A numerical investigation of scattering effects for teleseismic plane wave propagation in a heterogeneous layer over a homogeneous half-space

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SUMMARY
Numerical simulations of teleseismic wave propagation in a heterogeneous layer over a homogeneous half-space are conducted to further our understanding of teleseismic coda generation. Acoustic and elastic finite difference synthetics are generated for more than 150 different layer-over-a-half-space models. The models vary in scattering layer thickness \( L \), heterogeneity correlation distance \( a_c \) or \( a_z \), and heterogeneity standard deviation \( \sigma \). The synthetic data are analysed by examining coda intensity envelopes and frequency-wavenumber spectra.

The level of scattered energy is found to be controlled by the \( ka \) and \( \sigma \) values. Coda levels increase with increasing \( ka \) from \( ka < 1 \) to \( ka \sim 1 \), and decrease with increasing \( ka \) for \( ka > 1 \). Scattered energy levels always increase with increasing \( \sigma \). Models that vary in scattering layer thickness alone did not consistently produce changes in the coda level or rate of decay independently of the values of \( ka \) and/or \( \sigma \). The rate of coda decay is controlled by the heterogeneity aspect ratio \( (a_c/a_z) \). Models with spatially isotropic heterogeneities \( (a_c/a_z = 1) \) produce the slowest rate of decay, while those with an infinite aspect ratio (homogeneous, plane-layered models) produce the most rapid rate of decay. Any decay rate between these two extremes can be obtained by varying the heterogeneity aspect ratio. Acoustic and elastic models exhibit similar coda intensity envelope characteristics. Apparent scattering attenuation of the direct pulse is a function of \( ka \) and is strongest for models with spatially isotropic heterogeneities.

Frequency–wavenumber analysis showed that coda for models with spatially isotropic heterogeneities is composed largely of low apparent velocity energy in the form of \( P \)-to-\( S \) and/or body-to-surface wave scattered energy. Coda for models with spatially anisotropic heterogeneities is composed largely of vertically propagating layer reverberations. Coda for extreme anisotropic models is composed solely of vertically propagating layer reverberations. The onset time of low apparent velocity energy is also controlled by the heterogeneity aspect ratio. For models with anisotropic heterogeneities, low apparent velocity energy appears immediately after the first arrival, for models with an infinite heterogeneity aspect ratio \( (1 \text{-} D \text{ models}) \), low apparent velocity energy never appears.

Key words: \( f-k \) analysis, heterogeneity aspect ratio, \( P \)-to-\( S \) scattering, randomly heterogeneous media, relative coda intensities, spatially iso/anisotropic heterogeneities, teleseismic coda.

INTRODUCTION
The use of teleseismic waveform modelling to determine the location of major discontinuities in the Earth’s lithosphere was introduced over a decade ago by Burdick & Langston (1977). Their receiver function modelling procedure allows us to use teleseismic data from a single, isolated receiver to determine structural features of crust and upper mantle below the receiver. A disadvantage of this method is that we must assume that the structure beneath the receiver consists of homogeneous, plane layers (Burdick & Langston 1977;
complex modelling procedures that employ dipping layers propagation phenomena responsible for the observed widespread application to the teleseismic problem. For the teleseismic waveform modelling are complex P-wave a great deal of the observed coda and complex particle motions cannot be modelled using simple techniques (Langston 1979, 1989; Owens et al. 1987).

Attempts to help explain and understand the wave propagation phenomena responsible for the observed teleseismic particle motions and teleseismic coda began in the early 1980s. One of the first studies addressing the teleseismic coda problem was that of Richards & Menke (1983) who investigated scattering of teleseismic waves for propagation through a complex, plane-layered structure. It was approximately at this time that the large body of work on coda generation for local earthquakes began to see widespread application to the teleseismic problem. For the local earthquake case, the interest in coda, the scattered waves arriving after the major, deterministic seismic phases, began in the late 1960s and early 1970s with the introduction of the single backscattering model (Aki 1969; Aki & Chouet 1975). Since that time many studies, both theoretical and numerical, have been conducted to determine the relationship between the degree of heterogeneity of the Earth’s crust and upper mantle and the behaviour of seismic coda. Theoretical models include the single backscattering model (Aki 1969, 1980, 1982; Aki & Chouet 1975), diffusion models (Dainty et al. 1974; Nakamura 1976, 1977; Dainty & Toksöz 1977; Kopnichev 1977; Toksöz et al. 1988), energy-flux models (Frankel & Wennerberg 1987), energy-flux with radiative diffusion models (Langston 1989; Korn 1990), and transport theory models (Wu 1985, 1988). Numerical investigations into the mechanisms responsible for coda generation include those for models with complex plane-layered structures (Richards & Menke 1983; Langston 1989), models with irregular surface and subsurface layers (Hill & Levander 1984; Levander & Hill, 1985), and models with 2-D inhomogeneous half- and whole-spaces (Frankel & Clayton 1984, 1986; McLaughlin, Anderson & Der 1985; Dougherty & Stephen 1988; Korn 1990).

