A Composite Theory of Elementary Particles

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Abstract

A composite theory of particles is investigated, based on six fundamental particles (\(p, n, \Lambda; \nu, \bar{\nu}, \epsilon\) and \(\mu\)). We assume three types of interactions, very strong, moderately strong and weak interactions (abbreviated as: VSI, MSI and WI), beside electromagnetic couplings. The VSI is global (i.e., completely symmetrical with respect to \(p, n\) and \(\Lambda\)), and gives rise to major parts of baryonic mass but is missing among the leptons. This is why leptons are so light. This VSI is also responsible for the creation of various bound states, pions, kaons, etc., from baryon-antibaryon pairs. If there were only VSI, masses of \(n, p, \Lambda\) and those of pion and kaon would be equal. The charge independent MSI splits the mass degeneracies between nucleons and \(\Lambda\), pions and kaons, etc., and also \((\epsilon\mu)\) and muon. We can conclude that the kaon is pseudoscalar and the relative parity must be odd, where the \(\Sigma\) or \(\Xi\) is the bound state of \(\Lambda + \bar{N} + N\) or \(\Lambda + \bar{N} + \Lambda\). There are open possibilities of existence of baryons and mesons with higher values of strangeness. The Feynmann-Gell-Mann theory of weak interactions can be consistently transferred into our scheme. Finally, the possible existence of extremely weak interactions is speculated (metastability of matter and charge non-conservation).

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§1. Introduction

Since the proposed mechanism, associated production of the strange particles, has received experimental verification, theories of elementary particles have been put forward in conformity with Nakano-Nishijima-Gell-Mann's scheme. The essence of this scheme can be seen in a much simpler form if the composite nature of most of the particles is granted. In this paper, we shall present some considerations on a composite theory—as far as the baryon family is concerned, our scheme is a specific case of the so-called Sakata model—of elementary particles. It seems unnecessary to say that one can give rise to many pessimistic criticisms on what we are going to mention here. We shall, however, simply ignore such comments and try to construct and pursue a composite theory guided by the “progressive” optimism.

We already know various kinds of baryons and mesons among which there exist strong interactions. All such elementary particles can be built up from three fundamental particles: the proton \( p \), the neutron \( n \) and the \( A \)-hyperon (Sakata model). Roughly speaking, the pion \( \pi \) (the kaon \( K \)) is a bound state of a nucleon \( N \) and an antinucleon \( \bar{N} \) (an antilambda \( \bar{A} \)). Likewise \( \Sigma \) or \( \Xi \) is composed of \( N + \bar{N} + A \) or \( A + A + \bar{N} \). [We shall denote protons and neutrons by the small latin letters \( p \) and \( n \) respectively, while for the general name, nucleon, we shall use the capital letter \( N \).]

We can take, of course, another kind of composite model (the Goldhaber model), in which \( N \) and \( K \) are chosen as fundamental particles. However, we shall not use this Goldhaber model because it requires more fundamental particles (\( n, p, K^+ \) and \( K^0 \)) than the Sakata model does. Moreover, fermions (more specifically, Dirac particles) can be regarded as more basic particles than bosons as was emphasized by Heisenberg (bosons can be built up from fermions but not vice versa).
We now turn to the leptons. We know three of them: the neutrino \( \nu \), the electron \( e \) and the muon \( \mu \). We have not yet understood the difference between \( e \) and \( \mu \)—except their mass difference, these two particles behave exactly in the same way as in electromagnetic and weak interactions. Therefore it is simpler to assume that \( \mu \) is as fundamental as \( e \), thereby avoiding any attempt to understand their difference. The lepton family is supposed to be built up again from three fundamental particles \( \nu \), \( e \) and \( \mu \).*

In this way we reach our compound theory of particle physics: Fundamental particles are \( p, n, A \) and \( \nu, e, \mu \). The remaining particles are considered as bound states composed of them. We do not know how photons come out (however, see Heisenberg⁴), so we shall add the photon \( r \) as the seventh member of the fundamental particles.

It is the purpose of the present note to pursue this type of composite theory. In practice, it is rather difficult to discriminate experimentally two types of theories: the composite model and the "elementary" particle model. By this we mean the following question: Is it possible to prove (or disprove) experimentally the composite nature of, say, pions? Since there are strong interactions among pions and baryons this is certainly not physically a meaningful way of asking. In fact, there is a theorem due to Nishijima⁵ for the local renormalizable field theory, which guarantees the complete equivalence of two types of theories: the composite and the elementary particle theories. Therefore the support of the composite theory must be searched for in different ways. We believe the following facts can be considered as support of our composite theory:

(a) the simplicity of the theory,
(b) existence of (scalarial, vectorial) additive quantities like the spins, isospins, strangeness, and baryonic number, which are most naturally understood by the composite theory.⁶

We must emphasize that:
(c) our scheme contains the necessary and sufficient number of fundamental particles.⁷

Furthermore, unless we meet definite disproofs of the composite theory, we should accept the simplest type of theory even though we do not know how to work out mathematically rigorous treatments in such a composite theory.

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* M. Goldhaber (Phys. Rev. Letters 1 (1958), 467) has recently made an interesting remark: \( \mu \) and \( e \) are remarkably similar, except for their different masses. We do not understand this. So we turn round the argument: We assume the miraculous doubling of charged leptons as one of the examples of the more general rule of doubling of all fermions. Thus there must exist a muonic analogue for each baryon (which is analogous to the electron). We do not, however, take this point of view in this paper. We shall also see later many evidences (notably in (a)…(e) of § 2) which are not necessarily in favour of this conjecture of M. Goldhaber.
theory. Some of our discussions given in this paper are naturally of provisional character and others are not necessarily characteristic of the (specific) composite theory (chosen here). Our aim in this paper lies in stimulating future discussions along these lines.

Considerations very similar to ours have recently been published by Ogawa, although his presentation was too short and apparently not too extensive as compared with the present paper.*

§2. Characteristic features of fundamental particles

Once we have granted such a composite model, we shall immediately see striking similarities and/or complementarities between baryonic and leptonic fundamental ("physical") particles, which are all of spin 1/2, the Dirac particles with positive parity (by definition):

(a) Lighter pairs \((p, n)\) and \((\nu, e)\) of the fundamental particles form "charge-doublets". The mass difference among them could be attributed to the electromagnetic origin.

(b) Heavier members \(A\) and \(\mu\) appear as "charge-singlets".

(c) Remarkable complementarity of electric charges between baryons and leptons:

\[
\begin{align*}
A & \quad \mu^- \quad \cdots \cdots \text{(isosinglet)} \\
\mu^+ & \quad \mu^- \\
\nu & \quad e^- \\
p & \quad \nu \quad \cdots \cdots \text{(isodoublet)} \\
\nu & \quad e^-
\end{align*}
\]  

\[(2\cdot1)\]

(d) Baryons, \(N\) and \(A\), are very massive, while leptons are not.

(e) There is substantial mass difference between members of charge-doublet and charge-singlet.

From these facts we are going to construct a model of strong interactions. This we shall do in the next section.

In the strong interactions we know that the total isospin and the strangeness must be conserved. We have strict conservation laws of nucleonic number and electric charge. Moreover, the conservation of leptonic number seems to be strictly valid. We can amalgamate all these conservation laws in the following simple requirement in our composite theory: The strong interactions must conserve the total number of each fundamental particles (number of particles minus that of antiparticles). While in the weak interactions the requirement is greatly relaxed: Only the total number of baryons and that of leptons should be conserved. Furthermore, we have no evidence of any strong interaction between baryon and lepton

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* I have been informed that Ogawa, Ohnuki and Ikeda developed a mathematical analysis of the Sakata model (M. Ikeda, S. Ogawa and Y. Ohnuki, Prog. Theor. Phys. 22 (1959), 715).
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families (besides electromagnetic couplings) so that our strong interactions should contain no couplings between baryonic and leptonic fundamental particles.

Fig. 1. Schematical mass levels of the lowest three members of the physical fermions (quantum numbers to be assigned to each fundamental particles are as follows:)

\[ \begin{array}{c|c|c|c|c|c|c} 
\text{quantum number} & p & n & A & \nu & e^- & \mu^- \\
\hline 
\text{baryon number} & +1 & +1 & +1 & 0 & 0 & 0 \\
\text{lepton number*} & 0 & 0 & 0 & +1 & +1 & +1 \\
\text{strangeness**} & 0 & 0 & -1 & 0 & 0 & -1 \\
\text{charge} & 1 & 0 & 0 & 0 & -1 & -1 \\
\end{array} \]

§3. Strong Interactions

3.1. Very strong and moderately strong interactions

Since the mass difference between \( N \) and \( A \) is small compared with their individual mass, we can first neglect this mass difference. Then there must be very strong interactions (VSI) among \( p, n \) and \( A \), which create most of the baryon mass \( M_B \). The VSI is missing among the leptons, this is why leptons are so light. Furthermore we shall assume the VSI being "global",*** i.e., the VSI is not only charge independent (in the usual sense)

* This is purely conventional. We can equally well choose \(-1\) for all \( \nu, e^- \) and \( \mu^- \).

** This is also conventional. Our strangeness serves merely to distinguish between isodoublets and isosinglets.

*** Our global symmetry is entirely different from Gell-Mann's global symmetry (M. Gell-Mann, Phys. Rev. 106, (1957) 1296).
but also completely symmetrical with respect to the interchange between $n$ and $A$ (and also $p$ and $A$). Thus the self-mass $\Delta M_n$ caused by VSI is common to three baryons $p, n$ and $A$ (all leptons so far are, say, massless).

Now comes the fact (e). We first notice the following approximate relation among observed mass values:

$$m_A - m_N = 2(m_p - m_e).$$

This relation strongly suggests that both mass differences between charge-singlet and charge-doublet members of baryons and of leptons have a common origin (or at least closely connected origins). In other words, there must be some universal (or at least very similar) moderately strong interactions (MSI) which remove the degeneracy of masses of fundamental particles—the charge-singlet and -doublet are separated! Net effect of MSI must be small but not very small as compared to that of VSI, as is expected from the observed masses of $A$ and $N$.

