will result in compressive stresses, and research is being continued to discover what they are.

**Conclusions**

1. Grinding stresses in a ball-bearing steel, quenched and tempered to Rockwell hardness C90, do not exhibit the high tensile values found close to the surface in an annealed tool steel of comparable carbon content, ground under similar conditions. They also do not penetrate as deeply as in the annealed steel.

2. Compressive stresses can be developed in the surface of quenched and tempered ball-bearing steel by abrasive-wheel grinding.

3. The effect of wheel grade upon residual grinding stresses is small. Very soft wheels appear to reduce the magnitude and depth of penetration somewhat.

4. In general, increasing the unit downfeed increases both the magnitude and depth of penetration of the resulting stresses. The change is small for unit downfeeds up to 0.001 in. when grinding with a soft wheel.

5. Other conditions being the same, use of different grinding fluids can result in large differences in the residual-stress distributions.

**Acknowledgment**

The author wishes to acknowledge the very considerable contribution of the Norton Company in furnishing the test bars, and of Dr. L. P. Tarasov, under whose direction the bars were prepared and the grinding tests performed. He also wishes to acknowledge the assistance of Mr. R. C. Isler and Mr. J. M. Lesio in carrying out the stress analyses.

**Bibliography**


**Appendix**

Stäblein (5) showed that, in a bar containing a uniaxial residual stress, the stress at any level \( w \) is given by

\[
\sigma'(w) = \frac{E a^2}{6} \frac{dC(w)}{dw} + \frac{2Ew}{3} C(w)
\]

- \( \frac{E a^2}{6} C(0) - \frac{E}{3} \int_0^w C(z)dz \) \ldots \ldots \[2]\]

when the bar is held straight by external torques. If the stress is biaxial, each of the principal stresses can be represented by expressions similar to Equation [2]

\[
\sigma_1'(w) = \frac{E a^2}{6} \frac{dC_1(w)}{dw} + \frac{2Ew}{3} C_1(w)
\]

- \( \frac{E a^2}{6} C_1(0) - \frac{E}{3} \int_0^w C_1(z)dz \) \ldots \ldots \[3]\]

where \( C_1(w) \) is the curvature which \( \sigma_1'(w) \), acting alone, would produce. If \( C_2(w) \) is the corresponding curvature produced by \( \sigma_2(w) \), the actual curvatures resulting from the combined stresses are

\[
C_1(w) = C_1(w) - \nu C_2(w)
\]

and

\[
C_2(w) = C_2(w) - \nu C_1(w)
\]

provided that the deflections are small in comparison with the thickness of the specimen (6). Solving simultaneously

\[
C_1(w) = [(C_1(w) + \nu C_2(w))/(1 - \nu^2)]
\]

is obtained. Substituting into Equation [3] leads to Equation [1]. The principal stresses in a specimen free to bend to its equilibrium curvatures, unrestrained by external forces, are given by

\[
\sigma_i(w) = \frac{E}{6(1 - \nu^2)} \left\{ \frac{dC_i(w)}{dw} + \nu \frac{dC(w)}{dw} \right\}
\]

\[
+ 4\nu(C_1(w) + \nu C_2(w)) + 2(w_0 - 3w) [C_1(w_0) + \nu C_2(w_0)]
\]

\[
- 2 \int_0^w [C_1(z) + \nu C_2(z)]dz \ldots \ldots \[5]\]

and a similar expression for \( \sigma_2(w) \), with subscripts 1 and 2 interchanged (3). The difference between \( \sigma_1'(w) \) and \( \sigma_2(w) \) is seen to be

\[
\sigma_1'(w) - \sigma_2(w) = \frac{E}{1 - \nu^2} \left( \frac{w - w_0}{2} \right) [C_1(w_0) + \nu C_2(w_0)] \ldots \ldots \[6]\]

The principal stresses in the restrained condition thus exceed those in the unrestrained condition by just the stresses induced in straightening the specimen from its initially curved state.

**Discussion**

A. L. Christenson and W. E. Lettmann\* This paper contributes valuable information on a subject which has provoked much discussion but all too few experimental results. The high quality of the data reflects the precision which has become familiar in the author's work.

