

Water level stabilization in open channels using Chebyshev polynomials and teaching–learning-based optimization

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ABSTRACT

Water level fluctuations in open channels can cause serious problems for conveyance systems which lead to economical and performance issues. The problem of water level stabilization in open channels containing two pumping stations at both ends is investigated in this paper. The optimal control that should be imposed at one end of the channel to minimize water surface fluctuations due to sudden increase in water demand at the other end is determined.

Chebyshev polynomials are effectively hybridized with a modern teaching–learning-based optimization algorithm for the first time to estimate the optimal control. An efficient shock-capturing model is used to evaluate the value of the objective function. Results are compared with those obtained by the variational approach. The optimal control found by the proposed method shows fewer fluctuations in water level than that found by the variational method. Moreover, the proposed approach is easier to implement and to extend for solving more general problems.

Key words | optimal control, Saint-Venant equations, teaching–learning-based optimization (TLBO), unsteady open channel flow, water level stabilization

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INTRODUCTION

Minimization of water level fluctuations in open channels is of paramount practical importance. Large fluctuations in the water level in channels result in many problems such as severe loss of water, bank erosion, reduction in efficiency and equipment damage in the irrigation plant. Hence, in order to reduce water losses, conveyance costs and irrigation problems, and for better management of the irrigation system, effective control of such undesirable fluctuations is very important. As a practical instance, unsteady flow in an open channel occurs by an abrupt change in the flow rate at the downstream pumping station due to sudden increase in water demand. As a result, the upstream pumping station should also be engaged to increase the water supply at the other end of the channel. Consequently, sudden changes of discharge in both upstream and downstream ends of the channel develop undesirable waves which travel along downstream and upstream of the channel (Chow 1959). If the

upstream control is not optimized, the resulting waves produce large oscillations in the water level which can cause the aforementioned problems within the channel. According to the importance of monitoring water levels in open channels and sewer systems, water level control is usually made in real time using technology, based on water level sensors and related controllers (Campisano & Modica 2002a, b; Pleau *et al.* 2005; Campisano *et al.* 2009).

The problem of stabilization of water level in open channels can be considered as a practical optimization problem in water management. The optimal control that should be imposed at one end of the channel to minimize water surface fluctuations can be determined. In other words, undesirable waves developed in either end of the channel can be efficiently canceled by an appropriate flow control in the opposite end. Despite being an interesting and practical subject, the problem of finding the optimal control for water-level

stabilization has not been sufficiently studied. [Atanov & Borovik \(1995\)](#) utilized Lagrange multipliers on the basis of Saint-Venant equations to find an optimal control to decrease undesirable fluctuations within a channel involving two pumping stations at both ends. [Atanov et al. \(1998\)](#) followed a variational approach to minimize water-level deviations from a desired value to find an optimum inflow control. They idealized the problem to a frictionless channel with a trapezoidal cross-section and proposed a rather intricate approach by employing some techniques in calculus of variations to find a solution for such a simplified problem. Unfortunately, it is too formidable to extend their procedure to more general cases such as a channel with an arbitrary cross-section in which the friction is not negligible. Consequently, an alternative general approach should be sought.

Metaheuristic and evolutionary optimization algorithms such as ant colony, honey-bee mating optimization, and particle swarm optimizations are powerful tools to solve difficult engineering problems. These optimization methods have been already employed to solve practical problems in water engineering such as water distribution systems (e.g. [Farmani et al. 2006](#); [Keedwell & Khu 2006](#); [Suribabu & Neelakantan 2006](#)), wastewater collection networks (e.g. [Izquierdo et al. 2008](#); [Iqbal & Guria 2009](#)), reservoir operation and hydropower (e.g. [Haddad et al. 2008](#); [Mousavi & Shourian 2010](#); [Fu et al. 2011](#)), water quality management (e.g. [Margeta 1984](#); [Ostfeld & Salomons 2004](#); [Prasad & Walters 2006](#)), groundwater management (e.g. [Sedki & Ouazar 2011](#)) and operation of pumps (e.g. [Moradi-Jalal & Karney 2008](#); [Nourbakhsh et al. 2011](#)).

To extend the application of such metaheuristic algorithms to solve the problem of water level stabilization in open channels, two important points should be taken into account which are discussed in the following.

First, abrupt changes in flow rates in both ends of the channel develop strong shocks in the flow domain which should be treated accordingly in flow simulation models. Several well-known shock-capturing schemes can be used to deal with shocks ([Toro 1997, 2001](#); [Mingham & Causon 1998](#); [Fujihara & Borthwick 2000](#); [Liang et al. 2007](#); [Baghlani 2011](#)). Owing to the fact that in metaheuristic iterative algorithms, the value of the objective function should be evaluated several times; a numerical model with sufficient accuracy and low computational cost is recommended to simulate the flow with shock-capturing capacity.

Secondly, estimation of the optimal control in the optimization procedure should be carefully carried out. In a naive approach, the optimal control can be estimated by a polynomial of order n with unknown coefficients. The optimization algorithm can then be used to determine the optimal values for unknown coefficients in the polynomial, and a proper flow simulator is employed to determine the value of the objective function for each solution candidate. However, there are two main drawbacks in such a simple technique. First, most metaheuristic optimization algorithms search for optimal values within a specified range of design variables. In other words, the lower and upper limits of each unknown coefficient in the polynomial should be specified in advance. Depending on the order n , it is a formidable task to determine an appropriate range for each coefficient separately, since they may vary from large negative to large positive values. On the other hand, considering a wide range for each variable extends the searching space remarkably and makes the algorithm ineffective. Generally, a prior knowledge about the pattern of the optimal control is necessary which is not available in all cases. Second, shallow water equations cannot be solved with arbitrary boundary conditions. If the imposed control (as the upstream or downstream boundary condition) is inappropriate, zero or negative water depths might happen in the channel and numerical instabilities occur. Randomly changing the coefficients of the polynomial in the optimization algorithm increases the probability of generating such inappropriate boundary conditions. In this case, discarding these infeasible solutions reduces the efficiency of the searching procedure due to the waste of a large number of solution candidates.

