

Discussion

Bending Vibrations of a Pipe Line Containing Flowing Fluid¹

R. L. BISPLINGHOFF.² The authors should be commended for indicating a relatively simple plan of solution to a problem, which at first glance seems quite complicated. The fact that the significant effects of the internal flow apparently can be expressed by the simple expression of force per unit length of $-\sigma(\partial^2 z)/(\partial x \partial t)$ leaves the problem linear and thereby tractable. The power-series approximation of the deformed pipe illustrates how a solution of acceptable accuracy in an engineering problem can be obtained often with less labor by approximate methods than by the classical methods which usually come to one's mind at first. A solution of the Bernoulli-Euler beam equation containing an internal damping term according to Stokes' law also has been discussed by Sezawa in 1927, and more recently by Mindlin, Stubner and Cooper and Stowell, Schwartz and Houboldt. Although this damping term is not the same as that considered by the authors, Sezawa's work indicates a solution to a similar problem.

The last part of the paper, in which the traveling-wave point of view is adopted, is particularly illuminating in that it shows mathematically and discusses physically why standing waves are not possible. This conclusion of course can be drawn tentatively at the outset because of the existence of a damping term in the system. It is interesting to speculate on the importance of adding a term arising from the rotary motion of the fluid. This term, which would be proportional to the time derivative of the rate of change of curvature along the beam, would be of less importance than the term already considered but probably of greater importance than the Sezawa damping term already given considerable attention.

W. H. HOPPMANN, 2nd.³ The authors have presented a study of an interesting problem and have developed a differential equation which appears to have considerable importance for its description. However, the writer wishes to submit an objection to the use of the term "damping," applied to the resistance to vibration caused by change of momentum of the fluid in a direction perpendicular to the length of the pipe. Forces arising from this cause will resist vibrations of the pipe but only because the internal surface of the pipe imposes compressive forces upon the fluid to constrain it to move in the changing direction of the pipe.

It appears that one may consider a damping force, however, because of the relative transverse velocities of the fluid and this force would be given by $\mu (\partial^2 Z)/(\partial x \partial t)$ where μ is a viscosity coefficient. In this case it would be proper to speak of viscous damping. It is assumed that the authors ignored these forces because they are probably too small for practical consideration.

It is stated that supports are spaced about 66 ft apart and that they are assumed to be of the hinged type. This assumption appears to be reasonable but the writer cannot agree with the treatment of the pipe as if it were only a single span. The fre-

quencies and shapes for each span of a multispan beam are radically different from those of a single span.

It would be very interesting if the authors could find an opportunity to test a model pipe line in a wind tunnel and measure the vibration characteristics when a fluid is flowing through it.

AUTHORS' CLOSURE

The authors wish to express their appreciation to Professors Bisplinghoff and Hoppmann for several most instructive comments and suggestions. It certainly does appear that the importance of viscous-damping, structural-damping, and rotary-inertia terms to solutions of the pipe-line vibration equation should be fully investigated. The latter two effects were considered during detailed analysis of the problem but were omitted in the interests of simplicity and to emphasize what is believed to be the primary influence of the fluid flow.

It is agreed that the word "damping" is perhaps a poor choice to describe the transfer of energy from the pipe motion to the fluid. It was of course used loosely for lack of a better concise term. The authors concur with Professor Hoppmann, who is better qualified than they to write on the subject of multispan beams, that any possible asymmetry of support, inequality of support distance, and so on, will have a severe effect on the mode shapes and frequency. However, it was assumed on the basis of observation that alternate spans moved oppositely to one another and were hinged to the supports. Under these conditions there must be complete antisymmetry and the structural boundary conditions on any one span consist of zero deflection and bending moment at its extremities. This is in analogy with a single simply supported span.

The authors wish to call attention to an error in column 1, page 231, of the original paper: in the first two places where the symbol δ_i appears, its sign should be changed.

A Mechanical Analyzer for Computing Transient Stresses in Airplane Structures¹

WALTER RAMBERG.² The authors are to be congratulated for extending Dr. Biot's attractive torsion-pendulum analogy to obtain directly the transient response to a forcing function of fixed shape in space (e.g., impact at a fixed point) of a linear elastic system simultaneously in as many as three natural modes. The writer was especially pleased by the ingenious application of wire strain gages to record the forcing function as well as the response.

It is to be hoped that this highly flexible analyzer is to be used in future investigations of transient response. In particular, the writer would be interested in the authors' views on using the analyzer for determining the convergence of the response in terms of the natural modes, and also the response over a long period including several cycles of the fundamental mode. The solution of this latter problem for different cams representing forcing functions of different shape in time and for different rates of rotation

¹ By Holt Ashley and George Haviland, published in the September, 1950, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 72, pp. 229-232.

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¹ By R. L. Bisplinghoff, T. H. H. Pian, and L. I. Levy, published in the September, 1950, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 72, pp. 310-314.

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