In this manner we think that we can—at least qualitatively—understand the masses of three lowest level of physical fermions (the physical $p, n, A; \nu, e$ and $\mu$).

For simplicity we shall assume that our strong interactions, VSI and MSI, are parity ($P$) conserving, charge conjugation ($C$) invariant, as well as time reversal ($T$) invariant. We do, of course, assume the validity of the CPT theorem. If we choose some special form of the strong interactions, then combined $CP$ invariance (together with charge independence) might be sufficient to guarantee the $C$ and $P$ invariances separately. We shall not enter into such details here.

3.2. Leptons

We want also to add a few remarks about lepton families. First of all, leptons do not have VSI among themselves and are very light,* so that MSI—which is, of course, an agent of mass splitting between $\mu$ and leptonic doublet ($\nu, e$)—cannot form any stable (neglecting WI!) bound states consisting of lepton and antilepton.** Therefore there are no leptonic analogue of a pion or a kaon. This causes the fundamental disparity between the mass levels (fermions and bosons) of lepton- and baryon-families. MSI is, however, not too weak so that we meet a rather unpleasant problem as how to construct “physical leptons” out of three fundamental “bare” leptons. Frankly speaking, we have so far had no methods to attack at the many-body problems in relativistic quantum field

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* The fact that $m_\mu \approx m_e \approx 0$ may require something more to be assumed (e.g., $\eta$-invariance).

** Except the positronium, the muonium, etc., which are the bound states due to electromagnetic interactions.
theory. Hence we shall try to apply an analogue to what we have found in other fields of physics and conjecture some reasonable and most likely conclusions. We know that in many cases the strongly coupled system can be replaced by the assembly of “free particles” under certain conditions. (The spirit to find out appropriate collective coordinates in many-body problems is nothing but an example of this kind\(^\text{9}\)). Such an idea can be transferred to our problem, bare leptons have strong interactions (MSI), but “physical leptons” behave just like free Dirac particles at low energy regions. The electromagnetic properties of \(e\) and \(\mu\) can be predicted quite reliably quantum electrodynamics. However, such a “collective” picture of \(\mu\), \(e\) and \(\nu\) is valid up to some critical energy \(\varepsilon\). More precisely, if the Lorentz invariant momentum transfer of leptons in collision processes is larger than \(\varepsilon\), our concept of physical \(\mu\) and physical \(e\) in the sense that they are described by simple Dirac equations (or pure quantum electrodynamics in the conventional form) loses its validity. It is rather difficult to estimate, \(\varepsilon\), but it does not seem unreasonable to say that \(\varepsilon \gg m_\mu\) or \(\varepsilon\) is of the order of the nucleon mass. There are many investigations\(^\text{10}\) on the validity of quantum electrodynamics to \(e\) and \(\mu\), and our viewpoint is not inconsistent with experimental information we have had so far.

If the relativistic momentum transfer of leptons in the lepton-lepton or lepton-nucleon collisions exceeds the critical value \(\varepsilon\), the concept of weakly interacting physical leptons will certainly collapse. We can expect, under such a condition, rather “strong” productions of leptonic particles. The discrepancy* in energy balance between primary and secondary cosmic rays is as follows:

<table>
<thead>
<tr>
<th>ionization loss of</th>
<th>magnetic latitude</th>
<th>56°</th>
<th>28°</th>
<th>3°</th>
</tr>
</thead>
<tbody>
<tr>
<td>soft protons</td>
<td>54± 6</td>
<td>42± 6</td>
<td>30± 6</td>
<td></td>
</tr>
<tr>
<td>hard protons</td>
<td>30± 4</td>
<td>26± 2</td>
<td>22± 2</td>
<td></td>
</tr>
<tr>
<td>(\mu)-mesons in the atmosphere</td>
<td>80± 7</td>
<td>69± 5</td>
<td>61± 5</td>
<td></td>
</tr>
<tr>
<td>(\mu)-mesons underground</td>
<td>37± 4</td>
<td>37± 4</td>
<td>37± 4</td>
<td></td>
</tr>
<tr>
<td>electrons</td>
<td>340± 20</td>
<td>250± 20</td>
<td>160± 20</td>
<td></td>
</tr>
<tr>
<td>energy dissipated at nuclear disintegration</td>
<td>200± 40</td>
<td>81± 16</td>
<td>57± 12</td>
<td></td>
</tr>
<tr>
<td>(\pi^+\mu) decay</td>
<td>97± 5</td>
<td>86± 5</td>
<td>76± 5</td>
<td></td>
</tr>
<tr>
<td>(\mu)-electrons decay</td>
<td>240± 10</td>
<td>210± 10</td>
<td>180± 10</td>
<td></td>
</tr>
<tr>
<td>total</td>
<td>1080± 90</td>
<td>800± 50</td>
<td>650± 40</td>
<td></td>
</tr>
<tr>
<td>corrected for the lateral spread</td>
<td>1340±110</td>
<td>820± 50</td>
<td>660± 40</td>
<td></td>
</tr>
<tr>
<td>energy of primary cosmic rays</td>
<td>1910± 60</td>
<td>1150± 40</td>
<td>880± 40</td>
<td></td>
</tr>
</tbody>
</table>

* The energy balance of cosmic rays is as follows:

All numbers are given in MeV/cm\(^2\)/sec/sterad. This table is taken from H. Komori et al., Prog. Theor. Phys. 13 (1955), 205.

The discrepancy (amounting to ~30%) is usually attributed mainly to the albedo effect, and to a smaller extent (~10%) to the neutrino losses accompanied by the leptonic decay of strange particles.
radiations might be partially attributed to such a phenomenon: We have more energy losses into the form of neutrinos in leptonic collisions than those accepted so far. [Conventional (usual) neutrino losses are found in \( \pi^-\mu^- \) decay, \( \mu^-e^- \) decay, \( K_{e2}, K_{e3}, K_{e3} \) and possible leptonic decay of hyperons.]

So much for the leptons, let us now discuss the baryon family.

3.3. The baryon family

The Sakata model of "strongly interacting particles" (or baryon family) is a straightforward generalization of the Fermi-Yang theory\(^{11}\) of the composite pion. We can therefore accept many statements made by Fermi and Yang,\(^{11}\) later by the Nagoya group\(^{12\sim14}\) and by others.\(^{15}\) We shall just mention several important items.

3.3.1. Baryon-(anti)baryon forces

We must construct pions, kaons, \( \Sigma \)'s and \( \Xi \)'s and we have therefore to postulate that the force between baryon-antibaryon is of very short range and very strongly attractive, while the baryon-baryon or antibaryon-antibaryon force must be repulsive at small distances. Thus the hard core of nuclear forces (we mean by this the force between two nucleons) is naturally understandable. The (relatively weak) attractive (or repulsive, dependent on the states) force between two nucleons at "large" distance is caused by the exchange of "composite" pions or kaons [See the argument given in Ref. 11]. The baryon-(anti)baryon forces discussed here are of course the resultant effect of both VSI and MSI.

3.3.2. Mesons

We begin our discussion with the mesons as bound states. For this purpose let us imagine a hypothetical case in which we have switched off the MSI. In this case we have only the VSI. We shall refer to such a situation as the "global" approximation. There we find the equality of baryonic masses:

\[
m_p = m_n = m_\lambda \quad (\equiv M_B).
\]

This is guaranteed by the global symmetry of our VSI. Due to the strong attraction between fundamental baryon-antibaryon pairs, we find many bound states; notably we must have (at least) \( 9(=3\times3) \) degenerated "ground" levels [see also Okun', Ref. 15]. We assume that other mesic levels lie at much higher energies or are unstable. We can identify these degenerated ground states as three pions \( (\pi^+, \pi^0, \pi^-) \), four kaons \( (K^+, K^0, \bar{K}^0, \bar{K}^-) \) and two neutral mesons \( (\pi^0, \pi') \).* This degeneracy is again due

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* More precisely. see the caption of Fig. 2. Also see the forthcoming paper by the present author.
to the globalness of VSI. We know that the pion is pseudoscalar so that our baryon-antibaryon force must be most strongly attractive in the $^1S$-state; this is useful information for us in choosing or restricting the type of VSI. N and A are different only in the internal degrees of freedom: the isospin and the strangeness. We notice again that p, n and A all have spin 1/2 and also—by definition—positive parity! These facts, together with the global approximation, guarantee that all composite "mesons", π, K and π' must have identical spatial configurations (including ordinary spins). Therefore, we can conclude that K and π' are all pseudoscalar. Using the language of isospin, we may visualize our mesons as follows:

\[ \begin{array}{|c|c|c|c|c|c|} 
\hline
\text{Assignment} & \text{Constituents} & \text{Spin} & \text{Parity} & \text{Isospin} & \text{Strangeness} & \text{Multiplicity} \\
\hline
\pi & N + \bar{N} & 0^- & 1 & 0 & 3 \\
\pi^0 & N + \bar{N} & 0^- & 0 & 0 & 1 \\
K & N + \bar{A} & 0^- & 1/2 & +1 & 2 \\
\bar{K} & A + \bar{N} & 0^- & 1/2 & -1 & 2 \\
\pi' & A + \bar{A} & 0^- & 0 & 0 & 1 \\
\hline
\text{Total} & 9 \\
\end{array} \]

We are now going to switch on the MSI—we shall refer to this case as the charge independent approximation (we are still neglecting electromagnetic and weak interactions). First of all this splits the nucleon (isodoublet) mass and the lambda (isosinglet) mass. At the same time MSI also removes the degeneracy of masses of 4 mesons. The baryon-(anti)baryon forces depend upon the relative sign of the coupling constants of VSI and MSI. Therefore we can choose this relative sign, as well as the type of MSI, so as to weaken the attractions between N-A, A-A, isosinglet N-N, as compared to that of the isotriplet N-N. In this way we can make K, π^0 and π' considerably heavier than π. Thus two isosinglet neutrals, π^0 and π', even if created by energetic nuclear collisions, will immediately decay into lighter particles (2π decay is forbidden by parity conservation, but 2π+γ or 3π decay is allowed—π^0 or π' decays are caused by the strong interactions!) and escape experimental detections. The π^0, π' can, of course, appear as the resonance levels of scattering processes. We must furthermore assume that no drastic change (e.g., crossover) of mass levels of mesons takes place during the passage form global to charge independent approximations. Thus the spatial configurations of

* See the footnote, p. 8.
pions and kaons change smoothly (or, more appropriately, adiabatically) from global to charge independent cases when the MSI is switched on, and we can still conclude that pions and kaons are pseudoscalar!