In this article, a new interesting approach is used to solve the problem of water level stabilization in open channels involving two pumping stations in both ends by employing a modern metaheuristic algorithm. Chebyshev polynomials are used to approximate controls and to remove the aforementioned drawbacks of utilizing an n th order polynomial. For this purpose, a new metaheuristic algorithm called teaching-learning-based optimization (TLBO) is effectively hybridized for the first time with Chebyshev polynomials to find the optimum control. The effectiveness of this technique in water level stabilization is investigated through minimizing fluctuations in an open channel and comparing the results with those found by a

variational approach (i.e. Atanov *et al.* 1998). The results show that the optimal control found by the proposed approach decreases water surface fluctuations compared to the previous method and also improves the simplicity of the optimization procedure.

PROBLEM FORMULATION

The channel shown in Figure 1 has a finite length of L . The non-uniform water depth for steady state flow is $H_0(x)$ in which x denotes the longitudinal direction. In the unsteady flow case, the channel depth (and velocity) is a function of both distance x and time t and it can be denoted by $H(x, t)$.

The unsteady flow is governed by well-known Saint-Venant equations as follows (Chanson 2004):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad (1a)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} (QV + gA\bar{H}) = gA(S_0 - S_f) \quad (1b)$$

in which $A(x, t)$ is the cross sectional area of the flow, $Q(x, t)$ is the flow rate, $V(x, t)$ is flow velocity, \bar{H} is the vertical distance between water surface and the centroid of the cross section, S_0 is the channel's longitudinal slope, S_f is the energy slope which represents the effect of friction and g is gravitational acceleration. The first equation is the continuity equation and the second one is the momentum equation.

In an unsteady flow, the problem is subjected to the following initial conditions:

$$Q(x, 0) = \phi_1(x) \quad (2a)$$

$$H(x, 0) = \phi_2(x) \quad (2b)$$

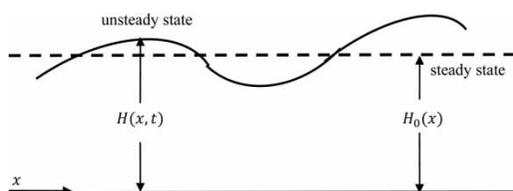


Figure 1 | Definition of parameters in open channel flow.

where both $\phi_1(x)$ and $\phi_2(x)$ are known functions since initial conditions are specified. In the case of subcritical flow, either the water depth or discharge rate must be specified on both ends of the domain to determine the water surface profile within the domain. In this study, the flow regime is considered to be subcritical and the channel is assumed to be bounded by pumping stations on both ends, imposing boundary conditions on the discharge as

$$Q(0, t) = \eta(t) \quad (3a)$$

$$Q(L, t) = \psi(t) \quad (3b)$$

It is assumed that the operation of the downstream pumping station is specified and hence the function $\psi(t)$ is known. For a specified function $\psi(t)$ acting on the downstream end, the problem is to find the optimal control (or controlling function) $\eta(t)$ subject to governing Equations (1), initial and boundary conditions (2) and (3b), such that the water level deviation from the initial and desired water depth $H_0(x)$ is minimized.

METHODOLOGY

A solution method for finding the optimal control, which should be applied at one end of the channel in order to stabilize the water level, is presented in this section. This is accomplished by first defining the objective function of the problem. Then, Chebyshev polynomials and the TLBO algorithm are hybridized to find the optimal control. These procedures are illustrated in the following subsections.

Objective function in water level stabilization problem

To represent the problem as an optimization one, the first step is to define an objective function. Once the objective function is determined, the optimization problem can be solved via any mathematical or numerical approach. Mathematically, the objective function is an averaged

value of deviations from $H_0(x)$ in all times and distances as

$$f = \frac{1}{Lt_f} \int_0^{t_f} \int_0^L |H(x, t) - H_0(x)| dx dt \quad (4)$$

in which t_f is a given transient duration (duration time) in which the solution is sought. Clearly, since $H(x, t)$ depends on the upstream boundary condition $\eta(t)$ defined in Equation (3a), the value of the objective function f also depends on $\eta(t)$.

As a mathematical solution, *Atanov et al. (1998)* developed a variational approach by making some simplifications on the problem such as assumption of frictionless channel with a trapezoidal cross section. After a relatively complicated procedure, they proposed a solution for optimum control $\eta(t)$. In this study, a general case is considered and a numerical-mathematical procedure is developed for finding the controlling function $\eta(t)$ based on Chebyshev polynomials and a population-based optimization algorithm. The proposed approach removes the limitations of the previous mathematical procedure in which simplified assumptions were made.

In a numerical solution of Equations (1a) and (1b), the domain of the problem is discretized both spatially and temporally. Therefore, the value of the objective function can be approximated as follows:

$$f = \sum_{j=1}^k \sum_{i=1}^n |H(x_i, t_j) - H_0(x)| \quad (5)$$

where $x_k = L$ and $t_n = t_f$.