Despite the numerous studies investigating seismic coda many questions about its origins and relation to the heterogeneities of the Earth’s crust and upper mantle remain unanswered, particularly for the teleseismic case. To help better understand how various heterogeneous structures affect teleseismic coda generation and waveforms in general, we have carried out a detailed numerical investigation into scattering effects for teleseismic wave propagation in a 2-D, randomly heterogeneous scattering layer over a homogeneous half-space. More than 150 models are tested to determine how the scattering layer thickness ($L$), the heterogeneity correlation distance ($a$, or $a_x$ and $a_y$), and the magnitude of the heterogeneity standard deviation ($\sigma$) affect teleseismic coda generation and apparent scattering attenuation.

Scattering effects for both acoustic and elastic media are studied by computing synthetic waveforms using finite difference (FD) techniques (Alford, Kelly & Boore 1974; Kelly et al. 1976; Frankel & Clayton 1984; Frankel & Clayton 1986; Kelly & Marfurt 1990). To simulate teleseismic wave propagation, we use a vertically propagating, broad-band, compressional plane wave source. The models have stress-free boundary conditions at the surface, and an absorbing boundary condition at the bottom. The absorbing boundary condition is meant to simulate the wave’s return into the mantle. Data are recorded at the free surface at 1 km intervals.

The two main questions for this study are: (1) Can the amplitude and rate of decay of the seismic coda be used to determine the degree of heterogeneity of the models? (2) What are the sources of the scattered energy?

The synthetic waveforms are analysed using two methods: (1) a comparison of relative coda intensity envelopes (Aki & Chouet 1975; Frankel & Wennerberg 1987; Langston 1989), and (2) frequency–wavenumber ($f$–$k$) analysis. Relative coda intensities for several frequency passbands of the data generated using each are compared in an attempt to reveal relationships that exist between the model parameters and the coda level and rate of decay. Frequency–wavenumber analysis is used to identify the wave types present in the scattered wavefield. This will, in turn, help identify the scattering sources and the mechanisms responsible for the production of seismic coda.

Comparison of results obtained in this study to observed data are discussed in two separate papers (Wagner & Langston 1991; Wagner & Owens, in preparation).

**NUMERICAL MODELLING**

Models used in the numerical simulations consisted of a randomly heterogeneous layer over a homogeneous half-space. The use of this type of model was motivated by the assumption that small-scale structural heterogeneities are generally confined to the Earth’s crust and upper mantle. Numerical techniques were used because they are, at present, the only methods available to compute waveforms in a medium that varies (randomly) both laterally and vertically.

The parameters needed to describe a randomly varying medium, in a statistical sense, are (Ishimaru 1978): (1) a correlation function; (2) a correlation distance ($\mathbf{a}$) and; (3) a standard deviation of the perturbations ($\sigma$). To describe accurately wave propagation through such a medium we also need to specify the length of the propagation path through the medium, $L$ (Chernov 1960; Ishimaru 1978). This distance is sometimes given in terms of the dimensionless normalized propagation distance, $L/\mathbf{a}$.

We generated more than 150 elastic and acoustic models using $a$, $\mathbf{a}$ and $L$ as variables. These parameters were varied in a systematic manner in order to span a large range of normalized wavenumbers, normalized propagation distance, and standard deviations. We tested three general types of models: (1) models with randomly varying, homogeneous plane layers; (2) models with randomly distributed, spatially
isotropic heterogeneities; and (3) models with randomly distributed, spatially anisotropic heterogeneities. Heterogeneities were in the form of velocity perturbations. The density was constant (2.5 g cm$^{-3}$) for both the layer and the half-space for all models. The mean layer velocity for the acoustic models is 5 km s$^{-1}$. The mean layer velocity for the elastic models is 6 km s$^{-1}$ for P-waves and 3.46 km s$^{-1}$ for S-waves. We assumed a Poisson solid when generating elastic models; the P and S velocity perturbations are, therefore, perfectly correlated. Half-space velocities are 8 km s$^{-1}$ for acoustic and P-waves, and 4.62 km s$^{-1}$ for S-waves.

The spatial distribution of the velocity perturbations was computed by filtering, in the wavenumber domain, a (spatial) grid of random numbers with a Gaussian correlation function (Frankel & Clayton 1986)

$$e^{-(x^2 + z^2)/(2a^2)} = e^{-\left(kf^2a^2 + kS^2a^2\right)/4},$$

spatial domain $\leftrightarrow$ wavenumber domain.

The decision to use a Gaussian correlation function was motivated by the availability of theoretical and numerical results from previous studies.

