Journal of Heat Transfer Discussion

Forced Convection in a Porous Channel With Localized Heat Sources[1]

K. Vafai[2] and S. J. Kim.[3] In this note we would like to show that the solution of Vafai and Kim (1989) presented in this journal is valid for Da < 1, which covers the entire range of all porous media. In a recent paper by Hadim (1994) he had mentioned that the results by Vafai and Kim (1989) are valid only when Da < 0.01. We believe this is because of typos in our paper. The second line of Eq. (11) in Vafai and Kim (1989) should read

$$+\frac{1}{D^2}\ln\frac{\cosh\,[D(y+C_1)]}{\cosh\,[D(1+C_1)]}\bigg)\frac{u_m}{\Gamma};$$

the last line defining Γ should read

$$+\frac{A+B}{A}\,\frac{2}{D^3}\,\frac{1}{\Delta}\,(D\tanh\,[DC_1]\Delta_2+D\Delta_2+\Delta_4)]),$$

and the sign at the begining of the third line defining gamma should be positive (+). It should also be noted that Eq. (9) should read

$$u=\frac{\cosh \mathrm{Da}^{-1/2}-\cosh\,[\mathrm{Da}^{-1/2}y]}{\cosh \mathrm{Da}^{-1/2}-1}$$

We assumed for our analytical solution that outside the momentum boundary layer, in the so-called core region,

$$\frac{d^2u}{dy^2}=0. \tag{1}$$

This is true as long as two momentum boundary layers do not meet. Mathematically, this is valid when

$$\mathrm{Da}^{1/2}<0(1) \tag{2}$$

The validity of this assumption was rigorously proven and established by comparing the exact solution from Vafai and Kim (1989) with the full numerical solution of the momentum equation

$$\frac{d^2u}{dy^2}-\frac{1}{\mathrm{Da}}\,u-\frac{\Lambda_I}{\mathrm{Da}^{1/2}}\,u^2-\frac{\delta H}{\mu_f u_\infty}\,\frac{d\langle p\rangle^f}{dx}=0 \tag{3}$$

and boundary conditions

$$\frac{du(0)}{dy}=0 \tag{4a}$$

$$u(1)=0 \tag{4b}$$

[1] By A. Hadim, published in the May 1994 issue of the ASME Journal of Heat Transfer, Vol. 116, pp. 465–472.

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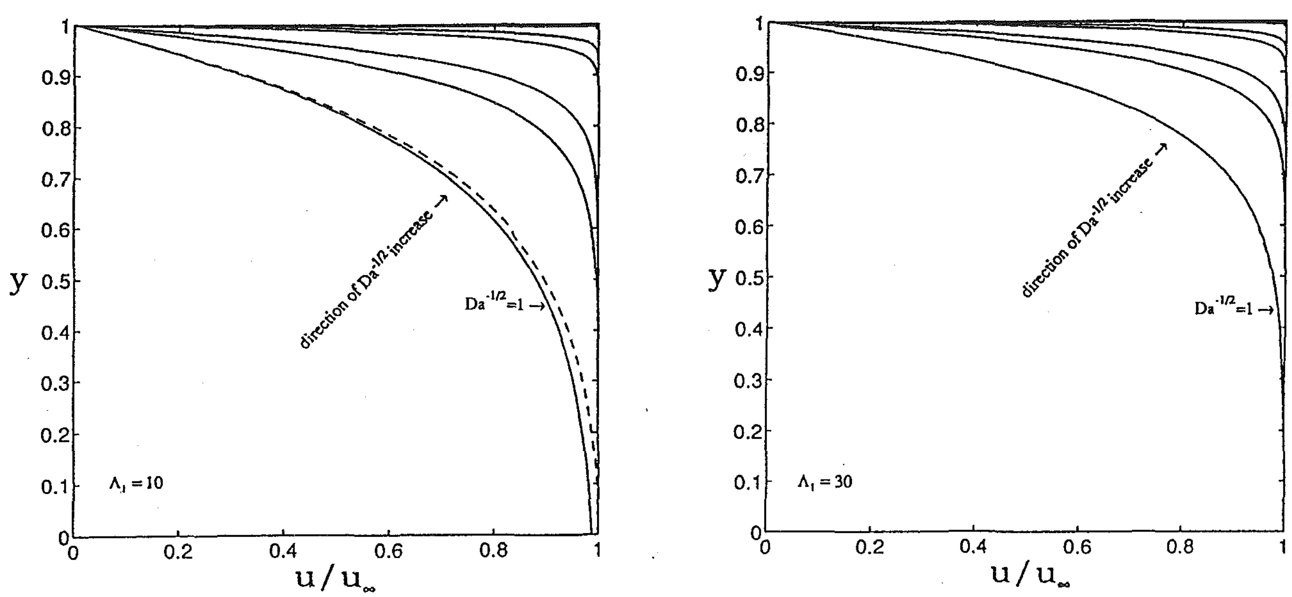