These optimistic expectations are not unlikely if one refers to the calculations by Fermi-Yang, Tanaka, and Maki from our revised point of view. In fact, one can construct—a theory in which only pseudoscalar pions and pseudoscalar kaons are—in the charge independent approximation—the only stable mesons. [See also §3.4.] Once we are able to construct mesons, we can effectively replace our composite theory by the conventional Yukawa type meson theories, at least for low energy regions [see Ref. 11].

We illustrate schematically our mass levels in Fig. 2.

![Fig. 2](https://example.com/f2.png)

**Fig. 2.** More precisely the convenient classification of meson states in the global approximation is given by \( \pi^+, \pi^0, \pi^-, K^+, K^0, \bar{K}^0, K^- \), and

\[
\rho_i^{\mp} = \frac{\bar{p}_p + \bar{n}_n \mp 2 \lambda A}{\sqrt{6}},
\rho_i^{\pm} = \frac{\bar{p}_p + \bar{n}_n \pm \lambda A}{\sqrt{3}}.
\]

By the permutation \( n \leftrightarrow A, \pi, K, \) and \( \rho_i \) mix together, but \( \rho_i^{\pm} \rightarrow \rho_i^{\mp} \). Hence we find \( m_{\pi^i} = m_K = m_{\rho_i} \) but the \( \rho_i \) mass could be different from all other meson masses. In the charge independent approximation \( \pi_i \) and \( \pi' \) are given by an appropriate combination of \( \rho_i \) and \( \rho_i' \) (as far as charge states are concerned). Similar notice can also be applied to "Y" states described below.
3.3.3. Physical baryons

Next we have to discuss the "physical baryons". There may be many possible kinds of physical baryons, to two of which we shall pay most of our attention. It is again convenient to consider three different stages of approximation:

- no interactions,
- global approximation, and
- charge independent approximation.

We start with the following two "bare" states:

\[
|B\rangle_0: \text{(state consisting of one bare fundamental particle)},
\]

\[
|Y\rangle_0: \text{(state consisting of two bare fundamental particles and one antiparticle; } B + \bar{B} + B; \text{ } B - \bar{B} \text{ and } \bar{B} - B \text{ are both } S\text{-states; total angular momentum } J=1/2),
\]

when there are no strong interactions, and we switch on first the VSI and then the MSI. Spin-parity of \(B\rangle_0\) is 1/2\(^+\), while that of \(Y\rangle_0\) is 1/2\(^-\) because the antibaryon is 1/2\(^-\). In the latter \(Y\rangle_0\) states we have restricted ourselves to totally \(S\)-states with \(J=1/2\) since we have learnt that the \(B - \bar{B}\) forces are most strongly attractive in \(^1S\)-states. Switching on all strong interactions in due course, we can trace smooth change and splittings of the degenerated wave functions \(B\rangle_0\) and \(Y\rangle_0\):

\[
|B\rangle_0 \rightarrow \begin{cases} \langle A \rangle \\ \langle n \rangle \\ \langle \rho \rangle \end{cases} \text{ these are symbolically written as } |B\rangle_{0,1}.
\]

\[
|Y\rangle_0 \rightarrow \begin{cases} \text{ (many different states) } |Y\rangle_{0,1}. \end{cases}
\]

no interactions charge independent approximation

During the passage of \(B\rangle_0 \rightarrow |B\rangle_{0,1}\) and of \(Y\rangle_0 \rightarrow |Y\rangle_{0,1}\), there must not occur any mixing between \(B\)-states (1/2\(^+\)) and \(Y\)-states (1/2\(^-\)) because of parity conservation in the strong interactions (VSI and MSI). Therefore the "physical particles" "\(B\)" and "\(Y\)" described by \(B\rangle_{0,1}\) and \(Y\rangle_{0,1}\) must have spin-parity 1/2\(^+\) and 1/2\(^-\) respectively. If we use the Fermi-type strong interactions, we can represent "\(B\)" and "\(Y\)" by the following Feynman diagrams (in the lowest order perturbation) [see Fig. 3].

There are three "\(B\)"-states, the physical proton "\(\rho\)" the physical neutron "\(n\)" and the physical \(A\)-particle "\(A\)". As we have already mentioned,
we have the iso-doublet ("p" and "n") and the iso-singlet ("A" only) in the charge independent approximation.

<table>
<thead>
<tr>
<th>physical particle</th>
<th>state vector</th>
<th>Feynman diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;B&quot;</td>
<td>$</td>
<td>B\rangle_{o.t.}$</td>
</tr>
<tr>
<td>&quot;Y&quot;</td>
<td>$</td>
<td>Y\rangle_{o.t.}$</td>
</tr>
</tbody>
</table>

Fig. 3. Feynman diagrams of physical "B" and "Y" particles for Fermi-type strong interactions

Table II. "Y"-states (totally S-state with 1/2-)

<table>
<thead>
<tr>
<th>state description</th>
<th>I</th>
<th>S</th>
<th>multiplicity</th>
<th>assignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N+\bar{N}+N$</td>
<td>3/2</td>
<td>0</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$N+\pi$</td>
<td>1/2</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$N+\pi^0$</td>
<td>1/2</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$N+\bar{K}+A$</td>
<td>1</td>
<td>-1</td>
<td>3</td>
<td>$\Sigma$</td>
</tr>
<tr>
<td>$N+K+\pi^0+A$</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$N+\bar{N}+N$</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$N+\pi^0$ or $K+A$</td>
<td>1/2</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$A+\bar{N}+N$</td>
<td>1</td>
<td>-1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$A+K+\pi^0+A$</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$A+\bar{K}+A$</td>
<td>1/2</td>
<td>-2</td>
<td>2</td>
<td>$\Xi$</td>
</tr>
<tr>
<td>$A+K$ or $\pi^0+N$</td>
<td>1/2</td>
<td>0</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$A+\pi^0$</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| total 27          |    |    |              |            |

Next we want to investigate the "Y"-particle (roughly speaking the bound states of $B+B+B$). There are $3 \times 3 \times 3 = 27$ different states of "Y" [see Table II], which are degenerated in the global approximation, but
which split into many iso-multiplets in the charge independent approximation. We can now make the following statement: The two lowest levels of "\(Y\)"-particles are \(\Sigma\) and \(\Xi\)-hyperons.* Therefore \(\Sigma\) and \(\Xi\) must be of \(1/2^+\). We illustrate the mass levels of physical baryons in Fig. 4. As we shall show later [§3.5], these splittings of \(Y\)-levels are rather naturally understood from the pattern of meson-splitting in the same approximation.

We can make an important comment on conventional theories of elementary particles, particularly those given by Schwinger,¹⁶ Tiomno,¹⁷ Gell-Mann¹⁸ and others,¹⁹ in which \((N, \Xi)\) and \((A, \Sigma)\) are treated as two sets of 4-component spinors in 4-dimensional charge space and the relative \(A \cdot \Sigma\) (and sometimes \(N \cdot \Xi\)) parity is assumed to be even. In our theory both the \(A \cdot \Sigma\) and the \(N \cdot \Xi\) relative parity are odd, so that the above-mentioned conventional theories are not consistent with our particular composite model.**

<table>
<thead>
<tr>
<th>spin parity</th>
<th>no interactions</th>
<th>global approximation</th>
<th>charge independent approximation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1/2^+)</td>
<td>(B + \bar{B} + B)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1/2^+)</td>
<td>(B)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 4. Mass levels of physical baryons

3.3.4. Possible existence of other strange particles

If there are (rougely speaking, of course) bound systems of more than three baryons and antibaryons, there can exist some new strange particles. Such examples are given in Table III. Laboratory experiments in the multi-BeV region and cosmic ray events have shown that the ratio of \(K^-\) to \(K^+\)-mesons produced by nuclear collisions with total strangeness zero is always

* For strangeness \(-1\) and \(-2\), respectively.
** The composite theory is so rich in its contents that one can easily deduce other possibilities (e.g. \(\Sigma:3/2^+\), etc.) than those preferred here. More detailed discussion on the classification of "\(Y\)"-particles will be given in the forthcoming paper.
very small. Therefore, very few of $H_1$, $H_0$ and $X_1$, $X_0$ and their antis, if they ever exist, should be produced by $N\cdot N$ and $\pi\cdot N$ collisions.

### Table III.

<table>
<thead>
<tr>
<th>constituents</th>
<th>isospin</th>
<th>strangeness</th>
<th>notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N+\bar{A}+N$</td>
<td>1</td>
<td>+1</td>
<td>$H_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>+1</td>
<td>$H_0$</td>
</tr>
<tr>
<td>$A+\bar{N}+A+\bar{N}+N$</td>
<td>3/2</td>
<td>-2</td>
<td>$\Sigma^+_1$</td>
</tr>
<tr>
<td></td>
<td>1/2</td>
<td>-2</td>
<td>$\Sigma^0_2$</td>
</tr>
<tr>
<td>$A+\bar{N}+A+\bar{N}+A$</td>
<td>1</td>
<td>-3</td>
<td>$E_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-3</td>
<td>$E_0$</td>
</tr>
<tr>
<td>$A+\bar{N}+A+\bar{N}$</td>
<td>1</td>
<td>-2</td>
<td>$X_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-2</td>
<td>$X_0$</td>
</tr>
<tr>
<td>$N+\bar{X}+N+\bar{X}$</td>
<td>1</td>
<td>2</td>
<td>$\bar{X}_1$</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>$\bar{X}_0$</td>
</tr>
</tbody>
</table>

### 3.4. A model of strong interactions

In this sub-section, we give an example of the strong interactions (VSI and MSI), which share the properties described above. For definiteness, we choose $C$, $P$ and $T$-conserving strong interactions of Fermi-type, which must conserve the total number of protons, neutrons and $A$'s separately.

