On Fermi's Theory of High Energy Nucleon-nucleon Collisions

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At present there is no really satisfactory of multiple production of pions in a high energy nucleon-nucleon collision due to certain inherent difficulties in the meson field theories. Among the various attempts to bypass them the procedures adopted by Lewis et al.¹ and Fermi² deserve special attention. Some of the basic assumptions of Fermi's theory have been criticized by Takagi³, Lewis⁴, and more recently by Bhabha⁵. The object of the present note however is to show that, even if Fermi's assumptions are accepted in their entirety, the final results are considerably modified by following a more rigorous thermodynamic procedure.

According to the usual Bose-Einstein theory, \( N_r \), the number of particles in the state of energy \( \varepsilon_r \) is given by

\[
N_r = \frac{1}{e^\frac{\varepsilon_r}{kT} - 1},
\]

where the degeneracy parameter \( \lambda = 1 \) for the pion assembly. In the extreme relativistic case Fermi uses the expression

\[
N = \frac{4\pi g}{(Ch)^3} \int_0^\infty \frac{e^3 d\xi}{e^{\xi/kT} - 1},
\]

for the total number of pions, where \( \sim \) is the volume in which the energy of the impinging nucleons is concentrated. Its value is given by

\[
\sim = (4\pi/3)R^3(2M^2/W),
\]

where \( R = \xi \delta/\mu c \), \( \xi \) being a dimensionless quantity of the order of unity and \( \mu \) the pion mass. It might be noted that in obtaining (2) from (1) the summation has been replaced by integration and for the
density of energy levels the asymptotic expression
\[ a(e)de = -4\pi g e^4/(\varepsilon H)^3 \]  
has been employed. A more precise result\(^6\), however, gives
\[ a(e)de = \left\{ -\frac{4\pi g}{(\varepsilon H)^3} e^4 + \theta \frac{\pi g e^4}{2 (\varepsilon H)^3} + \ldots \right\}, \]  
where the second term is of the nature of a correction, \( \theta \) being -1 or +1 depending on whether the wave function or its derivative vanish on the boundary. If we substitute \((4b)\) in \((1)\) and carry out the integration the contribution of the second term for \( W \gg M^2 \), indicating that equation \((2)\) and hence all the results obtained from it are invalid for high energies. This necessitates the reconsideration of the whole question.

The final result depends very strongly on the boundary condition which is chosen. For the case of a particle in a box none of the three quantum numbers is permitted to have zero value. We therefore get a very large null point energy and \( N \leq 4.3 \varepsilon \) always, as has already pointed out by Auluck and Kothari\(^7\). Fermi, however, maintains that it is possible for pions even with zero kinetic energy to be formed inside the small volume (see Lewis\(^4\) in this connection). This is equivalent to using the free boundary condition which permits zero value for the quantum numbers.

For collisions of very high energy the spheroid is contracted to a disc. It can be readily shown the states which are now important are those for which the quantum number corresponding to the contracted dimension is set equal to zero. This makes the pion gas effectively two-dimensional. We therefore have
\[ N = \frac{2\pi g}{(\varepsilon H)^3} \int_0^{(\varepsilon H)^3} e^3 de = 6\pi g \left( \frac{kT}{\varepsilon H} \right)^2, \]  
(5)
and
\[ W = \frac{2\pi g}{(\varepsilon H)^3} \int_0^{(\varepsilon H)^3} e^3 de = 12\pi \Gamma \left( \frac{kT}{\varepsilon H} \right)^2, \]  
(6)
which gives
\[ N \sim 3.7\varepsilon^{3/2} (W/M^2)^{3/4}. \]  
(7)
The error involved in replacing summation by integration in \((5)\) and \((6)\) is easily seen to be negligible.

It is interesting to note that Lewis et al.\(^1\) on the basis of their theory obtain
\[ N \sim \left( \frac{\Gamma M}{\pi \mu} \right)^{3/2} (W/M^2)^{3/4}, \]  
(8)
for multiplicity results from the two theories inspite of their different starting points. This close resemblance may not be fortuitous. We are examining this point and hope to come back to it in a later publication along with the problem of non-central collisions.

2) E. Fermi, Prog. Theor. Phys. 5 (1950) 570.
3) S. Takagi, Prog. Theor. Phys. 7 (1952), 123.
7) F. C. Auluck and D. S. Kothari, Phys. Rev. 90 (1953), 1002.