2 Reissner, E., and Stavsky, Y., "Bending and Stretching of Certain Types of Heterogeneous Aeolotropic Elastic Plates," JOURNAL OF APPLIED MECHANICS, Vol. 28, TRANS. ASME, Vol. 83, Series E, 1961, pp. 402–408.

3 Stavsky, Y., "Bending and Stretching of Laminated Aeolotropic Plates," *Proceedings of ASCE, Journal of the Engineering Mechanics Division*, Vol. 87, 1961, pp. 31–56.

4 Whitney, J. M., and Leissa, A. W., "Analysis of Heterogeneous Anisotropic Plates," JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, 1969, pp. 261–266.

Author's Closure

The author would like to thank Dr. Whitney for his comments. The author agrees that for more general laminates the plate theory may become laborious and the suppression of the microstructure may become desirable. Such simplicity is achieved at the expense of accuracy, however.

It has been shown in [5][4] that the effect of the microstructure in a composite beam is very substantial and that the model without microstructure is adequate only in the range of very long wavelengths. The same can be said about the laminated plate. The examples given by the discusser's reference [1] deal with the laminates in which the difference in shear rigidities of the layers is small, and, consequently, the shear deformation induced by the heterogeneity is negligible. A more detailed discussion on this point can be found in [6].

With regard to the last comment, we should notice that the couple stresses in the microstructure plate theory are originated by the introduction of some independent kinematic variables employed to describe the local deformations in the layers which are different from the gross deformation of the plate. In contrast, the couple stresses in the discusser's references [1, 2, 4] are not distinguishable from the gross moments of the plate.

References

5 Sun, C. T., "Microstructure Theory for a Composite Beam," JOURNAL OF APPLIED MECHANICS, ASME Paper No. 71-APM-S.

6 Sun, C. T., "Incremental Deformations in Orthotropic Laminated Plates Under Initial Stress," to appear in JOURNAL OF APPLIED MECHANICS.

[4] Numbers in brackets designate References at end of Closure.

Lateral Structure Interaction With Seismic Waves[1]

TEH H. LEE[2] and D. A. WESLEY.[3] The authors have successfully solved the problem of lateral structure-ground interaction by using a transient formulation. They have also made efforts, in their subsequent studies, to refine the analysis by including the effects of large base masses [1].[4] The discussers have conducted a similar type of analysis for nuclear power plants and the authors' techniques were adopted in one phase of the study. Some comments concerning their approach may be appropriate.

In the results obtained for the Alexander Building model on the stiffest soil (b = 2000 fps) subjected to excitation input based on the March, 1957, San Francisco earthquake, the authors reported that the response spectra values were increased over those obtained without interaction effects (free-wave spectra). When the discussers applied the authors' equations to determine the response of a large prestressed concrete reactor vessel (PCRV) subjected to the El Centro (May, 1940, Calif.) earthquake input, an increase in the peak base acceleration over the free-field input maximum was also observed for several cases. The authors attributed such an increase mainly to the lack of radiation of energy into the soil. While this is an important consideration, the discussers wish to point out that aside from the factor of the radiation damping, another important reason for the response to be significantly amplified is that the author's formulation cannot consider damping in the structure. However, the Volterra type of interaction equation as formulated by the authors can be modified to account for the existence of structural modal damping. Using the author's notation and considering the effects of a large base mass, the modified interaction equation may be put in the form:

$$\ddot{u}(t) = \frac{\mu A}{bM_0}\int_0^t [\ddot{u}_p(\tau) - \ddot{u}(\tau)]d\tau - \frac{b}{2\pi c}\int_0^t \ddot{u}(t-\tau) \times \mathrm{Im}\left[g\left(\frac{b\tau}{c}\right)\right]d\tau - \frac{1}{M_0}\sum_j M_j\omega_j K_j(t) - \frac{b}{2\pi c M_0}\sum_j M_j\omega_j\int_0^t K_j(t-\tau)\,\mathrm{Im}\left[g\left(\frac{b\tau}{c}\right)\right]d\tau \qquad (1)$$