Approximation of the optimal control by Chebyshev polynomials

A special class of polynomials is Chebyshev polynomials which are especially suited for approximating other functions. Chebyshev polynomials are extensively used in many areas of numerical analyses such as uniform approximation, least-squares approximation, numerical solution of ordinary and partial differential equations, and so on. The

Chebyshev polynomials of the first kind of order n are defined as (*Snyder 1966; Rivlin 1974*):

$$T_n(t) = \text{Cos}[n\text{Cos}^{-1}(t)], \quad t \in [-1, 1], \quad n = 0, 1, 2, \dots \quad (6)$$

Chebyshev polynomials $T_n(t)$ satisfy the following recurrence formula:

$$T_0(t) = 1, \quad T_1(t) = t, \quad T_{n+1}(t) = 2tT_n(t) - T_{n-1}(t) \quad (7a-c)$$

For the function $\eta(t)$ defined at $n+1$ selected points $t_0 < t_1 < \dots < t_n$, the basic idea is to approximate the optimal control $\eta(t)$ by $Q_n(t)$ using Chebyshev polynomials as

$$Q_n(t) = a_0T_0(t) + a_1T_1(t) + \dots + a_nT_n(t) = \sum_{k=0}^n a_kT_k(t) \quad (8)$$

such that

$$Q_n(t_i) = \eta(t_i) \quad \text{for } i = 0, 1, \dots, n \quad (9)$$

and accordingly, the unknown coefficients a_k will be optimized properly.

Although Chebyshev polynomials are basically defined in the interval $t \in [-1, 1]$, approximation of a function on an arbitrary interval $\tilde{t} \in [t_0, t_f]$ is a simple task by transforming the function to translate the t -values into $[-1, 1]$ by

$$t = 2 \frac{\tilde{t} - t_0}{t_f - t_0} - 1 \quad \tilde{t} \in [t_0, t_f] \quad (10)$$

Therefore, without loss of generality, the approximation can be done in the interval $[-1, 1]$ and then the values are readily transformed to the interval $[t_0, t_f]$ by means of Equation (10).

It is known that equally spaced nodes, on which the interpolation is carried out, develop very strong oscillations near the endpoints of the interval for some functions. The uniformity of the error in the interval of interpolation can be remarkably improved by selecting the interpolation nodes t_i carefully on the interval $[-1, 1]$ as (*Snyder 1966*)

$$t_i = \text{Cos} \left[\left(i + \frac{1}{2} \right) \frac{\pi}{n+1} \right], \quad i = 0, 1, \dots, n \quad (11)$$

This selection of nodes extends the range of functions for which convergence takes place and eliminates the problem of bad behavior of approximation near the endpoints of the interval.

As mentioned in the introduction, it is awkward to consider the unknown coefficients a_k in Equation (8) as design variables. Alternatively, suppose that the approximation of optimal control $Q_n(t)$ is known at $n + 1$ selected nodes t_i as

$$Q_n(t_i) = Q_i \quad i = 0, 1, \dots, n \tag{12}$$

From Equations (8) and (12) the following relations hold:

$$\begin{aligned} a_0T_0(t_0) + a_1T_1(t_0) + \dots + a_nT_n(t_0) &= Q_0 \\ a_0T_0(t_1) + a_1T_1(t_1) + \dots + a_nT_n(t_1) &= Q_1 \\ \vdots & \\ \vdots & \\ a_0T_0(t_n) + a_1T_1(t_n) + \dots + a_nT_n(t_n) &= Q_n \end{aligned} \tag{13}$$

The system of equations in (13) can be written in a matrix form as:

$$\begin{bmatrix} T_0(t_0) & T_1(t_0) & \dots & T_n(t_0) \\ T_0(t_1) & T_1(t_1) & \dots & T_n(t_1) \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ T_0(t_n) & T_1(t_n) & \dots & T_n(t_n) \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_n \end{bmatrix} \tag{14}$$

Considering the values of optimal control at selected nodes, i.e. Q_i as design variables rather than unknown coefficients, a_k is more reasonable in an optimization procedure:

$$X = [Q_0 \quad Q_1 \dots Q_n]^T \tag{15}$$

in which X is the vector of design variables.

Since the amount of abrupt changes in one pumping station (e.g. water demand increases 50%, at the downstream station) and the initial state of the flow rate in steady state are usually both known, one could have a reasonable estimation of extreme values of the optimal

inflow control, whereas approximating the range of changes of coefficients a_k in the polynomial is very difficult. Moreover, a unique interval such as

$$Q_i \in [Q_{\min}, Q_{\max}] \quad i = 0, 1, 2, \dots \tag{16}$$

can be specified for all variables Q_i , contrary to a_k .

In a population-based optimization algorithm, when the initial population is generated in the specified interval $[Q_{\min}, Q_{\max}]$, the unknown coefficients a_k can be obtained for each design vector X by solving the system of Equation (14). Once the coefficients are determined, the approximation of optimal control in any specified time t_s can be readily accomplished by

$$\eta(t_s) = [a_0 \quad a_1 \dots a_n] \begin{bmatrix} T_0(t_s) \\ T_0(t_s) \\ \vdots \\ \vdots \\ T_n(t_s) \end{bmatrix} \tag{17}$$

which can be used in flow simulation to impose upstream boundary conditions at each time step. Then, the value of the objective function, i.e. the amount of water surface fluctuations within the channel can be calculated by means of Equation (5) for this design vector. In the next step, the optimization algorithm is followed to obtain optimum values for Q_i and hence an approximation of $\eta(t)$ is made.

The aforementioned procedure overcomes the problem of generating unphysical boundary conditions in numerical solution of flow as well.

Optimization algorithm

Although any efficient population-based optimization algorithm can be employed with the proposed method, a modern TLBO is used in this study. TLBO is a recently developed metaheuristic approach which was first proposed by Rao *et al.* (2011, 2012). Most aspects of TLBO are similar to evolutionary algorithms (Crepinsek *et al.* 2012).