We begin with the VSI. We postulate that the VSI acting between $p$, $n$ and $A$ must satisfy:

(a) charge independence (in the usual sense),

(b) invariance for the exchange $n\leftrightarrow A$.

We can construct "two" such interactions of Fermi-type ("two" refers only to isospin dependence):

\[
\text{VSI: } F \left[ (\bar{p}\Gamma p)(\bar{n}\Gamma n) + (n\Gamma p)(\bar{p}\Gamma n) + \frac{1}{2} \left\{ - (\bar{p}\Gamma p) + (n\Gamma n) \right\}^2 \right] \\
+ \frac{1}{6} \left\{ (\bar{p}\Gamma p) + (n\Gamma n) - 2(\bar{A}\Gamma A) \right\}^2 + (\bar{p}\Gamma A)(\bar{A}\Gamma p) \\
+ (\bar{A}\Gamma p)(\bar{p}\Gamma A) + (\bar{n}\Gamma A)(\bar{A}\Gamma n) + (\bar{A}\Gamma n)(\bar{n}\Gamma A) \\
F'\{(\bar{p}\Gamma p) + (n\Gamma n) + (\bar{A}\Gamma A)\} \{(\bar{p}\Gamma p) + (n\Gamma n) + (\bar{A}\Gamma A)\}
\]

(3.1)

where $\Gamma$ is a product of Dirac $\tau$-matrices. We do not specify the form of $F'$, provided that we do remember that $\Gamma$ should be chosen so as to give the strongest attractive forces for $^1S$-states of the baryon and antibaryon,
while giving repulsive forces (otherwise very weak attractive) for two (anti)baryons. [Fermi-Yang\textsuperscript{11} have chosen a vector type $I = \tau_\mu$ while Maki\textsuperscript{14} has taken a pseudoscalar type $I = \tau_5$.]

Next we are going to discuss the MSI for which we only impose the charge independence. Therefore we can have a general linear combination of four Fermi-type interactions

\[
\begin{aligned}
&\langle \vec{N}\tau\Gamma'N \rangle \langle \vec{N}\tau\Gamma'N \rangle \\
&\langle \vec{N}\Gamma'N \rangle \langle \vec{N}\Gamma'N \rangle \\
&\langle \vec{N}\Gamma'N \rangle \langle \vec{A}\Gamma'\vec{A} \rangle \\
&\langle \vec{A}\Gamma'\vec{A} \rangle \langle \vec{A}\Gamma'\vec{A} \rangle ,
\end{aligned}
\]

where $\Gamma'$'s can be chosen in a different manner for each case, and $N = \begin{pmatrix} n \\ \bar{n} \end{pmatrix}$, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are the usual $2 \times 2$ $\tau$-spin matrices. MSI should be chosen so as to satisfy the conditions mentioned in the preceding sections:

\[
\text{(attraction between isotriplet } N - \bar{N} \text{) must be stronger than}
\]

\[
\begin{cases}
\text{isosinglet } N \cdot N \\
\text{attraction between } N \cdot \vec{A} \text{ or } \bar{A} \cdot \bar{N} \\
\text{or } A \cdot \bar{A}
\end{cases}
\]

(3.3)

If we accept the brave approximations given in Ref. 11), 13) and 14) we can fix the strengths and types (isospin dependence and explicit forms of $\Gamma'$'s) of VSI and MSI. We shall not enter into such details here.

### 3.5. Masses of physical baryons and physical mesons

We would like to give a phenomenological description of masses of physical particles which belong to the baryon family [also see Ref. 12]. The discussions given in this sub-section are so extremely simplified that their reliability should be taken seriously.

The physical mass $m_B$ of one "$B$"-particle ($p$, $n$, and $A$) can be simply expressed by the empirical relation

\[
m_B = |n_x|m_x + |n_z|m_z , \tag{3.4}
\]

where $m_x$ and $m_z$ are the masses of one physical nucleon and of one physical $A$-particle, and $n_x$ (or $n_z$) is the "number" of nucleons (or $A$-particles)—for example, $n_x = 1$ and $n_z = 0$ for one nucleon states.

The mesons ($\pi$, $n_0$, $\pi'$, $K$ and $\bar{K}$) are assumed to be bound states of one physical baryon pair. Then the mass of these mesons may be expressed phenomenologically by a single formula
\[
m_m = \frac{1}{2} m \left[ 1 + a_1 P_1 + a_0 P_0 + a_s P_s + a_K P_K \right] P_s
\]  
(3·5)

where \(P_1, P_0, P_s\) and \(P_K\) are the projection operators to pick up special charge states of baryon pairs and \(P_s\) is the projection operator for \(1S\)-state of baryon pair [see Table IV]:

<table>
<thead>
<tr>
<th>state</th>
<th>isospin</th>
<th>projection operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N + \overline{N})</td>
<td>1</td>
<td>(P_1)</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>(P_0)</td>
</tr>
<tr>
<td>(A + \overline{A})</td>
<td>0</td>
<td>(P_A)</td>
</tr>
<tr>
<td>(N + \overline{A}) and (A + \overline{N})</td>
<td>(1/2)</td>
<td>(P_K)</td>
</tr>
</tbody>
</table>

In the global approximation we have

\[
a_1 = a_0 = a_s = a_K
\]

while these four \(a\)'s are different in the charge independent approximation. For example, if we choose

\[
m = (\text{pion mass}) = m_\pi
\]

\[
a_1 = 1
\]

\[
a_0 \geq 5
\]

\[
a_s \geq 5
\]

\[
a_K = 6
\]

(3·6)

we find from Eq. (3·5):

\[
m_\pi = m
\]

\[
m_\pi \geq 3m
\]

\[
m_\pi' \geq 3m
\]

\[
m_K = 3.5m
\]

(3·7)

In this way we have a correct mass ratio between \(K\) to \(\pi\) and two isosinglet neutrals (\(\pi_0^0\) and \(\pi'\)) will not be seen because of extremely fast decay processes:

\[
\begin{align*}
\pi_0^0 & \quad \text{or} \quad \pi' \\
\rightarrow & \quad 3\pi
\end{align*}
\]

Finally, we must discuss the masses of "\(Y\)"-particles. For simplicity,
A Composite Theory of Elementary Particles

we assume "Y"'s to be bound states of two physical "B"'s and of one anti "B":

\[ "Y" = "B" + \bar{B} + B. \quad (3\cdot8) \]

where the position of antiparticle \( \bar{B} \) should be noticed: The order in writing the right-hand side of Eq. (3·8) is chosen to emphasize the facts:

- strong attraction between \( B \cdot B \), and
- repulsion between \( B \cdot B \).

Therefore one can take an analogue of linear molecules as a model for "Y" particles [see Fig. 5]. We shall refer to such a model as: "the linear molecule model" for "Y"-particles. We can think about two special configurations of our linear molecules: "\( \bar{B} \)" is very close to one of the two "B"'s. If "B" and "\( \bar{B} \)" are close enough, we can regard a pair "B" + "\( \bar{B} \)" as a meson \( m \):

\[ "Y" = B_1 + \bar{B} + B_2 \]

\[ = (B_1 + \bar{B}) + B_2 \quad \text{or} \quad B_1 + (\bar{B} + B_2) \]

\[ = m_1 + B_2 \quad \text{or} \quad B_1 + m_2. \]

Therefore the "Y"-states \( |Y\rangle_{c.l.} \) can be well approximated by two configurations (cf., the configuration mixing of the nuclear states in the shell model):

\[ |Y\rangle_{c.l.} = f_1|m_1B_2\rangle + f_2|B_1m_2\rangle. \quad (3\cdot9) \]

More appropriately, this wave function can be compared to that of positive ion states of the hydrogen molecule. To the approximation in which we neglect the "overlap":

\[ \langle m_1B_2 | B_1m_2 \rangle \approx 0, \quad (3\cdot10) \]

we obtain

\[ f_1^2 + f_2^2 \approx 1, \quad (3\cdot11) \]

where we have used the reality of \( f_1, f_2, |m_1B_2\rangle \) and \( |B_1m_2\rangle \) guaranteed.

---

Fig. 5. Linear molecule model for a "Y"-particle. This type of configuration is not the one we are going to use [we prefer to consider "Y" as being a composite system consisting of one "B" and one "meson" \( \bar{B} + B \)--see Fig. 7]. The reason for showing this figure is to emphasize the important polarization effect of a "constituent meson", see the discussion given in §3.6.1. It might be worthwhile to call the attention to the analogue of this model: the strongly deformed nuclei (of small spin 0, 1/2, ...),
by the (assumed) time reversal invariance of our strong interactions. The masses of \( \text{"Y\"}-\)states (3·9) can be evaluated using the perfect analogue to the well-known calculation of energy levels [Heitler-London\(^\text{20}\)] of the hydrogen molecule ion. In this way we may find the mass of \( \text{"Y\"}-\)particles

\[
m_Y = f_1^2 (m_m + m_n) + f_2^2 (m_p + m_m) - \sqrt{f_1^2 f_2^2} \Delta m_Y ,
\]

where \( m_m \) and \( m_n \) are given by \([\text{the direct integral of}]\) (3·5) and (3·4). The last term is the \("\text{Heitler-London correction}\" to the \(Y\)-mass due to the overlap integral:

\[
\Delta m_Y = < m, B_2 | \text{mass operator} | B_1 m_2 > + h.c. \]  

(3·13)

Let us examine the special cases. We shall insert empirical values for \( B\)- and meson-mass in Eq. (3·12).