Models were constructed using nearly every possible combination of the parameters. The scattered layer thickness ($L$) was 15, 30 or 45 km. The standard deviation of the velocity perturbations ($\sigma$) was 5, 10, 15 or 20 per cent of the mean layer velocity. The correlation distance for models with spatially isotropic heterogeneities was 1, 2.5, 5, 7.5 or 10 km. The correlation distances for models with spatially anisotropic heterogeneities were $a_x$ of 1, 2.5 or 5 km for $a_x$ of 10, 25, 40, 50, 80 or 125 km.

Figs 1 and 2 show examples of the velocity perturbations for models with spatially isotropic and anisotropic heterogeneities, respectively (please note the difference in the scale of the two figures). These figures show only the spatial distribution of the perturbations. The actual models used in the numerical simulations were constructed by truncating the heterogeneous layer to the desired $L$, scaling the perturbations according to the desired

Figure 1. Spatially isotropic velocity perturbations generated using an isotropic Gaussian correlation function with correlation distance of 1, 5 and 10 km.
Numerical modelling for teleseismic scattering effects

Finite difference modelling

Finite difference modelling techniques, accuracy, stability conditions, and boundary conditions have been discussed in many previous papers (Alterman & Karal 1968; Boore 1972a, b; Landers & Claerbout 1972; Alford et al. 1974; Alterman & Loewenthal 1970, Smith 1974; Ilan, Ungar & Alterman 1975; Ilan & Loewenthal 1976, Kelly et al. 1976; Clayton & Engquist 1977; Reynolds 1978; Levander 1988; Stephen 1988; Daudt et al. 1989; Kelly & Marfurt 1990). We used fourth-order accurate, inhomogeneous, displacement finite difference approximations to generate synthetic waveforms. No intrinsic attenuation was included in any of the models ($Q = \infty$). Based on our own accuracy and stability tests, we choose to use one-sided stress-free boundary conditions at the surface (Ilan & Loewenthal 1976), absorbing boundary conditions at the base of the model (Clayton & Engquist 1977), and transparent boundary conditions at the sides (Wagner 1991). The grid spacing and time increments were 0.25 km and 0.02 s, respectively. These increments allow frequency resolution to ~4 Hz. The source pulse used for all modelling was, in the time domain, a Gaussian shaped, 1 s, vertically propagating compressional plane wave. In frequency this corresponds to a Gaussian with an upper half-power frequency at -1.5 Hz. Acoustic models were 512 km wide. Elastic models with spatially isotropic heterogeneities were 256 km wide, while those with spatially anisotropic heterogeneities were 430 km wide. All models had a 15 km thick homogeneous 'half-space' which consisted of a homogeneous layer with an absorbing boundary (Clayton & Engquist 1977). The computer time required to calculate all of the waveform data used in this study (using individually, but simultaneously, a SUN 3/280, 4/280, and TAAC-1 application accelerator board) can be measured in months. The elastic

Figure 2. Spatially anisotropic velocity perturbations generated using a anisotropic Gaussian correlation function.

$a$, and then superimposing these perturbations on a homogeneous layer-over-a-half-space model.

$\sigma$.
models took, at a minimum, 1 hr to compute 1 s of surface recorded data.

PART I: INTENSITY ENVELOPE ANALYSIS

The goal of the relative coda intensity envelope analysis is to reveal relationships that exist between the size, shape, extent and magnitude of the heterogeneities, and the level and rate of coda decay. The energy in the scattered wavefield is compared to that in the incident pulse by generating averaged intensity envelopes computed by stacking the relative intensity envelopes from 10 receivers. We used the method introduced by Frankel & Wennerberg (1987) to compute these power-spectral-density-with-respect-to-time envelopes.

Power spectral densities, \( P(\omega, t) \), were calculated using the following procedure (Fig. 3).

1. Filter raw data using two passes of a two-pole Butterworth filter (non-causal, zero phase, 3 dB down at the corners). Corners were at \( \omega_0 + 0.25 \) Hz and \( \omega_0 - 0.25 \) Hz. Data were examined using \( \omega_0 \) of 0.5, 1 and 2 Hz.

2. Each bandpassed trace is normalized to the intensity of its initial arrival. The normalization constant is the area of the squared amplitude of the initial pulse (\( \sim \text{intensity} \))

\[
\int_{t_1}^{t_1 + \Delta t} |\phi(t)|^2 \, dt,
\]

where \( \Delta t \) is the duration of the initial arrival. These intensity envelopes show the intensity of the coda relative to the intensity of the initial arrival at a particular frequency.

3. The envelope of each bandpassed, normalized trace is calculated using its analytic time signal.

Coda Intensity Calculation

![Coda Intensity Calculation](image)

Figure 3. Procedure used to calculate relative coda intensity envelopes. The envelope above the bandpass filtered trace has been offset slightly for this figure.