TLBO mimics teaching and learning capabilities of a teacher and learners in a classroom. It is a population based method consisting of two stages, i.e. the teacher

phase and the learner phase. In the teacher phase, learners learn through the teacher. In the whole population, the best solution is considered as the teacher (X_{teacher}). Then, the teacher attempts to increase the mean result of the class to his/her level. For stochastic purposes, two randomly-generated parameters r in the range of 0 and 1 and T_F are applied in the updated formula for the solution X_i as

$$X_{\text{new}} = X_i + r \cdot (X_{\text{teacher}} - T_F \cdot X_{\text{mean}}) \quad (18a)$$

$$T_F^i = \text{round}[1 + \text{rand}(0, 1)\{2 - 1\}] \quad (18b)$$

where X_{new} and X_i are respectively the new solution and the existing one of i , and T_F is a teaching factor which can be either 1 or 2 as indicated by Equation (4b) (Rao *et al.* 2011, 2012; Crepinsek *et al.* 2012).

In the learner phase, learners can increase their knowledge by interacting with others. Hence, a student will learn new information if others have more knowledge than him/her. During this stage, the student X_i interacts randomly with another student X_j ($i \neq j$) in order to improve his/her knowledge. In the case that X_j is better than X_i (i.e. $f(X_j) < f(X_i)$ for minimization problems), X_i is moved toward X_j . Otherwise it is moved away from X_j

$$X_{\text{new}} = X_i + r \cdot (X_j - X_i) \quad \text{if } f(X_i) > f(X_j) \quad (19)$$

$$X_{\text{new}} = X_i + r \cdot (X_i - X_j) \quad \text{if } f(X_i) < f(X_j) \quad (20)$$

If the new solution X_{new} is better, it is accepted in the population. The algorithm will continue until the termination condition is met. The pseudo code of TLBO is shown with more detail in Table 1.

UNSTEADY FLOW SIMULATION

In order to find the optimal control $\eta(t)$ for the specified simulation time T and channel length L , an unsteady non-uniform flow simulation with shock-capturing capability is necessary. This numerical model is used to evaluate the objective function f in Equation (5) in the optimization algorithm. The numerical scheme should be shock-

Table 1 | The pseudo code for TLBO

```

Set  $k = 1$ ;
Objective function  $f(x)$ ,  $x = (x_1, x_2, \dots, x_d)^T$   $d = \text{no. of design variables}$ 
Generate initial students of the classroom
  randomly  $X^i$ ,  $i = 1, 2, \dots, n$   $n = \text{no. of students}$ 
Calculate objective function  $f(X)$  for whole students of the classroom
WHILE (the termination conditions are not met)
  {Teacher Phase}
  Calculate the mean of each design variable Mean
  Identify the best solution (teacher)
  FOR  $i = 1 \rightarrow n$ 
    Calculate teaching factor  $T_F^i = \text{round}[1 + \text{rand}(0,1)\{2 - 1\}]$ 
    Modify solution based on best solution(teacher)
     $X_{\text{new}}^i = X_k^i + \text{rand}(0, 1)[X_{\text{teacher}} - (T_F^i \cdot X_{\text{mean}})]$ 
    Calculate objective function for new students  $f(X_{\text{new}}^i)$ 
    IF  $X_{\text{new}}^i$  is better than  $X^i$ , i.e.  $f(X_{\text{new}}^i) < f(X_k^i)$ 
       $X_{k+1}^i = X_{\text{new}}^i$ 
    END IF {End of Teacher Phase}
  {Student Phase}
  Randomly select another learner  $X_j$ , such that  $j \neq i$ 
  IF  $X^i$  is better than  $X^j$ , i.e.  $f(X^i) < f(X^j)$ 
     $X_{\text{new}}^i = X_k^i + \text{rand}(0, 1)(X_k^i - X_k^j)$ 
  Else
     $X_{\text{new}}^i = X_k^i + \text{rand}(0, 1)(X_k^j - X_k^i)$ 
  END IF
  Calculate objective function for new students  $f(X_{\text{new}}^i)$ 
  IF  $X_{\text{new}}^i$  is better than  $X^i$ , i.e.  $f(X_{\text{new}}^i) < f(X_k^i)$ 
     $X_{k+1}^i = X_{\text{new}}^i$ 
  END IF {End of Student Phase}
END FOR
Set  $k = k + 1$ 
END WHILE

```

capturing since abrupt changes of flow rates at upstream and downstream pumping stations cause strong shocks within the flow domain. On the other hand, since the simulation of flow should be carried out several times in the optimization procedure, a robust and satisfactorily accurate model with low computational cost is recommended. In this study, a TVD-MacCormack scheme is adopted for this purpose. TVD-MacCormack is a shock-capturing scheme with satisfactory accuracy and low computational cost compared to other numerical schemes.

To simulate the flow with the TVD-MacCormack scheme, first, the one-dimensional governing Equations (1a) and (1b) are written as

$$\frac{\partial S}{\partial t} + \frac{\partial F}{\partial x} = C \quad (21)$$

in which

$$S = \begin{pmatrix} A \\ VA \end{pmatrix}, \quad F = \begin{pmatrix} VA \\ V^2A + gA\bar{H} \end{pmatrix}, \quad C = \begin{pmatrix} 0 \\ gA(S_0 - S_f) \end{pmatrix} \quad (22a,b,c)$$

S is the vector of conserved variables, F is the flux vector and C is the source term. The energy slope can be expressed by means of Manning roughness coefficient n as

$$S_f = \frac{n^2 V^2}{R^{4/3}} \quad (23)$$

The TVD-MacCormack scheme combines the well-known MacCormack scheme (MacCormack 1971) with the total variation diminishing (TVD) approach to build a robust high-resolution scheme. The MacCormack method is a two-step predictor-corrector scheme. In the predictor stage, a backward difference discretization is used whereas in the corrector stage a forward difference discretization is employed. This type of discretization agrees well with both upstream and downstream travelling waves associated with positive and negative eigenvalues of the problem. A TVD stage of the procedure has been added to the MacCormack scheme by Davis (1984) and developed by others (Mingham & Causon 1998; Liang *et al.* 2006). The whole procedure can be summarized as follows.