where the function K_j appears twice with different arguments and is defined, for argument s, as

$$K_j(s) = \frac{1-2\zeta_j^2}{\sqrt{1-\zeta_j^2}}\int_0^s \ddot{u}(\eta)e^{-\zeta_j\omega_j(s-\eta)}\sin\omega_j\sqrt{1-\zeta_j^2} \times (s-\eta)d\eta + 2\zeta_j\int_0^s \ddot{u}(\eta)e^{-\zeta_j\omega_j(s-\eta)} \times \cos\omega_j\sqrt{1-\zeta_j^2}\,(s-\eta)d\eta \qquad (2)$$

The new symbols ζ_j and M_0 denote, respectively, the jth modal damping factor and the base mass. In writing the foregoing equation, it has been assumed that the system was initially at rest at t = 0 and the inclusion of a large base mass was handled in the same way as the authors did in [1]. Fig. 1 shows the

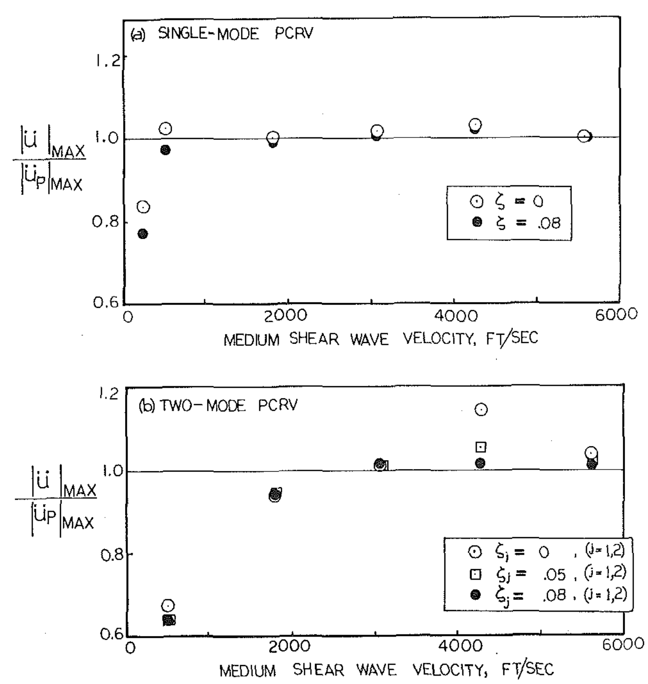

Fig. 1 Amplification factors of PCRV subjected to El Centro type earthquake

[1] By R. J. Scavuzzo, J. L. Bailey, and D. D. Raftopulos, published in the March, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 38, TRANS. ASME, Vol. 93, Series E, pp. 125–134.

[2] Staff Engineer, Structural Mechanics Branch, Gulf General Atomic Company, San Diego, Calif.

[3] Manager, Structural Mechanics Branch, Gulf General Atomic Company, San Diego, Calif.

[4] Numbers in brackets designate References at end of Discussion.

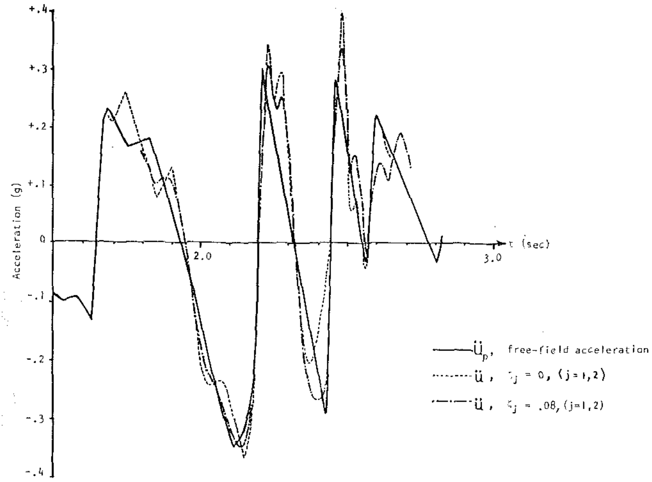

Fig. 2 Comparison of response for the two-mode PCRV with medium shear wave velocity at 4200 fps