(4) For each model and each passband, envelopes from 10 stations, each 10 km from its neighbour, are stacked to give the final plots of log intensity versus time.

To avoid a deluge of figures only a few selected figures will be shown. Individual figures may exhibit certain characteristics more clearly than others, but attributes of a single model are not used as evidence for phenomena that do not occur consistently. Because acoustic and elastic envelopes frequently exhibit similar characteristics, only one figure (acoustic or elastic) may be shown to illustrate some points.

Models with isotropic heterogeneities

The following four sections discuss the analysis of data produced by varying, independently, \( a, \sigma, L \) and the frequency passband for models with spatially isotropic heterogeneities. Most of our results are rather simple and agree with results from previous theoretical and numerical studies (Frankel & Clayton 1984; McLaughlin et al. 1985; Frankel & Clayton 1986; Dougherty & Stephen 1988; Korn 1990).

Changes in correlation distance

The heterogeneity correlation distance \( (a) \) is directly proportional to the size of the heterogeneities and inversely proportional to their number density per unit area. A simple relationship exists between the scattered energy level and the \( ka \) value \( (k = \text{wavenumber} = 1/\lambda, \lambda = \text{wavelength}) \): the level of scattered energy increases with increasing \( ka \) for \( ka < 1 \) (wavelength greater than the size of the heterogeneities), reaches a maximum near \( ka \approx 1 \) (wavelength approximately equal to the size of the heterogeneities), and begins to decrease with increasing \( ka \) for \( ka > 1 \) (wavelength less than the size of the heterogeneities). This relationship holds regardless of the value of \( \sigma \) or \( L \). Fig. 4 shows the relative coda intensity envelopes for \( ka \leq 1 \) and \( ka > 1 \). Fig. 5 shows the vertical and horizontal displacement envelopes for elastic models that varied in \( a \) alone (note that the \( k \) in the elastic models assumes that waves are travelling at P-wave velocities). Changes in \( a \) had no noticeable effect on the rate of coda decay for either the acoustic or elastic case. This is evident in both figures and was true for all of the models.

Changes in standard deviation

Changing \( \sigma \) had a very simple effect: increasing \( \sigma \) increases the level of scattered energy. This effect holds regardless of the value of either \( ka \) or \( L \). Fig. 6 shows relative vertical and horizontal displacement intensity envelopes for elastic models that varied in \( \sigma \) alone. Changes in \( \sigma \) had no noticeable effect on the rate of coda decay for either the acoustic or elastic case.

Changes in scattering layer thickness

Our modelling revealed no simple, unambiguous relationships between \( L \) and either the coda level or rate of decay. Any coda level or decay rate changes that occurred when varying \( L \) did not occur independently of the value of \( ka \).
Numerical modelling for teleseismic scattering effects

Numerical modelling for teleseismic scattering effects.

_envelopes from models with different α_

(Fixed parameters: α=5%, L=45 km, 1 Hz pass-band)

- 5.0 km (ka-1.0)
- 2.5 km (ka-0.5)
- 1.0 km (ka-0.2)

layer reverberation

acoustic

- 5.0 km (ka-1.0)
- 7.5 km (ka-1.5)
- 10.0 km (ka-2.0)

layer reverberation

acoustic

Figure 4. Relative coda intensity envelopes for acoustic models with spatially isotropic heterogeneities with different correlation distances. Correlation distances and the corresponding line weights are shown in the upper right corner of the figure. The passband, scattering layer thickness, and standard deviation are also listed on the figure.

and/or α. In some cases, changing L produced no marked difference in either the relative coda level or rate of decay (Fig. 7, part a), while in other cases both the level and rate of coda decay were affected (Fig. 7, part b). Nearly all of the data generated using models that vary in L alone showed no consistent or systematic differences in either the coda level or decay rate.

Varying pass bands

Looking at different passbands of data generated using a single model is analogous to examining identical passbands of data from models with different α. Results from this section are, therefore, similar to those obtained in the section where only α was varied. Namely, that coda level increases with increasing ka (decreasing frequency) when ka<1 and begins to decrease with increasing ka when ka>1. These ka relationships were most evident for ka<1, and less evident for larger ka. These ka relations held regardless of the value of L or α. Figs 8 and 9 show the vertical and horizontal intensity envelopes for elastic simulations where ka<1 and ka>1, respectively. The bandpassed data exhibited no noticeable differences in the rate of coda decay.

Models with anisotropic heterogeneities

With the models tested to this point we could easily and systematically alter the coda level but had no way of consistently altering the rate of coda decay. Our inability to control the rate of coda decay motivated a further modification of the model parametrization. The following section discusses results obtained using data generated using models with spatially anisotropic heterogeneities (α̸=α̸).