We first consider the \(\Sigma\)-hyperon: \( \Sigma' \) can be either \( A + \pi^+ \) or \( K^0 + p \), and

\[
| \Sigma' > = f_\pi | \pi^+ > + f_K | K^0 p > .
\]

(3·14)

We may assume:

\[
\left( \frac{f_\pi}{f_K} \right)^2 = \left( \frac{m_\Sigma - m_K - m_\pi}{m_\Sigma - m_\pi} \right)^2 \approx 4^2 ,
\]

(3·15)

a relation motivated from perturbation theory.* All masses in (3·15) should be taken as empirical values.

\((\Sigma\text{-mass}) \approx m_\Sigma + m_\pi - \frac{1}{4} \Delta m_\Sigma \)

where we have used (3·15), and \( \Delta m_\Sigma \) is the Heitler-London correction for \( \Sigma \)-mass. Inserting the known mass values,

\[
\Delta m_\Sigma \approx 280 \text{ MeV} .
\]

(3·16)

Next we want to discuss the \( \Xi \)-particle. We have immediately

\[
| \Xi > = \frac{1}{\sqrt{2}} \left[ | \pi K^0 > - | K^0 \pi > \right] ,
\]

where the coefficients are fixed from the symmetry argument. The \( \Xi \)-mass is given by

\[
(\Xi\text{-mass}) \approx m_\pi + m_K - \frac{1}{2} \Delta m_\Sigma .
\]

(3·17)

If one uses the global approximation, one obtains

\[
\Delta m_\Sigma = \Delta m_\Sigma .
\]

(3·18)

* For the same reason, we have neglected many other configurations in Eq. (3·14).
However, Eq. (3.17), together with (3.16), (3.18), predicts too large values for the $\Xi$-mass. Therefore one must go to the charge independent approximation also for the Heitler-London correction. Naively one can expect

$$|\langle A\pi|\overline{K}N\rangle|>|\langle A\overline{K}|\overline{K}A\rangle|,$$

so that one can tentatively take

$$\Delta m_{\Xi} \approx 2\Delta m_{\Sigma} = 560 \text{ MeV}, \quad (3.19)$$

then the predicted $\Xi^0$-mass would be 1330 MeV, a surprisingly good value, considering the crudeness of our argument.

Masses of remaining "$Y$"-states can be evaluated similarly, provided the masses of $\pi'$ and $\pi$ are known. If we assume, for example, (3.6), we can show that the $\Sigma$ and $\Xi$-particles are in fact lowest mass levels for $S=-1$ and $S=-2$ among many "$Y$"-states. It would be very interesting to evaluate similarly the masses of "new" strange particles listed in Table III. The $H$-hyperons can be very easily treated. The results are as follows: First of all $H_0$ would be unstable, $H_0 \rightarrow H_1 + \pi$. While $H_1$ ($= N + K$ or $K + N$) would have a mass value between 1154 MeV and 1294 MeV from simple application of a formula analogous to (3.17), together with the Heitler-London correction $\Delta m_H$:

$$\Delta m_{\Sigma} \ll \Delta m_{\Xi} \ll \Delta m_{\Xi}.$$

[cf. Eqs. (3.16) and (3.19)]. This result does not seem to be reasonable.*

We believe that this is certainly due to the crudeness of our approximation, so that we shall not take this as an evidence against our composite model. All other particles listed in Table III are so complicated that their masses can hardly be guessed.

3.6. Simple consequences of the composite model

We shall give several discussions which could be regarded as favourable to the composite nature of the physical particles. All examples given below do not necessarily mean to be in favour of our composite model. Most of them are, more or less, typical to any theories of (or systems coupled by) very strong interactions (possibly the conventional Yukawa type theory for meson-baryon systems would be contained in such categories of interactions).

3.6.1. The electric structure of physical neutron

Let us begin with the consideration of the physical neutron, whose

---

* On the contrary, if the existence of $H_1$ were established, we could rely very much upon the mass-arguments given in this section. We agree that such a situation would occur with small probability.
main configurations will be given by

\[ n \]
\[ \bar{p} + \bar{p} + n \}
\[ n + n + n \}
and
\[ A + \bar{A} + n \].

(3·20)

(3·21)

(3·22)

The last three configurations suggest again the linear molecule model [see Fig. 6]. Notably (3·21) can be replaced by

\[ \pi^0 + n \text{ or } \bar{p} + \pi^- \]

in the case of asymmetrical arrangement of the intra-baryonic distance; [for example, see Fig. 7]. Because of the even parity of the physical neutron, \( p \) and \( \pi^- \) (or \( n \) and \( \pi^0 \)) must be in the \( p_{1/2} \)-state with \( I = 1/2 \). In this way we can express the physical neutron state as follows:

\[
|\text{physical neutron}\rangle = f_\pi |n\rangle + f_\pi \frac{1}{\sqrt{3}} \left[ |n\pi^0; \bar{p}_{1/2}\rangle - \sqrt{2} |p\pi^-; \bar{p}_{1/2}\rangle \right] + f_\kappa |K^0A; \bar{p}_{1/2}\rangle + \ldots .
\]

(3·23)

The second and the third terms—namely the configurations \( N + \pi \) and \( K + A \)—may be compared with what Wheeler\(^{21\text{)}\} \) called the resonating group structure in nuclear configurations. Such configurations, as explicitly written in Eq. (3·23) are expected to be important from the energetical point of view.

Let us now discuss the electric charge distribution \( \rho_n(r) \) in the physical neutron state.\(^{22\text{)}\} \) At large distances from the center, \( \rho_n(r) \) must evidently be given solely by one configuration: \( (p + \pi^-) \) illustrated in Fig. 7. While,

The configuration

![Fig. 6. The linear molecule model of the neutron](image)

and

![Fig. 7.](image)
at (and within) relatively small distance, this configuration \((p+\pi^-)\) will neither be legitimate nor play any important role. The reasons for this are very simple: Firstly, at small \(\pi^-\cdot p\) distance \(\pi^-\) is highly polarized by strong \(N\cdot\bar{N}\) attraction so that the picture of "one pion" will lose its meaning. [There the model illustrated in Fig. 6 is more appropriate than that of Fig. 7.] Secondly, many other configurations carrying many charged particles become equally as important (or more) as \((p+\pi^-)\). All these effects tend to neutralize the electric charge at inner parts of the physical neutron. We now conclude that the \(\rho_n(r)\) is very near to zero over the relatively wide range \((r \lesssim 1/m_\pi)\) of the neutron structure.\(^{32}\) \(\rho_n(r)\) \((r > 1/m_\pi)\) must be negative and its asymptotic value \((r \gg 1/m_\pi)\) must be given by the simple configuration \((p+\pi^-)\) [Fig. 7]. Thus the physical neutron looks like electrically neutral to a surprisingly good extent. Such a picture for the electric structure of the neutron is, even though quite naive, in a good direction, if one recalls the difficulties\(^{24}\) met in understanding the electromagnetic structure nucleon (or more generally meson-baryon) system.

### 3.6.2. Antinucleon annihilation cross-section

According to the Sakata model the electric charge distribution in the physical proton is nothing but the distribution of "nucleonic charge", or "baryonic element". Therefore the Hofstadter (electric charge) radius\(^{32}\) \(r_n\) for the proton can also be regarded as the mean square root radius of the nucleonic charge. Hence, if an antiproton approaches a proton within the distance \(2r_n\), then the annihilation process (or more generally reactions) will immediately follow. Thus the annihilation cross-section will be given in the classical limit

\[
\sigma_{\text{ann}} \approx \pi (2r_n)^2 \tag{3.24}
\]

and the total cross-section turns out to be

\[
\sigma_{\text{tot}} \approx 2\sigma_{\text{ann}}, \tag{3.25}
\]

because of the diffraction scattering. If one inserts the empirical value \(\sim 1/2m_\pi\) for \(r_n\), \((3.24)\) and \((3.25)\) are consistent with the experimental information\(^{55}\) (within the accuracy of our reasoning), as has been noticed some time ago.\(^{33}\)

### 3.6.3. \(\pi\pi\) interaction cross-section and the high energy limit of nuclear cross-sections

The \(\pi\pi\) interaction cross-section will be given, in the classical approximation, by

\[
\sigma_{\pi\pi} \approx \pi (2r_\pi)^2,
\]
where \( r_\pi \) is the "size" of a composite pion. \( r_\pi \) is supposed to be

\[
\frac{1}{m_\pi} \quad \text{(half of the pion Compton wavelength)}
\]

and

\[
60mb \geq \sigma_{\pi^-\pi} \geq 5mb.
\]

Similarly, the high energy limit (i.e., classical limit) of the total interaction cross-sections of \( \pi^-N \) and \( \pi^-N \) collisions will be equal to \( \pi(2r_\pi)^2 \) and \( \pi(r_\pi + r_N)^2 \), respectively. These results are in fair agreement with evidence coming from artificial beam experiments in the multi-BeV region and extremely energetic cosmic radiations.

We have seen that the strange particle production is rather infrequent over a very great interval of energies [from multi-BeV (accelerators) to the extreme high energy region given by cosmic rays]. Cocconi\(^{29}\) has remarked that this fact is in favour of the composite nature of strange particles. Unfortunately we cannot agree with him: The so-called final state interactions are so important over the energy regions mentioned above that one can hardly conclude anything like the composite nature of strange particles. In other words, we must take into account the gradual expansion and cooling down of the "hot spots"—the collision complex produced by the energetic nuclear collisions—as was emphasized by Landau\(^{27}\) in his revised version [of the Fermi theory\(^{28}\)] of multiple production. Therefore the main feature of multiple production can essentially be fixed at [the late stage after the collision process actually began and at] the regions of relatively "low temperature". At "low temperature" we know that strange particle production is relatively rare.