amplification factors (ratio of peak base response to the maximum input value) obtained by using the foregoing modified equations for a PCRV. Both a low frequency, single-mode model (fixed-base natural frequency = 0.5 cps) and a two-mode model (fixed-base natural frequencies = 6.39 cps and 24.2 cps) were considered. Since both these models included large base masses, the cases with zero modal damping were computed by using the authors' improved equations given in [1]. The time-history curves in the vicinity of the peak are also displayed in Fig. 2 for the case where the amplification exceeds unity by a significant amount. It is interesting to note that the reduction on the base acceleration due to the consideration of damping in the structure appears to be more pronounced in those areas where the response peak greatly exceeds that of the input. Although not treated by the authors, significant interaction effects due to inclusion of the rocking and vertical modes, and to a lesser extent, the torsional modes, have been observed by the discussers.

Reference

1 Scavuzzo, R. J., Raftopulos, D. D., and Bailey, J. L., "Lateral Structure-Foundation Interaction of Nuclear Power Plants With Large Base Masses," USAEC Contract No. AT-(40-1)3822, Technical Report No. 3, The Research Foundation, University of Toledo, Sept. 1969.

J. H. RAINER.[5] A method of analysis for determining the interaction between structure and ground under dynamic loading was presented by the authors. The solutions are applicable to planar problems and include the influence of horizontal base motion only. Although the method is useful for assessing structure-ground interaction effects, the manner in which the results have been presented and interpreted can lead to erroneous conclusions.

1 A basic phenomenon that accompanies structure-ground interaction is that for a given structure the resonance frequency ω_1 of the interaction system is less than the resonance frequency of the fixed-based case ω_0. This occurs when the horizontal base displacement is accounted for [2][6] as well as when rocking of the base is permitted [2, 3]. The observation by the authors that "resonance does not occur even though the structure has the same frequency as the input wave" can be explained by the fact that the interaction system does not in fact possess the same frequency ω_0 as the disturbance anymore, but a resonance frequency $\omega_1 < \omega_0$. If a ramp function of frequency ω_1 had been chosen a resonance condition in the sense of large amplifications would still have been observed.

2 The practice of plotting the response of interaction structures on response spectra at the fixed-base natural frequency ω_0 of the structure is misleading. From a dynamical point of view, the frequency plotted should be the fundamental frequency of the interaction system ω_1, and not the fixed-base frequency. The natural frequency of the interaction structure for which the response has been computed is ω_1 and not ω_0, since the additional degree of freedom of horizontal base motion has reduced the system frequency to ω_1. Had the authors chosen a fixed-based structural frequency of about 5.5 or 6 cps, an amplification of response would probably have been encountered in Fig. 6 since the resonance frequency of the interaction system would be near the ramp frequency of 5 cps. The limited parameter study and the authors' presentation of interaction response relative to the fixed-based frequency ω_0 explains in part the large reductions of acceleration response presented in Table 1 and Fig. 6.

3 The analysis presented accounts for some radiation of energy from the structure into the half space. This introduces a damping type of phenomenon into the dynamic behavior of the interaction structure, commonly referred to as "geometric damping." Although it has not been explicitly stated in the authors' presentation, the acceleration response spectra for the ramp pulse, Fig. 6, and the Golden Gate Park earthquake, Figs. 9 and 11, are computed for zero interstory structural damping. As the interaction systems contain geometric damping, the large response reductions in going from the spectrum obtained from free-field motions to the response of the interaction system are therefore primarily due to a comparison between an undamped and a damped system. A more meaningful comparison could be made by including in the analysis small but realistic levels of structural damping, say, 2–5 percent of critical. Then the additional geometric damping introduced by the structure-ground interaction would be a much smaller percentage of the overall system damping and consequently the reductions in response would be considerably less.