We generated synthetic data for approximately 45 acoustic models with spatially anisotropic heterogeneities. The results from the acoustic modelling were used to select four elastic models that would best characterize the heterogeneity aspect ratio range. Time limitation prevented us from computing a complete complement of elastic models. The most interesting results were for models with a fixed α̸ and varying α̸. The end member models for such a case are those with spatially isotropic heterogeneities (a̸\_α_\_\_α = a̸\_\_α_\_\_\_), and models with homogeneous, plane layers (a̸ = ∞). The spatially anisotropic models mimic plane layered models with layers of varying lateral extent. This type of model is intuitively appealing because it agrees with geological models where layers do not have infinite lateral extent, but are truncated by faulting, folding or pinch outs.
envelopes from models with different $\sigma$
(fixed parameters: $a=1$ km, $L=30$ km, 1 Hz pass-band)

vertical

- 15%
- 10%
- 5%

layer reverberation

horizontal

- 15%
- 10%
- 5%

log amplitude

Figure 6. Relative coda intensity envelopes, for the vertical and horizontal components of displacement, for elastic models with spatially isotropic heterogeneities with different standard deviation magnitudes. Per cent standard deviation and the corresponding line weights are shown in the upper right corner of the figure. The passband, scattering layer thickness, correlation distance and displacement component are also listed on the figure.

envelopes from models with different $L$
(fixed parameters: $\sigma=5\%$, $a=7.5$ km, 1 Hz pass-band)

vertical

- 45 km
- 30 km
- 15 km

log amplitude

(a)

horizontal

- 45 km
- 30 km
- 15 km

log amplitude

(b)

Figure 7. Relative coda intensity envelopes for acoustic models with spatially isotropic heterogeneities that have different scattering layer thicknesses. Scattering layer thicknesses and the corresponding line weights are shown in the upper right corner of the figure. The passband, correlation distance, and standard deviation are also listed on the figure.

Figs 10 and 11 show the acoustic and elastic intensity envelopes from models with spatially anisotropic heterogeneities. These envelopes clearly illustrate how the heterogeneity aspect ratio controls the rate of coda decay. The decay rates, like the models themselves, are bound by the spatially isotropic heterogeneity case ($a_x=a_z$), which exhibits the slowest decay rate, and the homogeneous plane layer case ($a_z=\infty$), which exhibits the most rapid rate of decay.

PART II: FREQUENCY-WAVENUMBER ANALYSIS

The graphical display of frequency-wavenumber spectra shows, directly, the propagation direction and apparent velocity of any coherent plane wave energy crossing an array of receivers (Robinson 1967; Gold & Rader 1969; Beaudamp 1973, 1975; Claerbout 1976; Ingate, Husebye & Christofferson 1985; Kværna & Ringdal 1986). Frequency-wavenumber analysis was used to identify the waves present in the scattered wavefield. This will, in turn, help identify the scattering sources and the mechanisms responsible for the production of seismic coda. We used the vertical component of the elastic data in the $f-k$ analysis.

To avoid contamination of the $f-k$ results with energy produced by FD edge effects, we conducted $f-k$ analysis only on data that arrives before any edge effects. The spatially isotropic heterogeneity models are 256 km wide. The earliest arrival time (centre receiver) for any $S$-wave edge effect energy for these models should be $\sim 37$ s (the reason for our concern with only $S$-wave energy will become obvious shortly). Our 11 stations array should, therefore, be free of $S$-wave edge effects for $\sim 35$ s. For models with spatially anisotropic heterogeneities (430 km wide), the earliest arrival time for $S$-wave edge effect energy is $\sim 60$ s. None of the analysis windows for models with spatially anisotropic heterogeneities goes beyond 45 s.

Spatially isotropic heterogeneities

The scattered wavefield produced by the models with spatially isotropic heterogeneities contains a great deal of low apparent velocity (LAV) energy in the form of $S$ and possibly surface waves. This LAV scattered energy is
Numerical modelling for teleseismic scattering effects

envelopes for different pass-bands

(fixed parameters: \(\sigma=10\%\), \(a=1\) km, \(L=30\) km)

vertical

- \(2.0\) Hz
- \(1.0\) Hz
- \(0.5\) Hz

ka-2.5
ka-5.0
ka-10

layer reverberation

horizontal

- \(2.0\) Hz
- \(1.0\) Hz
- \(0.5\) Hz

ka-2.5
ka-5.0
ka-10

Figure 8. Relative coda intensity envelopes for different passbands of elastic data generated using models with spatially isotropic heterogeneities. The passband and the corresponding line weights are shown in the upper right corner of the figures. The correlation distance, standard deviation, scattering layer thickness and displacement component are also listed on the figures.

present in all of the elastic data generated using models with spatially isotropic heterogeneities. The heterogeneity correlation distance appears to play the major role in the production and frequency/wavenumber content of this LAV energy.