### 3.7. Electromagnetic mass splitting of the isomultiplets

In the charge independent approximation, masses of all members of the same isomultiplets are equal. The electromagnetic interactions are responsible to their mass splitting. It is hopelessly difficult to predict the mass differences from a theory, therefore we shall not discuss them at all, except for one important comment. It is perhaps commonly believed that the spinless neutral meson should be lighter than the charged meson, which belongs to the same isomultiplet. The pion is an (and only one known) example of this kind. Most naive explanation of this fact would be the "Coulomb energies" of the composite pions (cf., the semi-empirical formula for the masses of atomic nuclei, where we have the Coulomb term which makes the nucleus of higher \( Z \) heavier among nuclei of an isomultiplet). In practice, such an over-simplification would not be allowed and a more refined investigation of the problem is required. We have again learned
that $\pi^0$ should be lighter than $\pi^\pm$.* The argument used to derive this result cannot, however, be applied to $K^+ - K^0$ mass difference. This is because the anti $\pi^0$ is identical to $\pi^0$ (consequently $\pi^0$ is completely neutral for one gamma processes) while the anti $K^0$ ($\bar{K}^0$) is different from $K^0$ ($K^0$ and $\bar{K}^0$ cannot be regarded as electrically neutral for single virtual gamma processes). More precisely, a matrix element of the electric current operator $J_a$ taken between two physical one $\pi^0$-states with different momenta $q$ and $q'$ is identically zero:

$$\langle \pi^0, q \mid J_a \mid \pi^0, q' \rangle = 0 \quad \text{for all } q, q'. $$

On the other hand, an analogous matrix element for $K^0$ is in general

$$\langle K^0, q \mid J_a \mid K^0, q' \rangle \neq 0 \quad \text{for } (q - q')^2 \neq 0$$

even though

$$\langle K^0, q \mid J_a \mid K^0, q \rangle = 0,$$

i.e., the total charges and the static magnetic moments of $K^0$ and $\bar{K}^0$ must be zero. Hence we can well except the similar situation as we met in the case of the nucleon (the neutron is heavier than the proton): $K^0$ could be heavier than $K^+$.** Recent experiments have shown that this is really the case.***

§4. Weak interactions

In this section we discuss weak interactions (WI) and possible existence of extremely weak interactions (weaker than usual WI).

4.1. Weak interactions

We shall here describe an example of how we can formulate the weak interactions in our framework of the composite model. We shall use the similarities and complementaries of leptonic and baryonic fundamental particles emphasized in § 2.

We may begin our discussion in the following manner: Particle-number conservation laws are very much relaxed for the WI and there are only two conservation laws:

$$\begin{align*}
(baryon \ number) &= n_b = \text{const} \\
(lepton \ number) &= n_l = \text{const}
\end{align*}$$

---

* For example, see S. Gasiorowicz and A. Petermann, Phys. Rev. Letters 1 (1958), 457.
where \( n_b \) (or \( n_i \)) is the sum of three: proton-, neutron- and \( \Lambda \)-numbers (or the sum of three: neutrino-, electron- and \( \mu \)-numbers). The interactions (including WI) try to convert all particles into their ground levels (i.e., the protons and the neutrinos), provided there are no restrictions imposed by the electric charge conservation or by the energetic reason. [Stability of electrons and of non-radioactive nuclei are such "exceptional" examples]. Therefore, we argue that WI must contain the forms:

\[
\left( \overline{p} \Gamma p \right), \left( \overline{p} \Gamma \Lambda \right)
\]

and

\[
\left( \overline{\nu} \Gamma e \right), \left( \overline{\nu} \Gamma \mu \right)
\]

where \( p, n, \ldots \) designate the field operators of the bare proton, of the bare neutron, \( \ldots \); and \( \Gamma \) is an appropriate matrix constructed from the usual Dirac matrices. These bilinear forms in field operators evidently satisfy the conservation law (4·1). Since bare fundamental particles show complete symmetries among themselves, there are no \textit{a priori} reason to prefer one of two \((n, \Lambda)\) and \((e, \mu)\) [and thus one of the 4 bilinear forms]. In fact, this is supported by the equality of vector constants of nuclei \( \beta \)-decay, \( \mu \)-decay, and \( \mu \)-capture. Thus we arrive at the universal* weak interactions of Fermi type

\[
\frac{G}{\sqrt{2}} \left[ \left( \overline{p} \Gamma p \right) + \left( \overline{p} \Gamma \Lambda \right) + \left( \overline{\nu} \Gamma e \right) + \left( \overline{\nu} \Gamma \mu \right) \right] \\
\times \left[ \left( \overline{n} \Gamma p \right) + \left( \overline{\Lambda} \Gamma p \right) + \left( \overline{e} \Gamma \nu \right) + \left( \overline{\mu} \Gamma \nu \right) \right].
\]

(4·2)

Experiments have shown that neutrino obeys the two component theory, and the nuclear \( \beta \)-decays must be V-A. These empirical results, together with complete symmetry among bare fundamental particles, suggest the choice of \( \Gamma \): \n
\[
\Gamma = \gamma_\alpha (1 + \gamma_5)
\]

and

\[
\left( \overline{p} \Gamma n \right) = \left( \overline{p} \gamma_\alpha (1 + \gamma_5) n \right), \text{ etc.}
\]

(4·3)

Our WI (4·2) with \( \Gamma = \gamma_\alpha (1 + \gamma_5) \)—in (4·2) the sum over \( \alpha \) (\( \alpha = 1 \) to 4) should be taken—is invariant against the so-called Fierz-transformation, and is essentially equivalent to the Feynman-Gell-Mann\(^{29}\) theory of WI.

In our theory, strangeness conserving and violating (baryonic) currents have very simple forms:

\[
J_{so} = \left( \overline{r} \gamma_\alpha (1 + \gamma_5) n \right) = J_{so}^a + J_{so}^s.
\]

(4·4)

* If experiments disprove the universality, we may simply introduce different weak constants for various terms in (4·2).
A Composite Theory of Elementary Particles

and

\[ J_{\text{SNK}} = (\overline{p}r_a(1 + r_b)A) = J_{\text{SNK}} + J_{\text{SNK}}. \] (4.5)

The pionic and kaonic terms are missing here because pions and kaons are considered as composite systems of baryon-antibaryon. However, the decay rates of

\[ \pi^+ \rightarrow \pi^0 + e^+ + \nu \]

\[ K^0 \rightarrow K^+ + e^- + \bar{\nu} \]

\[ \Sigma^- \rightarrow \Sigma^0 + e^- + \bar{\nu}, \text{ etc.}, \]

evaluated from (4.2) [such evaluations can be done in the same way as, say, the nuclear $\beta$-decay case], are exactly the same as in the Feynman-Gell-Mann theory.\(^{29}\) Moreover, we can prove the (approximate) conservation laws:

\[ \delta_a(J^a) = 0 \quad \text{(valid in the charge independent approximation)} \] (4.6)

and

\[ \delta_a(J^a) = 0. \quad \text{(valid in the global approximation)} \] (4.7)

The axial currents $J^a$ and $J_{\text{SNK}}$ fail to satisfy the conservation laws.* These results are quite satisfactory since:

\[ \pi^+ \rightarrow \{ \mu^+ + \nu \} \]

\[ K^+ \rightarrow \{ \mu^+ + \nu \} \]

are allowed\(^{31,32}\) and the inequality,

\[ \frac{K^+ \rightarrow \pi^0 + e^+ + \nu}{K^+ \rightarrow \pi^0 + \mu^+ + \nu} > 1, \] (4.8)

holds to the extent that the global approximation is valid. [(4.8) is

* If the bare masses of baryonic fundamental particles are zero, $J^a$ and $J_{\text{SNK}}$ obey the conservation laws.

\(^{31,32}\) The fact that $\pi^\mu$ and $K_{\mu}$ decay processes are allowed, can be regarded as the proof of non-zeroness of "bare" baryon mass $m_0$ (we must have the equality of masses for $p$, $n$ and $A$). The bare mass $m_0$ should not be too small to allow the $\pi^\mu$ or $K_{\mu}$ decay, while it should not be too large to be consistent with the approximate equality:

\[ -(\text{renormalized Axial vector constant of nuclear } \beta\text{-decay}) \]

\[ \approx (\text{renormalized vector constant of nuclear } \beta\text{-decay}) \] (see Ref. 6). We may guess that the bare mass $m_0$ is of the order of a tenth of the physical proton mass.
consistent with experimental information, although—peculiar enough—Goldberger-Treiman\(^{32}\) have made an opposite statement.\) Clearly many results obtained by Feynman-Gell-Mann,\(^{2}\) Okubo et al. and others\(^{33}\) can be directly applied to our weak interactions \((4\cdot2)\). In spite of this fact, we shall add a few comments on the pionic decay processes of \(\Sigma\)-hyperons.

\(K\)-mesons.

The fact that the \(A\)-decay \((A\rightarrow N+\pi)\) is successfully explained from our decay interaction, Eq. \((4\cdot2)\), is well known,\(^{34}\) so no further comments on it are necessary.

The pionic \(\Sigma\)-decay is rather obscure.* Suppose we make the following assumptions:

i) The \(\Sigma\)-state is represented by a simple configuration, the \(S\)-state consisting of one \(A\) and \(\pi\), likewise the physical nucleon is represented by the bound state of \((N+\pi)\) in the \(P_{3/2}\) state,

ii) our WI \((4\cdot2)\) is replaced by the phenomenological decay interactions for \((A\rightarrow N+\pi)\) decay

\[
\left(\bar{\rho}_T(1+\gamma_5)A\right) \frac{\partial \pi}{\partial X_a} + \frac{1}{\sqrt{2}} \left(\bar{\eta}_T(1+\gamma_5)A\right) \frac{\partial \pi^0}{\partial X_a}.
\]

iii) baryons are infinitely heavy ("static approximation"), and

iv) there are no strong \(\pi\)-\(\pi\) interactions.