The predictor step:

$$S_i^p = S_i^n - (F_i^n - F_{i-1}^n) \cdot \Delta t / \Delta x + C^n \cdot \Delta t \quad (24)$$

The corrector step:

$$S_i^c = S_i^n - (F_{i+1}^p - F_i^p) \cdot \Delta t / \Delta x + C^p \cdot \Delta t \quad (25)$$

The TVD step:

$$S_i^{n+1} = 0.5(S_i^p + S_i^c) + [G(r_i^+) + G(r_{i+1}^-)] \cdot \Delta S_{i+1/2}^n - [G(r_{i-1}^+) + G(r_i^-)] \cdot \Delta S_{i-1/2}^n \quad (26)$$

where Δx and Δt are spatial and temporal steps, respectively. The superscript n denotes time step and the subscript i the

node number. Moreover, the following relations hold:

$$\Delta S_{i+1/2}^n = S_{i+1}^n - S_i^n, \quad \Delta S_{i-1/2}^n = S_i^n - S_{i-1}^n \quad (27a,b)$$

$$r_i^+ = \frac{\langle \Delta S_{i-1/2}^n, \Delta S_{i+1/2}^n \rangle}{\langle \Delta S_{i+1/2}^n, \Delta S_{i+1/2}^n \rangle}, \quad r_i^- = \frac{\langle \Delta S_{i-1/2}^n, \Delta S_{i+1/2}^n \rangle}{\langle \Delta S_{i-1/2}^n, \Delta S_{i-1/2}^n \rangle} \quad (28a,b)$$

Each pair of brackets indicates the scalar product of the two vectors inside.

The function G which has been employed to ensure TVD property of the scheme is defined as:

$$G(x) = 0.5 \times \sigma \times [1 - \phi(x)] \quad (29)$$

in which the flux limiter function $\phi(x)$ has been employed to suppress the spurious numerical oscillations and is defined as follows:

$$\phi(x) = \max(0, \min(2x, 1)) \quad (30)$$

and the parameter σ is:

$$\sigma = \begin{cases} CFL \times (1 - CFL) & CFL \leq 0.5 \\ 0.25 & CFL > 0.5 \end{cases} \quad (31)$$

where CFL is the well-known Courant–Friedrichs–Lewy number defined as

$$CFL = \frac{\Delta t}{\Delta x} (|u| + c) \quad (32)$$

where $c = \sqrt{gH}$ is the wave celerity.

Boundary conditions

Although there exist various boundary conditions in open channel flow problems depending on the flow regime, in most practical cases the flow rate in the upstream boundary and the water depth in the downstream boundary are specified in the case of subcritical flows. In the current problem with pumping stations at both ends, proper boundary conditions should be developed which are discussed as follows.

Upstream boundary

According to the theory of characteristics, Riemann invariants are defined as follows (Wang *et al.* 2000; Chanson 2004):

$$R^- = u + 2c, \quad R^+ = u - 2c \quad (33a, b)$$

which are constant on characteristics $dx/dt = u + c$ and $dx/dt = u - c$. Neglecting the contribution of the source term in short distance between boundaries and adjacent nodes, R^- is constant for the upstream boundary. Hence:

$$u_R + 2\sqrt{gH_R} = u_B + 2\sqrt{gH_B} \quad (34)$$

where subscript B is used for boundary and subscript R is used for the adjacent node on the right of the boundary. This equation can be used if water depth H_B is known at the upstream boundary. In the case of known discharge:

$$q_{in} = H_B u_B \quad (35)$$

in which

$$q_{in} = \eta(t)/W \quad (36)$$

is discharge per unit width of the channel and W is the channel width.

Combining Equations (34) and (35) and eliminating u_B yields

$$q_{in} - H_B u_R - 2H_B \sqrt{g} (\sqrt{H_B} - \sqrt{H_R}) = 0 \quad (37)$$

For each design vector in the aforementioned optimization procedure, $\eta(t)$ is computed in each time step using Equation (17) and hence q_{in} is determined by means of Equation (36). Then, the nonlinear Equation (37) is solved using the Newton–Raphson method to obtain H_B . Consequently, the boundary velocity u_B is calculated using Equation (35) and the boundary flux can be then evaluated by these known values.

As illustrated in this section, the choice of control $\eta(t)$ directly affects boundary conditions of the problem and hence fluctuations of water surface.

Downstream boundary

Analogous to the upstream boundary by conserving R^+ in the downstream boundary, the following equation is derived after a simple manipulation

$$q_{out} - H_B u_L - 2H_B \sqrt{g} (\sqrt{H_L} - \sqrt{H_B}) = 0 \quad (38)$$

where subscript L is used for adjacent node on the left side of the downstream boundary and

$$q_{out} = \psi(t)/W \quad (39)$$

is the outflow discharge per width of the channel.

APPLICATION

Application of the proposed technique in stabilization of water surface fluctuations in an open channel is investigated through a design example. This problem has been already studied by Atanov *et al.* (1998) using a variational approach and their results are compared with the results of the current study.