It would be interesting to know whether the abrupt maxima or minima in the stress-distribution curves for cross-feed grinding will also be present in plunge-ground samples. Does the author have any information on stress distribution in plunge-ground-hardened steel? Was cross-feed used in the preparation of the sample of Fig. 8?

X-ray measurements\* of surface stress in the direction of grinding have been made on samples carburized and hardened or through-hardened to 60 Rockwell C and ground according to good commercial practice.\* The results are similar to those shown in Fig. 8. An interesting observation is that a given grinding condition appears to establish a characteristic surface stress regardless of the sign or magnitude of the prior stress due to heat-treatment. For example, for a prior surface stress of 130,000 psi in compression, grinding produced about 50,000 psi in compression, and for a prior stress of 50,000 psi in tension, grinding produced 30,000 psi in compression.

An over-all view of the stresses produced by grinding leads one to speculate as to their origin. Temperature measurements

\* The Timken Roller Bearing Company, Canton, Ohio.


\* Unpublished research of A. L. Christenson.
within ground samples show that grinding can produce sufficient nonuniform thermal expansion in the work to cause compressive yielding of the material just beneath the ground surface, thus generating a residual tensile stress. If heating is insufficient to cause yielding, the cold work of the surface in the chip formation may produce the compressive stresses which have been observed by the author and the writers. The fact that similar grinding conditions impose less penetration of tensile stresses in hardened steel than in annealed steel is consistent with the foregoing concept, because a higher peak temperature would be required to cause compressive yielding in hardened steel.

J. F. Frisch. The author has presented an interesting study of residual grinding stresses which are found to be of a compressive nature under certain conditions. However, residual stresses, induced in a stress-free material by grinding only, have in the past been shown to be tensile stresses on and immediately below the ground surface. The residual stresses which were determined by the author in hardened and ground specimens sectioned from fatigue coupon must be two or more superimposed residual stresses induced by a combination of processes including grinding.

This writer's recent experience in finding considerable compressive residual stresses in high-strength alloy steels heat-treated to various ultimate strength up to 280,000 psi leads one to believe that residual tensile stresses caused by grinding such steels and superimposed on the existing compressive stresses may not necessarily be large enough to nullify them. The finally measured stress values would still be negative, i.e., compressive, but not necessarily pure grinding stresses.

Specimens A and B in Fig. 1 of the paper were removed by means of a cutoff wheel from the hardened and ground fatigue test bars for etching and curvature measurements. Since the residual stresses in manganese oil-hardening tool-steel specimens, which were cut to final shape before hardening and grinding, are reported to be tensile stresses, two questions arise with regard to the different results:

1. Does the use of the abrasive cutoff wheel cause residual stresses in the specimen, similar to those produced by a hacksaw or jeweler's saw, which may be large enough to influence the original stress patterns considering the 1 to 1 ratio of length to width of the specimens?

2. Does the removal of specimens from fatigue test bars cause them to relax before an etchant is applied, and have curvature changes been recorded during and after the use of the cutoff wheel?

Although it is most likely that proper heat-treatment and a unique combination of variables in a subsequent grinding process may produce final residual compressive stresses, the fact that abrasive-wheel grinding can develop compressive stresses would be verified only if either initially stress-free specimens are used, or the exact stress due to a process such as heat-treatment is known.

J. A. Mueller. It is gratifying to hear that grinding can produce stresses that not only fail to injure the surface but also leave it in a condition that may be desirable from the end use of the ground surface.

The pattern of stresses, as found by the author, has a remarkable parallel in the efficiency or grinding ratio of the grinding wheel. The writer would like to bring out this parallel in an attempt to correlate stress with grinding ratio and ultimately, if possible, to set up a means of choosing grinding wheels to produce favorable stresses or eliminate unfavorable stresses without the complexity of lengthy computations.

