biased rain data, found by calculating the intersections between the
NSE results using biased and displaced data, respectively. As an example, it can be seen from Figure 2 (left) that the use of 5 minute time displaced rain data leads to a NSE of 0.95 for a 20 minute forecast and this is also the case when using 40% biased rain data. Accordingly, Figure 2 (right) shows that 5 minute time displaced rain data correspond to a bias of 40%.

When looking at both panels in Figure 2 it can be concluded that, even though the time displaced data perform better in terms of NSE for the shorter forecast horizons, this is actually where time displacements have the biggest negative impact compared to biased input data. Note that the time displaced data perform worse than 100% biased rain data for forecast horizons shorter than approximately the time displacement of the data, meaning that use of the displaced rain data has actually made the model perform worse than using no rain data at all.

Figure 3 has been made in a similar way as Figure 2 (right) but with the distribution of an only event-wise constant (mis-)scaling of the rain data as reference instead of constant bias in percentage. The scaling has been chosen to lay in the interval $e^{\pm \ln(5)}$ ($[-0.2-5]$) and be log-normally distributed with zero mean and standard deviation $\sigma$ of the variables’ natural logarithm. On the vertical axis of Figure 3...
the exponential function of $\sigma$ is plotted. Approximately 68% of the events have been affected with a scaling between $1/\exp(\sigma)$ and $\exp(\sigma)$. This means that for a 10 minute forecast, radar data can be scaled with a factor of more than two for 32% of the events and still perform better than rain gauge data with a 10 minute time displacement.

From Figure 2 and Figure 3 it can be concluded that the importance of having accurate precipitation data grows with the forecast horizon, while for the very short forecasts the timing of the rain data is more important. This implies that the choice of model and data assimilation scheme is affected by the precipitation data available. If radar data are available the benefit of choosing a simple fast model that can be updated frequently becomes relatively larger than if only rain gauge data from gauges situated outside the catchment are available.

For big distributed models a data assimilation scheme is unlikely to be able to update the entire model, but this does not change the conclusion drawn above since the time displacement error will continue to exist at local scale in the model once the impact of the model forcing propagates down to the updated part of the model.

A real system is of course always more complex than the simplified setup used in this study. In reality the hydraulic response time will be dependent on the amount of water in the system and therefore some of the effects from time displacement of the input data will also be present for quantitative imprecise rain data. A similar study can also be made using a distributed model. This will, however, give rise to difficulties in terms of producing general interpretable results, since the properties of the individual model will affect the results.

Rain estimates from a well calibrated radar are unlikely to be constantly biased for longer periods in time, but will rather vary between over- and underestimation during events. This will result in a smaller negative impact on the model than a constant bias, as used in the current study, and therefore the results shown in this article can be interpreted as worst case results for radar rain estimates.

The speed, direction, spatial extent and correlations, as well as the temporal development of the rain cells might in many cases have bigger impact on the forecasted runoff than the time-offset of the rain input, but these effects have previously been investigated by others and are beyond the scope of the current study. Furthermore, the fact that the speed and direction of rain cells might change during an event is ignored in the current simplified setup. That this is a reasonable simplification is to some extent confirmed by Renard et al. (2012), who found that intense rainfall cells in France travel in a straight linear movement with a near constant speed.

**CONCLUSION**

For online models that are being updated from system measurements, the precision of the measurements of the rain input is not always as important as measuring at the right locations. Even when disregarding the spatial variability of the rain and the dynamic nature of rain cells, the consequence of using point measurements located outside catchments modelled is quite severe due to the lack of synchronization between the model forcing and the system data used to update the model. Weather radars measure rain at the right place at the right time but are seldom as accurate in terms of rain intensity as rain gauges. In the current study it was quantified how big an impact time displaced rain data have on the quality of the forecasts from updated models. This was quantified both in absolute terms as well as in equivalent performing biased rain data. By using simple time-area models and selecting a rain series as being the actual rain that was used for creating the true runoff and model states as well as various perturbed rain series, the impact of having time displaced rain data for an updated model was found. The results showed that for a 10 minute forecast, time displacements of 5 and 10 minutes compare to rain data biases of 60 and 100%, respectively, in terms of model forecast quality. The impact of the time displacement decreases as the forecast horizon increases. The results are independent of the catchment’s time of concentration. The results show that even in the case where precipitation estimates from a radar have substantially lower accuracy than rain gauge data, the radar data might still be the better choice as rain data for an online updated model.

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REFERENCES


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