Suppose a steady state flow in an open channel with a rectangular cross section having the length of $L = 20$ km and the width of $W = 30$ m. The flow rates of the supplying (upstream) and withdrawing (downstream) pumping stations are initially the same. The initial and desired water depth in the channel is $H_0 = 3.6$ m with an initial flow rate equal to $100 \text{ m}^3/\text{s}$. The unsteady flow in the problem starts by increasing downstream pumping station withdrawal rate. The downstream flow demand is increased by 50% so that $\psi = 150 \text{ m}^3/\text{s}$. The flow is simulated by TVD-MacCormack scheme with spatial discretization of $\Delta x = 400$ m. A finer grid has no significant effect on the accuracy of the results. Simulation starts at $t_0 = 0$ and appropriate time steps are automatically calculated using Equation (32) by imposing $CFL = 0.9$ to ensure stability of the scheme. Moreover, the total simulation time is considered to be $t_f = 4 \text{ hr} = 14,400 \text{ sec}$.

If the controlling upstream flow rate $\eta(t)$ is not optimized, the only way to maintain the water level in the channel is obviously matching the downstream flow rate,

i.e. to also increase the flow rate at the upstream (supplying) pumping station by 50%. The dashed line in Figure 2 shows this state. An alternative and more effective way is to impose an appropriate control on the upstream pumping station so as to cancel undesirable waves developing in the opposite direction from the downstream pumping station. As mentioned earlier, Atanov *et al.* (1998) proposed such an optimal control using a variational approach. The curve shown by solid line in Figure 2 depicts the upstream control obtained by Atanov *et al.* (1998).

For the case of no control at the upstream end, time series of the water surface elevation at the two ends of the channel are as shown in Figure 3. As shown in the figure, the abrupt change of flow rates at both pumping stations develops strong waves which move in opposite directions along the channel. A positive wave moves toward the

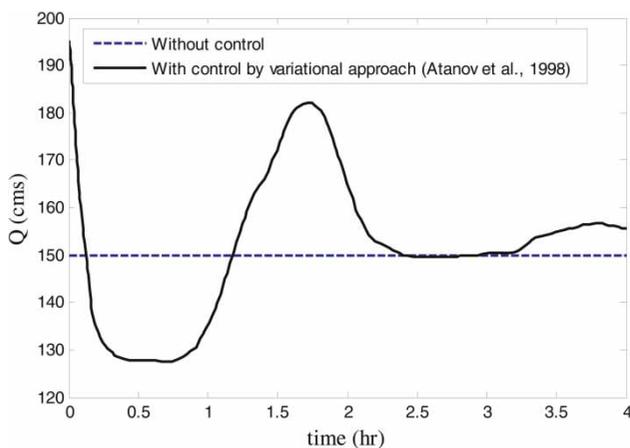


Figure 2 | Optimal control (after Atanov *et al.* 1998) versus no control at the upstream end.

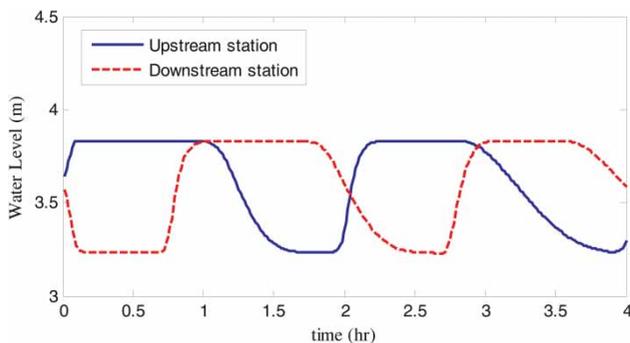


Figure 3 | Upstream and downstream time series of water surface level when no control (dashed line in Figure 2) is imposed at the upstream end.

downstream, while a negative wave moves toward the upstream. However, the waves do not simply cancel each other out; when they reach the end of the channel, they reflect. This process develops considerable fluctuations within the channel as shown in 3D view in Figure 4.

The value of the objective function in this case for the whole length of the channel and over the whole 4 hours of the simulation time is considerable ($f = 2396.33$ m). To compare the results found by the proposed technique with those obtained by the variational approach, the results of imposing the control found by Atanov *et al.* (1998) are presented as well. If this upstream control (solid line in Figure 2) is imposed, the desired flow rate starts at a larger value than the downstream flow rate as shown in the figure. As Atanov *et al.* (1998) have mentioned, this is necessary to compensate for the negative wave moving in the upstream. The upstream flow rate then oscillates around the value corresponding to the downstream flow rate. The time series of water level fluctuations at both ends are depicted in Figure 5

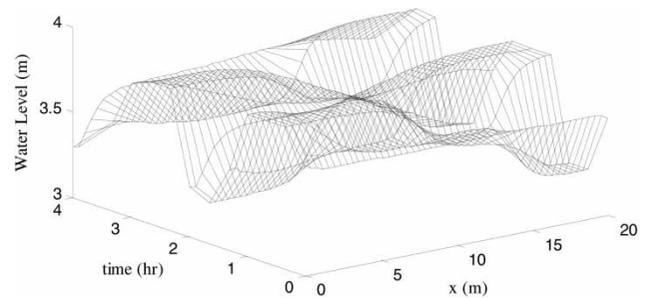


Figure 4 | A 3D view of water level fluctuations in the whole canal over simulation time when no control (dashed line in Figure 2) is imposed at upstream end ($f = 2396.33$ m).