The author indicates that the stresses produced by a change in wheel grade ranging from a shallow stress in the soft end of the grade range to a maximum in the middle of the hardness range and then to a lesser stress in the hard end. The grinding ratio, we have found, parallels this stress pattern remarkably. Fig. 10 of this discussion shows the grinding ratio of a range of surface-grinding wheels. The soft wheel produces a low grinding ratio, and as the grade of the wheel becomes harder the grinding ratio becomes greater until it reaches a maximum and then falls off. In the soft end of the grade range, continuous breakdown occurs and this continues until the wheel becomes too hard for the operation and then discontinuous breakdown sets in. The wheel loads and unloads and breaks out rather than wears away. The grinding ratio strikingly follows the stress pattern as the wheel grade is changed.

In the stress pattern at unit downfeeds ranging from 0.0001 in. to 0.002 in., the author found that there was little difference in the stresses up to a downfeed of 0.001 in. There was a significant change at a downfeed of 0.002 in. Fig. 11, herewith, shows the grinding ratio at various downfeeds. At the smallest downfeed the grinding ratio is small and at heavier feeds the grinding ratio becomes greater until a maximum is reached and then proceeds to fall off. Again the stress pattern shows a remarkable similarity to the grinding ratio.

The third parallel between stress and grinding ratio occurs when grinding with straight oil, an oil-water emulsion, and air. Grinding oil produced higher stresses than the oil-water emulsion.
Fig. 12 shows the grinding ratio for straight oil, and oil-water emulsion, and dry. Grinding oil produced the highest grinding ratio, air was the next highest, and the emulsion was the least efficient.

If the author were to have ground in air at the 0.002-in. downfeed the stress pattern might have fallen between the straight oil and the water-emulsion curves.

In summary there is a striking similarity between the stress pattern and the grinding ratio, and it leads to the conclusion that in any grinding operation a compromise must be effected between any or all of the multiplicity of operations that a grinding wheel is called upon to perform. We must compromise among production rate, wheel life, dimensional tolerance, stress generation, form-holding ability, surface finish, stock removal, and many other operations.

**Author's Closure**

The author wishes to thank the discussers for their interesting and helpful contributions to the paper.

The question of cross-feed grinding versus plunge grinding raised by Messrs. Christenson and Littmann is extremely pertinent. The author has no data on plunge-ground surfaces. Cross-feed grinding was normally employed in grinding the test bars to an intermediate thickness of 0.140 in.; however, the exact cross-feed procedure used for the final 0.0005-in. cut on each surface of bar No. 202 (Fig. 8 of the paper) is not known. The fact that such markedly different stress distributions were obtained with a 3-in.-wide wheel suggests that wheel width and cross-feed are important factors for future study.

It is encouraging to learn that another laboratory, using grinding procedures probably somewhat different from those described in the paper and determining stresses by an independent method, finds stress distributions similar to those in Fig. 8. The observation of Messrs. Christenson and Littmann that a given grinding operation establishes a characteristic surface stress, relatively independent of any initial stress resulting from heat-treatment, is substantiated by experience in our own laboratory. In fact, there is a good deal of evidence that each pass of the wheel wipes out existing residual stresses to the depth to which it plastically deforms the surface and introduces its own characteristic stress. If the stress initially present extends deeper than the plastic deformation by grinding, the stress distribution resulting from grinding joins that of the previously existing stress at a depth corresponding to the lower edge of the layer deformed by grinding.

The picture of the origin of grinding stresses proposed by Messrs. Christenson and Littmann is, in general, one to which the author subscribes. The substantial difference between $\sigma'$ and $\sigma''$ near the surface and their rather close agreement at depths below 0.001 in. is consistent with this picture.

Professor Frisch points out that tensile grinding stresses have previously been found on the surface of annealed steel (2,3) and suggests that the compressive surface stresses found in the present work on hardened steel are a composite of those caused by grinding plus those from some other source, probably heat-treating. Aside from the fact that there appears to be no inherent reason why the grinding stress distributions in quenched and tempered specimens should be identical with those found in annealed specimens of different chemical composition, the suspicion that the results presented in the paper are appreciably affected by initial thermal stresses is not confirmed by experimental data.

Fig. 13 of this closure shows the residual stress in the surface of a typical bar as heat-treated. Recalling that the intermediate grinding removed 0.010 in. and the test grinding another 0.010 in., the initial thermal stress to be considered is that at a depth of 0.020 in. and below in Fig. 13. In this range it does not exceed 1000 psi in absolute value, which is of no significance in com-
parison with the grinding stresses. A further indication of the insignificance of the initial stress is given by Fig. 8 which shows the initial stresses in both surfaces of a typical bar at the beginning of the test grinding. It is quite apparent that the removal of 0.010 in. from each surface gets well below any stress initially present.