Fig. 12 shows 35 s of the synthetic waveforms generated using models with spatially isotropic heterogeneities with correlation distances of 1 and 10 km. The correlation distance is the only parameter that differs in these two models (\(\sigma=10\) per cent and \(L=30\) km for both models).

Frequency–wavenumber spectra were calculated for several windows: (1) a window that includes all of the data (0–35 s), (2) a 5 s window containing data between the initial arrival and the first multiple (10–15 s), and (3) two 5 s windows containing data arriving after the first multiple but prior to any S-wave edge effect energy (20–25 s and 30–35 s). The \(f\)-\(k\) spectra for these data are shown in Fig. 13. Several important features are shown in this figure, some that are common to both models and several that are not.

A feature common to both data sets is that LAV energy begins to appear immediately after the passage of the initial pulse. Several of these waves are clearly visible in both the vertical and horizontal components of the \(a=10\) km data. This LAV energy continues to dominate the \(f\)-\(k\) spectra at later times and decays slowly with time (see max power on the figures).

envelopes from models with different \(a_x\)

(fixed parameters: \(\sigma=10\%\), \(L=30\) km, \(a_x=5\) km, 1 Hz pass-band)

5 km
10 km
20 km
40 km
80 km
1-D

acoustic

Figure 9. Relative coda intensity envelopes for different passbands of elastic data generated using models with spatially isotropic heterogeneities. The passband and the corresponding line weights are shown in the upper right corner of the figures. The correlation distance, standard deviation, scattering layer thickness and displacement component are also listed on the figures.

envelopes for different pass-bands

(fixed parameters: \(\sigma=10\%\), \(a=10\) km, \(L=30\) km)

vertical

- \(2.0\) Hz
- \(1.0\) Hz
- \(0.5\) Hz

ka-0.25
ka-0.50
ka-1.00

Figure 10. Relative coda intensity envelopes for acoustic models with spatially anisotropic heterogeneities that vary in \(a_x\) only (\(a_x=5\) km for all models). The various \(a_x\) and the corresponding line weights are shown in the upper right corner of the figure. The passband, scattering layer thickness and standard deviation are also listed on the figure.
envelopes from models with different $a_x$

(10\%\% parameters: $a=10\%$, $L=30$ km, $a=5$ km, 1 Hz pass-band)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure11.png}
\caption{Relative coda intensity envelopes for elastic models with spatially anisotropic heterogeneities that vary in $a_x$ only ($a_x=5$ km for all models). The various $a_x$ and the corresponding line weights are shown in the upper right corner of the figure. The passband, scattering layer thickness, standard deviation and displacement component are also listed on the figure.}
\end{figure}

These data sets also exhibit two significant differences. First, the amount of LAV energy produced in the model with $a = 10$ km is much greater than that produced in the model with $a = 1$ km. This fact is demonstrated by comparison of the 0–35 s windows and the later 5 s windows. The 0–35 s $f$–$k$ spectrum for the $a = 10$ km model exhibits LAV energy at a noticeable fraction of the energy of the direct wave. No LAV energy is apparent in the 0–35 s $f$–$k$ spectrum for the $a = 1$ km case. For the later 5 s windows, the maximum power estimates are nearly an order of magnitude greater for the $a = 10$ km case than they are for the $a = 1$ km case. Secondly, the frequencies and wavenumbers of the waves in the scattered wavefield are, in general, much higher for models with small heterogeneities. This effect is most obvious in the 10–15 s window and is directly related to the size of heterogeneities. Large heterogeneities produce low $k$, low $\omega$ energy, while small heterogeneities produce high $k$, high $\omega$ energy.

Spatially anisotropic heterogeneities

Like the models with spatially isotropic heterogeneities, models with spatially anisotropic heterogeneities also produced LAV scattered energy, but the onset time of this energy is delayed, the larger the aspect ratio of the heterogeneities the longer the delay of LAV energy onset. Two models will be discussed in this section: one with $a_x=25$ km and the other with $a_x=125$ km; $a_z=5$ km for both models. The heterogeneities in these model are 'stretched' in the horizontal direction; the greater $a_x$ the greater the stretch. The waveforms generated using these two models are shown in Fig. 14.

Frequency–wavenumber spectra were again computed for several windows, some from identical and some from analogous sections of data. The windows are: (1) a window that includes all of the data (0–35 s); (2) a 5 s window containing the data between the initial arrival and the first
multiple (10–15 s or 8–13 s); and (3) two 5 s windows containing data arriving after the first multiple but prior to any S-wave edge effects (25–30 s, and 40–45 s or 35–40 s). The $f-k$ spectra for these data are shown in Fig. 15.