The concept of the equivalent mass to represent a multidegree-of-freedom structure is, in general, not applicable to interaction problems. This was pointed out by Tajimi [4] in discussing Parmelee's multistory results (authors' reference [16]) for the rocking case. A similar conclusion applies when only horizontal base displacement is considered since the mode shapes $\bar{X}$ are not the same for the fixed-base and the interaction case. Consequently, the effective masses M_j in equation (2) vary for different amounts of base flexibility.

In conclusion it may be stated that in realistic situations structure-ground interaction effects are not as significant as the authors' study would indicate.

References

2 Jacobsen, L. S., and Ayre, R. S., *Engineering Vibrations*, McGraw-Hill New York, 1958, Chapter 7.

3 Merritt, R. G., and Housner, G. W., "Effect of Foundation Compliance on Earthquake Stresses in Multistory Buildings," *Bulletin of the Seismological Society of America*, Vol. 44, No. 4, Oct. 1954, pp. 551–570.

4 Tajimi, H., Discussion of "Building Foundation Interaction Effects" by R. A. Parmelee, *Journal of the Engineering Mechanics Division*, ASCE, Vol. 93, No. EM6, Dec. 1967, pp. 294–298.

Authors' Closure

The authors wish to thank Messrs. Lee and Wesley for their valuable extension to this problem by including structural damping in the interaction equations. However, there is one point to be made concerning the peak foundation acceleration shown in Figs. 1 and 2 of the Discussion. The authors also found that

[5] Building Physics Section, Division of Building Research, National Research Council of Canada, Ottawa 6, Canada.

[6] Numbers in brackets designate References at end of Discussion.

the peak acceleration of the containment vessel foundation was approximately equal to the peak free-field value for a soil with a shear wave velocity of 2000 fps [5, 6].[7] However, the spectrum calculated with the containment vessel foundation acceleration was found to be reduced at the fixed-base natural frequencies of the containment vessel where as for the Alexander building the spectrum was increased. Thus, in the first case, seismic inertia loads acting on the structure are reduced and in the latter case loads slightly increased for stiff soils.

Significant interaction effects caused by the vertical and rocking modes have also been observed by Isenberg [7] and Chiapetta [8] using finite-element methods. These trends are consistent with observations made by the discussers.

The authors wish to thank Professor Rainer for the careful review of their work. However, the authors do not feel that the conclusions are erroneous. A great deal of effort was spent comparing results of the presented analysis with finite-element results [7, 8]. Excellent agreement was obtained with the finite-element work of Isenberg, [7, pp. 132–137]. The authors realize that the discusser did not have the opportunity to study referenced document.

In the analysis of the subject paper, the acceleration term, $\ddot{u}$, denotes the structure acceleration at the ground. Given this ground motion, the force acting on each mass of the structure, m_i, can be calculated by the formula [9]

$$F_i = -m_i \sum_a \bar{X}_{ia} P_a \omega_a \int_0^t \ddot{u}(\tau) \sin \omega_a (t - \tau) d\tau \qquad (1)$$

The modal participation factor is defined by

$$P_a = \frac{\sum_i m_i \bar{X}_{ia}}{\sum_i m_i \bar{X}_{ia}^2} \qquad (2)$$

where $\bar{X}_{ia}$ and ω_a are the mode shapes and circular frequencies obtained with the structure fixed at the base. When normal mode theory is applied to a linear N-mass structure, the fixed-base natural frequencies appear in the resulting equation. It is for this reason that only the fixed-base frequencies are of significance in this analysis. The use of these fixed-base frequencies is well known in naval shock [10].

When study of this phenomenon was initiated, a true resonance of the structure was expected. Many input frequencies were employed attempting to obtain resonance in this study as well as previous work [11]. None were found. However, it should be pointed out that shock loads increase at the resonant frequency even though a true resonance was not obtained, Fig. 6.

In reference [6], a preliminary study of structural damping was made. Spectrum responses of both the free-field input $\ddot{u}_p(t)$ and $\ddot{u}(t)$ were calculated assuming the same damping applied to each motion. Interaction effects were still found to be significant. The work of the discussers Lee and Wesley would also apply to this point.