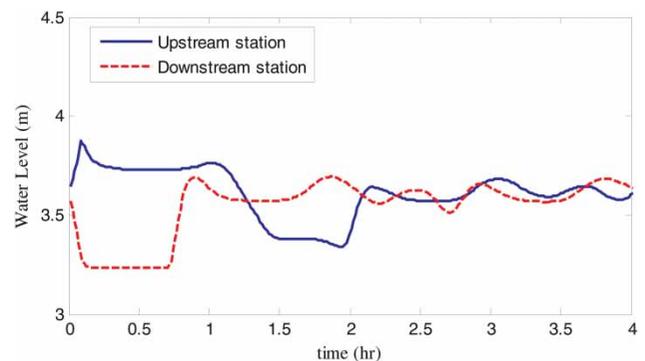


Figure 5 | Upstream and downstream time series of water surface level when the control found by Atanov *et al.* (1998) (solid line in Figure 2) is imposed at the upstream end.

for this case. According to the large length of the channel, approximately 1 hour is required for the two waves to contact each other. After this time, the fluctuations of the water level are small compared to the case of no upstream control (Figure 3). The fluctuations are around the desired water level as shown in Figure 5. A 3D view of water level fluctuations in this case is presented in Figure 6. Comparing Figure 3 with Figure 5 and Figure 4 with Figure 6 reveals that the water-level fluctuations at pumping stations and in the whole channel are reduced when an appropriate control is imposed at the upstream. The fact that the water level has been stabilized by imposing upstream control is also validated by noting that the value of the objective function has been considerably reduced to $f = 1107.39$ m in this case.

In the next step, the aforementioned problem is solved by the proposed technique using Chebyshev polynomials and the TLBO. First, the number of interpolation nodes should be selected. It is worth pointing out that more nodes will not necessarily lead to a more accurate solution and the optimum number of nodes should be usually obtained by trial. After some trials, the total number of 17 nodes on the interval $[-1, 1]$ (and accordingly on the interval $[t_0, t_f] = [0, 14400]$) was chosen. Therefore, the design vector consists of 17 variables including values of upstream flow rates at selected nodes which should be optimized. Secondly, the appropriate range of change of design variables should also be determined. This is necessary for initialization of the population. Since the initial condition of flow is $Q = 100 \text{ m}^3/\text{s}$ and downstream flow rate increases 50% ($\psi = 150 \text{ m}^3/\text{s}$), flow rates at various times oscillate around this increased value. Hence, it is reasonable to take the

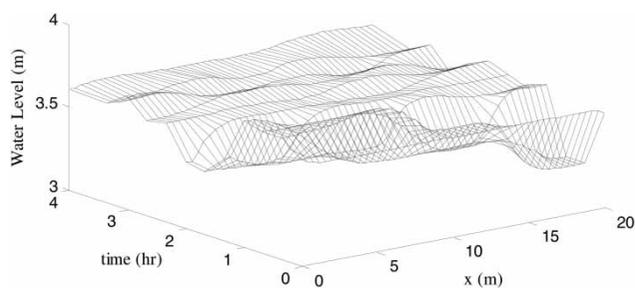


Figure 6 | A 3D view of water level fluctuations in the whole canal over simulation time when the control is found by Atanov *et al.* (1998) (solid line in Figure 2) is imposed at the upstream end ($f = 1107.39$ m).

values of flow rates as follows:

$$Q_i \in [Q_{\min} = 100, Q_{\max} = 200] \quad i = 0, 1, 2, \dots, 17 \quad (40)$$

In the TLBO procedure, 100 students are considered in the classroom and the following design vector is obtained after optimizing design variables:

$$X = \begin{bmatrix} 198.995 & 150.770 & 127.478 & 130.683 & 128.889 \\ 135.153 & 169.121 & 180.906 & 155.202 & \\ 149.835 & 149.208 & 149.924 & 148.691 & 150.047 \\ 154.768 & 152.354 & 152.626 & & \end{bmatrix} \quad (41)$$

which is related to the following coefficients after substituting in Equation (14):

$$a = \begin{bmatrix} 151.498 & 1.388 & -0.975 & -6.354 & 16.993 \\ -12.102 & 1.873 & 0.849 & 4.526 & -7.638 & 2.035 & 1.112 \\ -1.432 & -1.098 & 0.328 & 0.658 & 0.966 \end{bmatrix} \quad (42)$$

and representing the following optimal control:

$$\eta(t) = 151.498 T_0(t) + 1.388 T_1(t) - 0.975 T_2(t) - 6.354 T_3(t) \\ + 16.993 T_4(t) - 12.102 T_5(t) + 1.873 T_6(t) + 0.849 T_7(t) \\ + 4.526 T_8(t) - 7.638 T_9(t) + 2.035 T_{10}(t) + 1.112 T_{11}(t) \\ - 1.432 T_{12}(t) - 1.098 T_{13}(t) + 0.328 T_{14}(t) + 0.658 T_{15}(t) \\ + 0.966 T_{16}(t) \quad (43)$$

The optimal control described by Equation (43) is shown in Figure 7. The value of the objective function for this optimal control is $f = 989.45$ m, which is less than that found by the variational approach ($f = 1107.39$ m), which shows 10.65% reduction in water level fluctuations compared to the previous study and 58.71% reduction when no control is imposed.

Time history of convergence of the TLBO in conjunction with Chebyshev polynomials is depicted in Figure 8.

The trend of optimal control found in this study is similar to that found by the variational approach. The required flow rate starts with a higher value and oscillates around the line $Q = 150 \text{ m}^3/\text{s}$. However, the proposed control is more effective in stabilization of water level fluctuations than the optimal control proposed by Atanov *et al.* (1998).