Unlike the case for models with spatially isotropic heterogeneities, the scattered energy arriving immediately after the initial arrival for the spatially anisotropic heterogeneity models consists almost solely of vertically propagating energy (i.e. layer reverberations). There is only a slight amount of LAV energy for the $a_z = 25$ km case and none for the $a_z = 125$ km case. This trend continues further into the data with still no or very little LAV energy up to

Figure 13. Frequency–wavenumber spectra for the data generated using models with spatially isotropic heterogeneities (Fig. 12). Window durations, apparent velocities and estimated power maximums are shown with each spectrum.
models with spatially anisotropic heterogeneities; top set and spectrum exhibits only vertically propagating layer reverberations. Decreasing very rapidly (see later times, while for the 25 km case, LAV energy dominates the spectrum at \( \sim 25 \) s. At this point LAV energy begins to dominate the \( f-k \) spectrum for the \( a_x = 25 \) km data, but is still absent in the \( f-k \) spectrum for the \( a_x = 125 \) km data. The \( a_x = 125 \) km spectrum exhibits only vertically propagating layer reverberations and the energy carried by these waves is decreasing very rapidly (see max power in spectra). Windows further into each data set show that, for the \( a_x = 25 \) km case. LAV energy dominates the spectrum at later times, while for the \( a_x = 125 \) km case, the now almost negligible energy is still mostly in the form of vertically propagating layer reverberations.

**DISCUSSION**

**Modelling shortcomings**

Before we begin the discussion and start drawing conclusions from our results, it is important to recognize the shortcomings of our modelling procedure. A partial list of modelling assumptions that are of dubious validity includes: (1) perturbations in velocity only; (2) perfectly correlated \( P \) and \( S \) velocity perturbations; (3) constant and equal densities for both the layer and half-space; (4) the use of a Gaussian correlation function to parametrize the heterogeneity distribution; (5) no depth dependence for any model parameters; (6) sampling a \( a = 1 \) km model with a \( \Delta a = 0.25 \) km grid; and (7) all of the numerical, round off, boundary condition, edge effect (etc.) errors that are implicit in finite difference simulations. We need to make such assumptions and simplifications for several reasons: (1) computational reasons (hardware and software); (2) the study must be finished in a finite amount of time; and (3) models must be made simple enough to allow us to see simple cause and effect relationships. Unfortunately, some modelling shortcoming can be recognized only with the advantage of hindsight.

The goal of this study was not to determine the exact earth structure responsible for the production of seismic coda, true earth structure is far too complex, but to determine the effects of several structural parameters on the level and rate of coda decay. While the models used in this study may not accurately model true earth structure, the results obtained are no less valuable in answering important questions regarding the physics involved in seismic wave propagation and scattering.

**Relative coda intensity analysis**

The results from the relative coda intensity analysis can be summarized with two figures. Figs 16 and 17 are composite plots showing relative coda levels and decay rates, respectively, for 36 different models. The coda decay rate and level are the slope and the \( t = 0 \) intercept of the least-squares fit to the coda envelopes. The coda level gives an estimate of the intensity of the coda immediately following the passage of the initial pulse. The coda decay rate is inversely proportional to the amount of time required for the intensity of the coda to reach \( 1/e \) that in the initial pulse. All intensities are relative to the intensity of the initial pulse.

Fig. 16 shows coda levels as a function of both \( \sigma, \) \( ka \) and \( a_x:a_y \) (the heterogeneity aspect ratio). Changing these parameters has the following effects on the coda level: (1) increasing \( \sigma \) increases coda levels regardless of other model parameters. (2) Coda levels increase with increasing \( ka \) from \( ka < 1 \) to \( ka \sim 1 \), where levels reach their maximum, and then decrease with increasing \( ka \) for \( ka \sim 1 \). This effect occurred regardless of any other model parameters. (3) Coda levels are greatest for models with spatially isotropic heterogeneities \( (a_x = a_y) \); the location of the absolute maximum is a function of \( ka \). Many of these effects have been observed in previous numerical studies (Frankel & Clayton 1984, 1986; Dougherty & Stephen 1988).

It is interesting to note the remarkable similarities between the coda levels for the acoustic and elastic cases. These coda levels are similar despite the fact that the mechanisms responsible for coda generation cannot be the same (i.e. acoustic models are incapable of producing wide-angle \( S \)-wave scattering). It is also interesting to note that the coda levels for the vertical and horizontal
components of the elastic data are nearly identical. This similarity indicates that the scattered energy is equally partitioned between the vertical and horizontal components.

Fig. 16 also contains information about scattering attenuation. The $ka_0$ blocks indicate that scattering attenuation is greatest when $ka_0 \sim 1$. The $a_x:a_z$ block indicates that scattering attenuation is greatest for models with spatially isotropic heterogeneities. The absolute scattering attenuation maximum is a function of $ka$, $\sigma$ and $a_x:a_z$, while relative maxima vary as a function of $a_x:a_z$.