Then only the parity conserving parts of WI contribute to the \(\Sigma\)-decay. That is all right as far \(\Sigma^-\rightarrow n+\pi^-\) and \(\Sigma^+\rightarrow n+\pi^+\) are concerned, since they have such small asymmetry parameters.\(^{35}\) Under these assumptions, the decay rates would be, unfortunately,

\[
\begin{align*}
w(\Sigma^+\rightarrow p+\pi^+) : w(\Sigma^+\rightarrow n+\pi^+) : w(\Sigma^-\rightarrow n+\pi^-) \\
= 0 : 1 : 1.
\end{align*}
\]

Apparently a more refined treatment of the problem is required. We shall give a more detailed discussion in the near future.\(^{36}\)

The situation about the \(\beta\)-decay of \(\Sigma^+\) is very obscure.\(^{27}\) So we shall not discuss it at the moment. [If this \(\beta^+\)-decay were in fact present with the branching ratio of the order of 1\%, we would have to modify our theory of Feynman-Gell-Mann type to a great extent. Thus the check of \(\Sigma^+-\beta^+\)-decay is very urgently required.]

* B. d'Espagnat and J. Prentki (private communication) have found complicated Fermi-type interactions whose lowest order perturbation calculation leads to consistent results for all pionic decays of \(\Sigma\) and \(A\)-hyperons.
4. 2. Super weak interactions (SWI)

4. 2. 1. General discussion

Finally we would like to present a more drastic speculation. We claim, of course, by no means that our discussions exhaust all possibilities. Until now we have been discussing strong and weak interactions (VSI, MSI and WI). We know the presence of electro-magnetism. Then we naturally raise a question: "Are there any other type of interactions ?" Let us try to clarify—partially—this question.

First of all, the possibility of finding some basically new, but still important,* interactions is extremely unlikely (though we cannot say it is impossible). Therefore, at the moment, we should rather search for the possible existence of "extremely" weak interactions—hereafter referred to as "super weak interactions" (SWI).

We know from our experience the following important "hierarchies" of interactions according to their strengths; the weaker the interactions, the

<table>
<thead>
<tr>
<th>interaction symmetry property or conservation law</th>
<th>VSI (very strong)</th>
<th>MSI (moderately strong)</th>
<th>EM (electro-magnetism)</th>
<th>WI (weak)</th>
<th>SWI (extremely weak)</th>
</tr>
</thead>
<tbody>
<tr>
<td>global symmetry</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>charge independence</td>
<td>NO</td>
<td>NO</td>
<td>NO</td>
<td></td>
<td></td>
</tr>
<tr>
<td>strangeness conservation</td>
<td>NO</td>
<td></td>
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<tr>
<td>parity conservation</td>
<td>NO</td>
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</tr>
<tr>
<td>charge conjugation invariance</td>
<td>NO</td>
<td></td>
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</tr>
<tr>
<td>time reversal invariance**</td>
<td></td>
<td></td>
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<td></td>
<td>?</td>
</tr>
<tr>
<td>baryon number conservation</td>
<td></td>
<td></td>
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<td></td>
<td>?</td>
</tr>
<tr>
<td>lepton number conservation**</td>
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<td>electric charge conservation</td>
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<tr>
<td>energy-momentum conservation</td>
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<tr>
<td>angular momentum conservation</td>
<td></td>
<td></td>
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<td>?</td>
</tr>
</tbody>
</table>

* By "important" interactions, we mean that their effective strengths are larger than—or of the order of—the usual weak interaction constant.

** This is assumed to be valid in usual WI, since there is no evidence against it.
larger the number of symmetry properties and of conservation laws which
are violated. We can see this fact in Table V. Natural and straightforward
extrapolation of our knowledge on the properties of various interactions would
strongly suggest that

"SWI's, if they ever exist, must violate some of
the conservation laws valid up to the usual WI".

First, even if the energy-momentum or the angular momentum conserva-
tion law were violated, it could not be found by any feasible experiments.
So that we shall not discuss such a possibility. Theoretically speaking,
such a violation would be too catastrophic indeed.

Although violation of the time reversal invariance is far less drastic,
as compared with the one just mentioned, its test would also meet fantastic
difficulties in the experiments.* We decided not to discuss it here.

Then we are left with only a few possibilities: violation of either one,
or some combinations, of violation among

\[(b) : \text{baryon number } (= n_b) \text{ conservation},\]
\[(l) : \text{lepton number } (= n_l) \text{ conservation},** \text{and}\]
\[(Q) : \text{electric charge } (= Q) \text{ conservation},\]

which we are going to discuss. We must not forget the fact that three
laws, \((b), (l) \text{ and } (Q)\), guarantee altogether the stability of atoms consisting
of electrons and non-radioactive nuclei—more appropriately we may refer
to it as the "stability of matter". Moreover, these three laws are derived
from the invariance of the conventional theory against the "gauge" (or phase)
transformations.

We can think of many varieties, for example:

"Both \((b) \text{ and } (l)\) lose their validity but \((Q)\) must still hold" \ldots (A)

"\((b) \text{ and } (l)\) lose their separate validities but the combined law,
\[n_b + n_l = \text{const}\]  \((4\cdot9)\)

or
\[n_b - n_l = \text{const}\]  \((4\cdot10)\)

is valid, and \((Q)\) is still good" \ldots (B)

"Both \((b) \text{ and } (l)\) hold, but \((Q)\) fails to be valid" \ldots (C)

\* We are discussing the \(T\)-violation in SWI. \(T\)-violation in usual WI, if it exists, is easy
to see. [We have assumed \(T\)-invariance for WI]

\** Conventionally, we call \(\nu, e^+\) and \(\mu^-\) leptons (\(\bar{\nu}, e^-\) and \(\mu^+\) antileptons) and assign to
them lepton number \(+1\ (−1)\).
and many other cases. For definiteness we shall restrict our discussion to (A), (B) and (C).

4.2.2. Metastability of matter

The "metastability of matter" has already been discussed both experimentally\(^3\) and theoretically\(^{39,40}\) Hence we shall only say a few words on cases (A) and (B). Since we have the angular momentum conservation law, the change of \(n_s, n_l\) or \(n_s \pm n_l\) by an odd number (say, \(\pm 1\)) is strictly forbidden. Therefore the possible mechanisms of metastability of matter would occur through the following processes:* 

**Case (A):**

\[
p^+ + p^- \rightarrow \pi^+ + \pi^+,
\]

\[
n + n \rightarrow \frac{2\gamma}{2\nu}, \text{ etc.}
\]

**Case (B):**

**Case (B\(_\uparrow\)):** "\(n_s + n_l = \text{const}\)"

\[
\{ \begin{align*}
n & \rightarrow 2\nu + \bar{\nu}, \\
p + n & \rightarrow p + \nu, \text{ etc.}
\end{align*}
\]

**Case (B\(_\downarrow\)):** "\(n_s - n_l = \text{const}\)"

\[
\{ \begin{align*}
n & \rightarrow \nu + 2\bar{\nu}, \\
p & \rightarrow e^- + 2e^+, \\
e^- + p & \rightarrow (e^+ + e^-), \\
\nu + \bar{\nu} & \text{, etc.}
\end{align*}
\]

It should be noticed that in case (B\(_\uparrow\)) the proton cannot (directly) transform into leptons because of the charge conservation \((Q)\), while in the second case, (B\(_\downarrow\)), there is no such restriction. If future experiments may prove either one of the two alternatives, case (B\(_\uparrow\)) and case (B\(_\downarrow\)), then one could arrive at a "natural" definition of particle numbers: For example, if (B\(_\downarrow\)) were true, it would be more appropriate to call \(\bar{\nu}, e^+, \mu^+\) the "leptons" (rather than antis) and \((4\cdot 10)\) can be regarded as the conservation law of the "fermion number".

Experimental checks\(^{38}\) have shown that the lifetime of the nucleon must be longer than \(10^{22}\) yr. If we take a special example of Fermi-type SWI,

* It seems unnecessary to remark that what we are going to list is only illustrative examples. The "final state interactions" due to "strong interactions" (VSI, MSI, and usual WI) would affect the outlook of the final reaction products.
30

Y. Yamaguchi

\[
\frac{1}{\sqrt{2}} g (\bar{e}^+ \tau_a (1 + \tau_5) p) (e^- \tau_a (1 + \tau_5) e^-)
\]

(4.11)

for the spontaneous decay of the proton: \( p \rightarrow 2e^+ + e^- \), we can set an upper limit for \( g \)

\[ g < G \times 10^{-14} \]

(4.12)

where \( G \) is the usual weak constant which appeared in Eq. (4.2):

\[ G = 1.4 \times 10^{-49} \text{ erg cm}^3 = 1.0 \times 10^{-5}/M^2 \]

\( (M = \text{proton mass}) \).

For more details and related problems, we shall refer to the paper\(^{39}\) by the present author.

4.2.3. Violation of charge conservation law

Finally we shall give a brief discussion on case (C), a special example of charge non-conservation. Our discussion given below is based on dimensional arguments and is largely insensitive to the specific choice of a charge-violating interaction.

As was mentioned in §4.1, the usual WI's cause the conversion of all particles belonging to the baryon and lepton families into their ground states (the protons and the neutrinos). However, there was a restriction imposed by the charge conservation law \((Q)\) [beside the energetical restriction for the nuclei]. If one admits the non-conservation of electric charge, one can remove this restriction. Thus possible examples of charge non-conserving SWI of Fermi-type would be given by

\[
\frac{1}{\sqrt{2}} f_1 (\bar{p} \Gamma n) (\bar{p} \Gamma p),
\]

(4.13)

\[
\frac{1}{\sqrt{2}} f_2 (\bar{\nu} \Gamma e) (\bar{\nu} \Gamma \nu),
\]

(4.14)

\[
\frac{1}{\sqrt{2}} f_3 (\bar{p} \Gamma n) (\bar{p} \Gamma \nu)
\]

(4.15)

and

\[
\frac{1}{\sqrt{2}} f_4 (\bar{p} \Gamma p) (\bar{\nu} \Gamma e),
\]

(4.16)

where we have assumed that both \( n_0 \) and \( n_t \) are conserved ["case (C)"].