Fig. 1 Comparison between the numerical and analytical dimensionless velocity profile for $\mathrm{Da}^{-1/2}$ = 1, 5, 10, 50, 10^2, 10^3, 10^4 using Λ_I = 10, 30

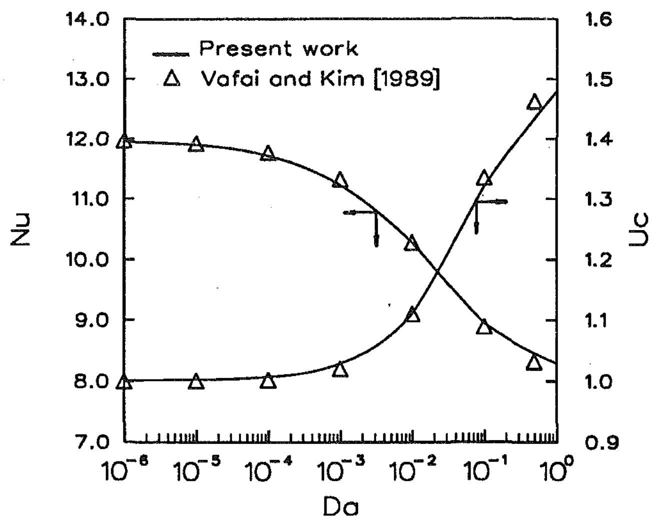

Fig. 2 Comparison of results with Vafai and Kim (1989)

Comparisons between the full numerical solution based on the momentum equation given by Eq. (3) along with boundary conditions (4*a*, *b*) and the exact solution given by Vafai and Kim (1989) are shown in Fig. 1. For brevity, the temperature distribution comparisons are not shown here. As shown in this figure the exact solution matches the numerical solution of the momentum equation given by Eq. (4) and boundary conditions (5*a*, *b*) of Vafai and Kim (1989). In fact, for all practical porous media, our exact solution matches the numerical solution within less than 0.001 percent. The exact solution starts deviating from the numerical solution for Da > 1. This translates to a permeability, K, of about 10^{-2} m^2 and larger. To the best of our knowledge, real porous media have permeabilities of at most 10^{-4} m^2 and smaller. That is for all practical porous media and even beyond, the exact solution given by Vafai and Kim (1989) precisely matches the full numerical solution of the problem. Furthermore, even for a permeability of 10^{-2} m^2 and even larger, the full numerical solution matches the analytical results extremely well as long as the inertia parameter, Λ_I, is larger than 30. Even for the extreme nonrealistic case of $K \sim 10^{-2}$ m^2 and $\Lambda_I = 10$ the agreement is still within 2.6 percent while for the nonrealistic case of $K \sim 10^{-2}$ m^2 and $\Lambda_I = 30$ the agreement is still within 0.7 percent. For all cases (other than $Da^{-1/2} = 1$) shown in Fig. 1 the agreement is within less than 0.001 percent. It should be noted that all the results presented in Vafai and Kim (1989) are already based on the solution without any typos.

In summary, the results presented by Vafai and Kim (1989) are valid as long as Da < 1, which covers all practical porous media and even beyond.

References

Vafai, K., and Kim, S., 1989, "Forced Convection in a Channel Filled With a Porous Medium: An Exact Solution," ASME JOURNAL OF HEAT TRANSFER, Vol. 111, pp. 1103–1106.

Author's Closure

After incorporating the corrections for the typos as suggested by Vafai and Kim for their analytical solution (Vafai and Kim, 1989), it was found that the numerical results presented in Fig. 2 (Hadim, 1994) were in excellent agreement with the exact analytical solution of Vafai and Kim (1989) for Da < 1.0 as shown in Fig. 2 in which the Darcy number is defined as in Vafai and Kim (1989). In fact using the same approach outlined by Vafai and Kim (1989) and after incorporating their suggested corrections for the typos, Eqs. (9) and (11) of Vafai and Kim (1989) have been rederived independently and identical expressions for velocity and temperature were obtained. It should be noted that all the graphic results presented in Vafai and Kim (1989) are based on the solution without any typos.