The effective mass presented in equation (2) of the subject paper is a result of the linear N-mass approximation. This equation is obtained by applying normal mode theory of the N-mass system and is not a concept. In the discussion by Tajimi referenced by the discusser, the same equation (equation (30)) is used in the analysis.

The authors feel that the greatest weakness of this analysis is that the foundation is assumed to be two dimensional. The effect of the third dimension on this transient analysis is not known. Because of the good agreement with finite-element results, the approximation of the constant stress boundary condition is felt not to be significant.

References

5 Scavuzzo, R. J., Bailey, J. L., and Raftopoulos, D. D., "Lateral Structure-Foundation Interaction of Nuclear Power Plants During Earthquake Loading," USAEC Contract No. AT-(40-1)3822, Technical Report No. 2, The Research Foundation, The University of Toledo, Aug. 1969, p. 44.

6 Scavuzzo, R. J., Raftopoulos, D. D., and Bailey, J. L., "Lateral Structure-Foundation Interaction of Nuclear Power Plants With Large Base Masses," USAEC Contract No. AT-(40-1)3822, Technical Report No. 3, The Research Foundation, University of Toledo, Sept. 1969, p. 59.

7 Isenberg, J., "Interaction Between Soil and Nuclear Reactor Foundations During Earthquakes," USAEC Contract No. AT-(40-1)3822, ORO-3822-5, Ababian-Jacobsen Associate, Los Angeles, Calif., June 1970.

8 Chiapetta, R., "Effect of Soil-Structure Interaction on the Response of Reactor Structures to Seismic Ground Motion," USAEC Contract No. AT-(40-1) 3822, ORO-3882-4, IIT Research Institute, Chicago, Ill., Apr. 1970.

9 O'Hara, G. J., and Cuniff, P. F., "Elements of Normal Mode Theory," NRL Report 6002, Nov. 1963.

10 O'Hara, G. J., "Background for Mechanical Shock Design of Ship Systems," NRL Report 6267, March 12, 1965.

11 Scavuzzo, R. J., "Foundation-Structure Interaction in the Analysis of Wave Motions," *Bulletin of the Seismological Society of America*, Vol 57, 1967, pp. 735–746.

[7] Numbers in brackets designate References at end of Closure.

Normal Impact of an Infinite Elastic Beam by a Semi-Infinite Elastic Rod[1]

A. J. Durelli.[2] The following comments refer to the experiment conducted by the author. The experiment was designed to obtain results in terms of bending moments. For this reason strain gages were affixed at opposite points on the top and bottom surface of the beam and the difference in their outputs was recorded. It is the discusser's opinion that this way of designing the experiment may have hidden some important features of the phenomenon.

The author assumes in his thoretical analysis that the dynamic response of a beam to normal impact consists entirely of bending waves. It would be interesting to verify this assumption experimentally by observing the top and bottom strain gage signals separately and noting whether the required antisymmetry between the output signals exists. If longitudinal waves are also present in the beam then the author's assumption of an infinite beam in his theoretical solution may no longer be valid since reflections of the faster longitudinal waves could occur before bending waves reach the ends of the beam. This possibility could be checked if the author would specify the length of the beam used in his experiment. The existence of higher symmetric and antisymmetric modes of bar waves might also be anticipated. All of these possibilities underline the importance of conducting an experiment from which the state of stress can be determined independently of assumptions about the types of waves propagating in the beam.

The author does not seem to be acquainted with a paper of the discusser in which results are given from a beam impacted by a striker [1].[3] Because of differences in several of the parameters a comparison with the author's results is not immediate, but it seems that the photoelastic approach would have given the author much more information, Figs. 1 and 2 of this Discussion. The strains observed photoelastically did not display everywhere and at all times antisymmetry assumed by the author and effects of the end supports on the stresses distribution were observed before stresses associated with a bending type of deformation of the

[1] By S. Ranganath published in the June, 1971, issue of the Journal of Applied Mechanics, Vol. 38, Trans. ASME, Vol. 93, Series E, pp. 455–460.

[2] Professor, Department of Civil and Mechanical Engineering, The Catholic University, Washington, D. C.

[3] Numbers in brackets designate References at end of Discussion.