![Fig. 13 Initial Stress in Typical Bar After Heat-Treatment](image)

It does not seem likely that the residual stresses induced in the edges of a specimen by the careful parting procedure described in the paper can influence the determination of stresses in the test surface appreciably. The greatest depth of stress penetration encountered in the surface-grinding experiments, even under the severest conditions tested, was only about 0.008 in. It seems reasonable that stresses set up in the edges of the specimen by the side of the cutoff wheel, operating as described, would be smaller and penetrate less deeply. Irrespective of their magnitude, the fact that they are confined to such a thin layer on the edges practically precludes the possibility of their affecting the determination of stresses in the surface of interest.

![Fig. 14 Above, Specimen Parted From Bar and Then Surface-Ground; Below, Bar Surface-Ground and Then Specimen-Parted](image)

This reasoning is substantiated by the graphs in Fig. 14 which also help to answer the question of relief of grinding stresses by the parting operation. The upper graph represents the stresses in a 2-in. square specimen, cut from a bar by the procedure outlined, and then surface-ground. The lower graph represents the stresses in a specimen of the same dimensions cut from another bar (No. 214, Fig. 7 of the paper) after surface grinding. The grinding conditions were nominally the same except that the upper curves were obtained with a 38A461-KEVBE wheel while the lower curves were obtained with a 38A46-M6VBE wheel. Taking the comparatively small effect of wheel grade upon residual grinding stresses into consideration, it is apparent that the distributions in Fig. 14 agree with each other as closely as those from the top and bottom surfaces of the same specimen. It is perhaps worth pointing out that the steep negative gradients close to the surface in Fig. 14 appear to be characteristic of dry grinding on steel of the composition and hardness used in the experiments.

Since the results presented in the paper do not appear to have been influenced appreciably either by initial stresses or by the parting procedure, it is logical to believe that the difference between grinding-stress distributions previously published (2, 3) and those in the present paper is primarily due to differences in grinding conditions and in the chemical composition and heat-treatment of the specimens. For purposes of clarification, it should be pointed out that all of the specimens discussed in references (2) and (3) were annealed and ground dry.

The parallelism in behavior between grinding ratio and residual stress pointed out by Mr. Mueller is indeed interesting. If one chooses either the peak tensile stress or the net area under the stress-distribution curve as his criterion, there is some indication that high stress is inevitably the consequence of high grinding efficiency. This conclusion is supported by a comparison of Fig. 5 with Fig. 10, and Fig. 6 with Fig. 11. Although, in the latter case, the range of unit downfeeds in Fig. 6 lies below that in Fig. 11, experiments in our laboratory with both annealed and fully hardened tool steel have shown that the area under the stress curve increases with unit downfeed and reaches a maximum in the neighborhood of 0.004 to 0.006 in.

The correlation is confused somewhat by the stress data obtained with different types of grinding fluid. Although the data for the 0.002-in. unit downfeed in Fig. 7 are consistent with the view that stress and grinding ratio go hand in hand, those for the 0.001-in. unit downfeed support the opposite conclusion.

The peak tensile stress is certainly to be regarded with suspicion in so far as its influence upon service life is concerned; nevertheless, there are other features of grinding-stress distributions which may be of comparable importance. For example, bar No. 216 (Fig. 7) has comparatively high tensile peaks at a depth of about 0.001 in. at the surface it has high compressive stresses. At present it is not known what effect, if any, these two features of the stress distribution, either individually or in combination, may have upon service life.

Irrespective of the criterion used, the variation of stress with either wheel grade or unit downfeed (Figs. 5 and 6) is modest in comparison with the differences exhibited by bar No. 204 (Fig. 6) and bar No. 202 (Fig. 8), yet both of these bars were ground with soft wheels at the same final unit downfeed. It is our lack of knowledge concerning the causes for large differences such as these which emphasizes the need for residual stress studies on a much wider range of grinding conditions.