Existence of such interactions gives rise to two immediate consequences: The one is a very small difference between electric charges of the proton \((Q_p)\) and of the electron \((Q_e)\):

\[
\Delta = \left| \frac{Q_e + Q_p}{Q_p} \right| \cong 0,
\]

(4.17)
and the other is an extremely small, but finite, rest mass of the photon (because of gauge non-invariance)

\[ m_r = 0. \] (4.18)

The neutron could also have a small amount of charge:

\[ |Q_n| = Q_n \cdot J. \] (4.19)

(This depends upon the choice of charge non-conserving interactions.) At present, empirical limits of \( J^{(1)} \) and \( m_r^{(2)} \) are given by

\[ J < 10^{-13}, \quad \frac{1}{m_r} > 10^9 \text{km} = 10^{10} \text{cm}. \] (4.20)

It should be noticed that the interactions (4.14) and (4.16) give rise to the metastability of matter. The former (4.14) predicts the decay process

\[ e^- \rightarrow 2\nu + \bar{\nu} \] (4.22)

with the lifetime

\[ \tau_e = \left( \frac{f^2 m_e^3}{3 \cdot 2^6 \cdot \pi^3} \right)^{-1} \] (4.23)

for \( \Gamma = \gamma_a(1 + \gamma_b)("V - A \text{ type})" \). Therefore "stable atoms" will lose their electrons by the spontaneous decay, thereby leaving stripped positive nuclei. Hence the matter will collapse. On the other hand, the latter choice (4.16) causes the conversion of \( e^- \) into \( \nu \) by the presence of \( p \rightarrow \bar{p} \) acts as "catalysis":

\[ p + e^- \rightarrow \bar{p} + \nu. \] (4.24)

The transition rate for 1s-electron in the atom (atomic number \( Z \)) is given by:

\[ \omega_e = \frac{2 f^2}{\pi^2} \left( \frac{1}{137} \right)^n Z^2 m_e^5 = \frac{1}{\tau_e} \] (4.25)

where we have again chosen \( \Gamma = \gamma_a(1 + \gamma_b) \), for simplicity sake, in comparison with the usual WI. The lower limit of the electron lifetime \( \tau_e \) was determined by direct experiment\(^\text{40}\)

\[ \tau_e > 10^{17} \text{yr}. \]

Notice that this limit, \( 10^{17} \text{yr.} \), is much longer than the age of our galaxy, \( \sim 10^{10} \text{yr.} \). From this lower limit we easily find:
where $G$ is again the usual WI-constant (= nuclear $\beta$-decay constant). On
the other hand, the interactions (4·13) and (4·15) do not lead to any
metastability of matter (though they can induce spontaneous nuclear mu-
tations, $A^{36} \rightarrow S^{36}$ etc).

The perturbation theory applied to the interactions (4·13)—(4·16) would
give the following results:* 

$$A \sim (fM^2)^2,$$

$$m_\gamma \sim \frac{1}{137} (fM^2)^2 m,$$

where we have used a cut-off method to avoid the divergent integrals [we
took a cut-off momentum $= $ proton mass $= M$]. (4·28) can be easily
obtained from dimensional argument, and, therefore, their validity—especially the
relation $m_\gamma \sim M^2 A/137$, where $M$ is a constant of dimension of mass, say,
proton mass—is very general (independent of the special assumption of
the charge-violating interactions) (though the results are sensitive to cut-off
momentum). Comparing these expressions with the empirical data (4·20)
and (4·21) we find upper limits on the SWI-constants $f$: 

$$f \lesssim 3 \times 10^{-22} G \quad \text{from } A,$$

$$f \lesssim 10^{-18} G \quad \text{from } m_\gamma.$$  (4·29)

(4·30)

For the SWI-Hamiltonian (4·14) or (4·16), the stability of matter
gives a limit on $f_3$ or $f_4$ [Eqs. (4·26) or (4·27)]. Therefore one can
predict $A$ and $m_\gamma$ in such a case:

For the choice (4·14): $A \lesssim 10^{-29}$, $(1/m_\gamma) \gtrsim 10^6$ cm,  (4·31)

For the choice (4·16) $A \lesssim 10^{-20}$, $(1/m_\gamma) \gtrsim 10^6$ cm. (4·32)

In all cases, (4·13) and (4·16), the lower limit of the photon Compton
wavelength gives a better limit for the strength $f$ of charge violating
SWI and that for $A$: $A \lesssim 10^{-46}$.

Lytleton and Bond* have recently discussed the cosmological implications
of finite $A$. Notably their choice of $A$ was

$$A = 2 \times 10^{-18}.$$  (4·33)

---

* The bare mass of the photon is assumed to be zero. $p$ and $e^+$, etc., are assumed to
have the same charge if we neglect all SWI's.

The author is indebted to Dr. G. Feinberg for pointing out the error in the original
expression for $m_\gamma$.

Goldhaber and Feinberg* have also considered a similar possibility described in §4,2,3
(private communication).
It would be interesting to remark about another limitation to the photon compton wavelength. There are hydro-magnetic turbulences of huge scale in interstellar gases, etc., and notably in Crab nebula. Such turbulent motions—a sort of cooperative phenomena—suggest that the Compton wavelength \(1/m_\gamma\) for the photon should not be smaller than the linear dimension of magnetic turbulence, \(\sim 0.1 \text{ l.y.} = 10^{17}\text{cm}\):

\[
1/m_\gamma > 10^{17}\text{cm}.
\]

If this is true, it turns out to be

\[
f < 10^{-20}G \quad \text{and} \quad 4 < 10^{-60}.
\]  

(4.34)

This result sharply contradicts the sensational proposal (4.33) due to Lyttleton and Bondi. The lower limit of \(1/m_\gamma\) is extremely large and of astronomical scale. This would mean that the electromagnetism with finite \(m_\gamma\) could not be closed in the domain of the particle physics and might require some unified description covering the microscopic world (particle physics), as well as the macroscopic world (cosmology).

Within such a high accuracy we have to discuss the mass of the neutrino also. Up to now we have known only poor limit on the upper limit of neutrino-mass, and it is impossible to discuss it further.

What we have tried here is to find a possible link between three items:

— non-conservation of electric charge,
— disparity of charges between the proton and the positron,
— a finite rest mass of the photon.

Sometimes stability of matter can also be connected to these three. Of course, there exist infinitely many other possibilities. For example, we may be able to construct such a modified theory of electromagnetism with a non-vanishing rest mass of photon as to be consistent with the strict charge conservation. In this paper we are not interested in such a possibility.

4.2.4. Discussion

We have just discussed the possibility of violating the baryon number, the lepton number and the charge conservations. The very reason for

* The following argument has been clarified during the course of discussions with Dr. W. Thirring.

** Of course we know many counter examples to such an argument: The short range interactions can give rise to large scale cooperative phenomena (spin wave, collective behaviour of nuclei, etc).

*** The size of Crab nebula is \(\sim 3\text{ l.y.}\).
doing this is nothing but our revolutionary experience: discovery of the parity non-conservation in usual WI. By this we have also learned that validity of any conservation laws (and corresponding symmetry properties) must be based on experimental verification, no matter how natural and absolutely true it may look. We hope that our foregoing discussions serve to warn of the possibility of some further break-down in conservation laws.

We may go on now to further speculation. The relative strengths of usual WI, in comparison with those of strong interactions, would be given by

$$R = G/F \sim 10^{-5}$$

which is a “mysterious number” and we have no idea where it comes from. We shall simply accept this “empirical” number and use it as guide for further extrapolation, we may assume that strengths of various SWI’s are given by

$$\sim R^n G$$

where $n$ is positive integer, $n = 2, 3, 4 \ldots$. For example, $(4 \cdot 12)$ and $(4 \cdot 30)$ or $(4 \cdot 34)$ would suggest:

$$\begin{cases} g/G \sim R^2 \\ f/G \sim R^4 \text{ or } R^5 \end{cases}$$

for spontaneous proton decay or charge non-conservation.

The experimental verification of such relations would be extremely interesting.*

It should be noticed that the cases corresponding to $n = 2$ might be very confusing, due to the presence of second order effects of usual WI. Double $\beta$-decay of nuclei or the decay with $\Delta S = \pm 2$, e.g., $\Sigma \to N + \pi$, would be such an example.

To be fair, we must not forget the existence of extremely weak, but well-known, interactions: the universal gravitation. The strength of the gravitational force is extremely small compared with all other known coupling constants. Nevertheless, the gravitational interaction shares many “good” features with strong (particularly, electromagnetic) interactions as far as the validity of most of the conservation laws is concerned. Moreover, we do not at all understand any possible connection of the gravitational interaction with other interactions known to us in the particle physics [however, see Ref. 39]). This is why we have not discussed the gravitation.

* The experiment to improve the lower limit on the lifetime of “stable matter” is in progress at CERN by H. Frauenfelder, B. D. Hyams and L. Koester.
Acknowledgements and Addenda

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22) See the review articles by R. Hofstadter. Rev. Mod. Phys. 28 (1956), 214; 30 (1958), 482.
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25) For example, see E. Segre, Annual Rev. of Nucl. Sci. 9 (1958), 127.
30) Y. Yamaguchi. Lecture notes on strange particle physics, CERN, 1959, chapter V (6).
36) In this connection, see: G. Takeda and M. Kato, Prog. Theor. Phys. 21 (1959), 441. What these authors have mentioned is extremely optimistic, so that reliability on such arguments is rather difficult to judge.
37) Private Communication. I have enjoyed discussions about this problem with Prof. N. Dallaporta, Dr. M. Baldo-Ceolin, Dr. Saladin and many others.
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(Note added in proof)

Addenda to References

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