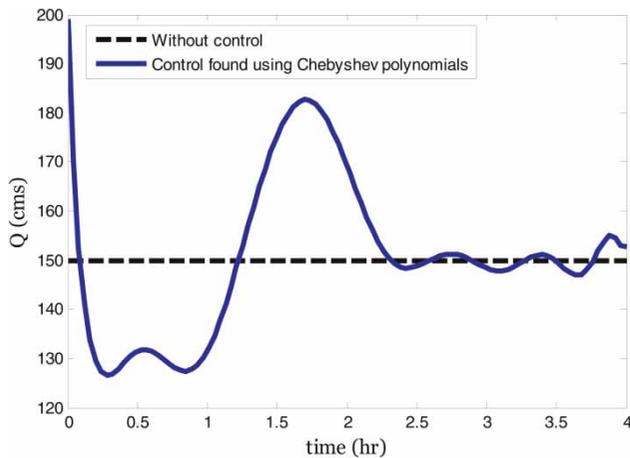


Figure 7 | Upstream control found using Chebyshev polynomials (solid line).

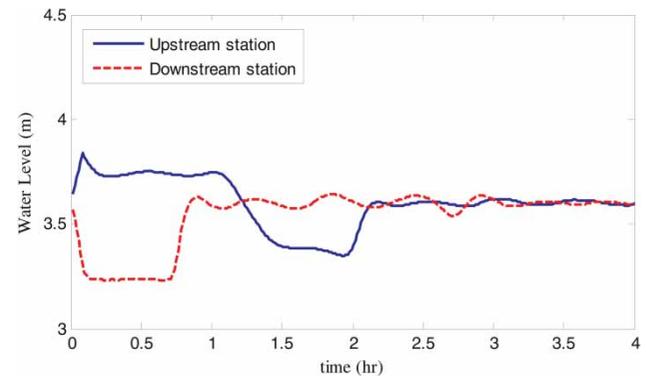


Figure 9 | Upstream and downstream time series of water surface level when the control found in this study (solid line in Figure 7) is imposed at upstream end.

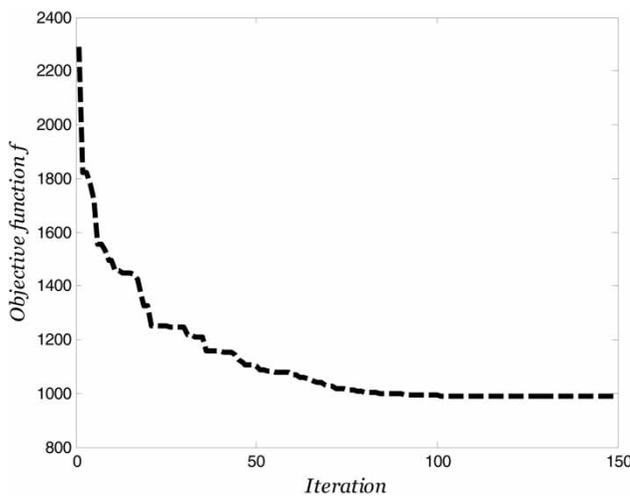


Figure 8 | Convergence history of TLBO.

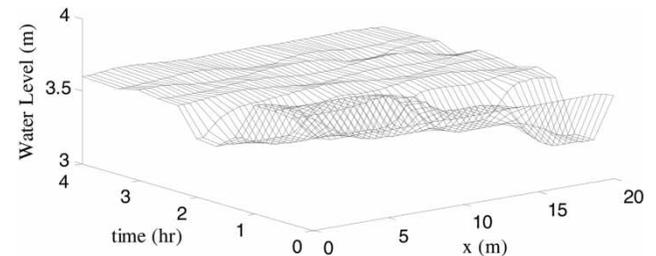


Figure 10 | A 3D view of water level fluctuations in the whole canal over simulation time when the control found in this study (solid line in Figure 7) is imposed at upstream end ($f = 989.45$ m).

In addition, time history of fluctuations in upstream and downstream pumping stations and a 3D view of the water level surface, when the proposed control is imposed at the upstream end of the channel, are shown in Figures 9 and 10, respectively. Comparing Figure 9 with Figures 3 and 5 shows that the proposed control stabilizes water surface fluctuations more effectively. Stabilization of the water surface level around the desired water level ($H = 3.6$ m) is more obvious in Figure 9 after the waves encounter. Moreover, the 3D view of the water surface in the channel depicted in Figure 10 shows the positive effect of imposing the proposed control in decreasing undesirable fluctuations within the channel.

CONCLUSION

Stabilization of water level fluctuations in open channels involving two pumping stations at both ends was investigated in this study. TLBO and Chebyshev polynomials were effectively hybridized to find an effective control for this purpose. In finding the unknown coefficients of the polynomial representing this control two major obstacles were taken into account. First, the coefficients of the polynomial vary within a wide unknown range and they are very hard to initialize in population-based optimization algorithms. Secondly, unphysical controls are generated by randomly altering the coefficients in the search space. As an efficient alternative, flow rates at specified times were chosen as design variables rather than the unknown coefficient in the polynomial. In this case, an appropriate range for all design variables can be readily selected by engineering judgment. In order to evaluate the fluctuations in the

channel as the objective function in the optimization algorithm, an effective and computationally efficient flow simulator with shock capturing capability was used. The control found by the proposed method for a given problem shows satisfactory improvement in reducing water surface fluctuations compared to the control obtained via a variational approach. Furthermore, the proposed approach is easier to implement and it can be readily employed to solve analogous cases. Such optimal controls can be used for water management in open channels in order to decrease water losses and avoid several serious problems in the system. A similar procedure can be used to solve other practical problems in water engineering context.

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