Fig. 17 shows coda decay rates as a function of both $\sigma$, $ka$ and $a_x:a_z$. It is clear from this figure that the heterogeneity aspect ratio is the dominant factor controlling the rate of coda decay. Variations of $ka$ and $\sigma$ have negligible and inconsistent effects of the rate of coda decay. The rate of decay can be controlled by simply varying the
Coda Level as a Function of $ka : \sigma$ and $a_x : a_z$

**Figure 16.** Relative coda levels for a range of models. The $a_x = a_z$ plots show 1 Hz passband data from models with $L = 30$ km. The $a_x \neq a_z$ plots show 1 Hz passband data from models with $\sigma = 10$ per cent and $L = 30$ km.

Coda Decay Rate as a Function of $ka : \sigma$ and $a_x : a_z$

**Figure 17.** Relative coda decay rates for a range of models. The $a_x = a_z$ plots show 1 Hz passband data from models with $L = 30$ km. The $a_x \neq a_z$ plots show 1 Hz passband data from models with $\sigma = 10$ per cent and $L = 30$ km.

Energy-flux with radiative diffusion models have suggested that scattering layer thickness plays a role in the production of coda (Langston 1989; Korn 1990). Our modelling revealed no simple, well defined or unambiguous relationships between $L$ and either the coda level or rate of decay. Any level or rate of decay changes that occurred in conjunction with variations of $L$ did not occur independently of the value of $ka$ and/or $\sigma$. The thickness of the scattering layer does, no doubt, play some role in coda production and the resulting level and rate of decay, but we were unable to resolve any clear cause and effect relationships in our modelling.

**Frequency–wavenumber analysis**

Frequency–wavenumber analysis revealed the physics responsible for the heterogeneity aspect ratio/coda decay rate relationship. For models with isotropic heterogeneities, the scattered wavefield was largely composed of horizontally propagating energy travelling near S-wave velocities. This observation agrees with theory which predicts that a $P$-wave incident on a velocity perturbations will produce wide-angle scattered S-waves and forward scattered $P$-waves (Wu & Aki 1985). Conversely, the coda in the models with anisotropic heterogeneities is largely composed of vertically propagating layer reverberations; the higher the heterogeneity aspect ratio the greater the percentage of the scattered wavefield travelling as layer reverberations. The heterogeneity aspect ratio also controls the onset time of LAV energy. For models with isotropic heterogeneities, LAV energy appears immediately after the first arrival. For models with an infinite heterogeneity aspect ratio, LAV never appears. The onset time of LAV energy increases with increasing heterogeneity aspect ratio.

The progression from low levels of scattering, to high levels of LAV scattering, to an entirely vertically propagating scattered wavefield reflects a progression through the three general scattering regimes: (1) isotropic Rayleigh scattering when $ka \ll 1$; (2) wide-angle Mie regime scattering when $ka \sim 1$; and (3) forward Frenzel regime scattering when $ka \gg 1$.

The $f-k$ analysis is illustrated the importance of $P$-to-$S$ and possibly body-to-surface wave scattering in the coda problem and, therefore, the importance of using an elastic formulation to investigate seismic scattering and coda generation. Coda intensity analyses exhibited essentially identical results for both the acoustic and elastic cases. This was true despite the fact that the physics responsible for generating the scattered wavefields in each model is completely different. This similarity exhibits one of the shortfalls of using a scalar measure (i.e. intensity) to analyse a vector process, and exhibits the limited validity of using an acoustic field to model what is truly a vector process.

The $f-k$ spectra also revealed a direct correlation between the frequency and wavenumber content of the scattered wavefield and the heterogeneity correlation distance. For models with spatially isotropic heterogeneities,
larger heterogeneities produce longer wavelength, lower frequency scattered energy, while smaller heterogeneities produce shorter wavelength, higher frequency scattered energy. This effect is most obvious in the window immediately following the direct arrival (Fig. 13, 10−15 s window).

Heterogeneity aspect ratio and the rate of coda decay

Heterogeneity spatial anisotropy and the physics controlling its effects provide a simple and intuitively appealing way to alter coda decay rates and provides a physically appealing method of parametrizing earth structure. How does the heterogeneity aspect ratio control the rate of coda decay? The physics behind this observation is rather simple: the heterogeneity aspect ratio controls the rate of coda decay by controlling the rate at which vertically propagating energy is scattered into the horizontal direction. This is a simple explanation but carries with it significant implications as to the way in which we model earth structure. If the effect of going from a 1-D model (homogeneous, plane layers) to a (truly) 2-D model (spatially isotropic heterogeneities) is any indication as to the effects that can be expected when going from a 2-D to a 3-D model, we can see that extreme models, such as some of those used in this study (i.e. α = 20 per cent), may not be required to reproduce observed coda levels. This study has shown that it is not only the correlation function, a, α and L that are important in the scattering problem, but also whether or not earth structure should be modeled with a 1-D, 2-D or 3-D geometry.

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