Calculation of Magnetic Penguin Amplitudes in $B \to \phi K$ Decays Using PQCD Approach

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New physics contributions to $B$ decays often arise through chromo-magnetic penguin operators. To look for new physics effects in $B$ decays, it is useful to be able to estimate the hadronic matrix elements for the chromo-magnetic operator. We compute this contribution to $B \to \phi K$ decays using PQCD methods. It is shown that, if the Wilson coefficient of the new physics is same order of magnitude as that of the Standard Model, this operator gives a non-negligible contribution compared to that of the Standard Model (about 30%). We also investigate the value of $q^2$, which is the momentum transferred by the gluon in the chromo-magnetic penguin operator. We find that the expectation value $\langle q^2 \rangle$ is approximately $M_B^2/4$, in agreement with a naive guess. This result, however, is very sensitive to the scale dependence of the Wilson coefficient. We also show that the matrix element for the chromo-magnetic penguin operator is independent of the choice of energy scale to a very good approximation.

§1. Introduction

$B \to \phi K_S$ decay plays an important role in the search for new physics beyond the Standard Model (SM), as this decay is a pure penguin process. In the SM, the time dependent $CP$ asymmetry for $B \to \phi K_S$ decay is the same as that of $B \to J/\psi K_S$ decay, which is a tree dominant process. Any difference between them would be a signal for physics beyond the SM. Recently, the BaBar¹ and Belle² collaborations reported $CP$ asymmetry in $B \to \phi K_S$ decay. If the present trend continues with additional data, the results suggest a deviation from the SM prediction.

In the effective Hamiltonian approach,³ the Hamiltonian is expressed as the convolution of local operators and the Wilson coefficients, which can include contribution from new physics. The effects of new physics change the Wilson coefficients. In many cases, the Wilson coefficient for the chromo-magnetic operator is most sensitive to new physics.⁴⁻¹⁰ The factorization approximation (FA) is often used in computing the hadron matrix elements. However, it is difficult to calculate this contribution in the FA, because the magnitude of $q^2$ of the virtual gluon in the chromo-magnetic penguin operator is unknown. In the FA, $q^2$ is often assumed to be $M_B^2/4$ or $M_B^2/2$. Then, the result is sensitive to the assumed value of $q^2$.

In this paper, we calculate the chromo-magnetic penguin using the perturbative QCD (PQCD) approach within the SM. We can predict not only the factorizable contributions but also non-factorizable contributions and annihilation contributions, which cannot be calculated in the FA. Furthermore, we can calculate strong phases,
so that it is possible to predict $CP$ asymmetries. PQCD has been applied to some exclusive decay modes, i.e. $K\pi$, $\pi\pi$, $KK$, $K\eta$, $D_sK$, $\rho(\omega)\pi$, $\phi K$, $K^*\pi$ and $\phi K^*$ and the results are consistent with experimental data. They were calculated to leading order in $\alpha_s$ in the PQCD approach. Therefore, the chromo-magnetic penguin was not included in the computation of the branching ratio for the $B \to \phi K$ decay. Obviously, there are many other higher-order diagrams that must be considered simultaneously if we are to add the magnetic penguin term to the SM computation. To estimate the new physics contribution, the magnetic penguin contribution can be singled out, as it is most sensitive to new physics. In this study, we also analyze the FA computation of the color magnetic moment using PQCD as a guide.

The outline of this paper is as follows. First, we present the effective Hamiltonian and the chromo-magnetic penguin operator. Then, we derive magnetic penguin contributions for hadronic two-body decays. Next, we calculate the chromo-magnetic penguin contributions in $B \to \phi K$ decays using the PQCD approach and we give the numerical result. We also derive the distribution of $q^2$. Finally, we summarize this study.

§2. Chromo-magnetic penguin operator

In the effective Hamiltonian approach, the Hamiltonian is expressed as the convolution of local operators and the Wilson coefficients. The effective Hamiltonian for $\Delta S = 1$ transitions is given by

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left[ \sum_{q' = u, c} V_{qs}^* V_{q'b} \left( C_1(\mu)O_1^{(q')}(\mu) + C_2(\mu)O_2^{(q')}(\mu) \right) - V_{ts}^* V_{tb} \left( \sum_{i=3}^{10} C_i(\mu)O_i(\mu) + C_{7\gamma}(\mu)O_{7\gamma}(\mu) + C_{8G}(\mu)O_{8G}(\mu) \right) \right] + \text{h.c.}, \quad (2.1)$$

where $V_{qs}^*$ and $V_{q'b}$ are the Cabibbo-Kobayashi-Maskawa matrix elements, $O_{1-10}$ are local four-fermi operators, $O_{7\gamma}$ is the photo-magnetic penguin operator, and $O_{8G}$ is the chromo-magnetic penguin operator. The local operators are given by

$$O_1^{(q')} = (\bar{s}_iq'_j)V_{-A}(\bar{q}'_jb_k)V_{-A}, \quad O_2^{(q')} = (\bar{s}_iq'_j)V_{-A}(\bar{q}'_jb_k)V_{-A},$$
$$O_3 = (\bar{s}_ib_k)V_{-A} \sum_q (\bar{q}_jq_i)V_{-A}, \quad O_4 = (\bar{s}_ib_k)V_{-A} \sum_q (\bar{q}_jq_i)V_{-A},$$
$$O_5 = (\bar{s}_ib_k)V_{-A} \sum_q (\bar{q}_jq_i)V_{+A}, \quad O_6 = (\bar{s}_ib_k)V_{-A} \sum_q (\bar{q}_jq_i)V_{+A},$$
$$O_7 = \frac{3}{2}(\bar{s}_ib_k)V_{-A} \sum_q e_q(\bar{q}_jq_i)V_{+A}, \quad O_8 = \frac{3}{2}(\bar{s}_ib_k)V_{-A} \sum_q e_q(\bar{q}_jq_i)V_{+A},$$
$$O_9 = \frac{3}{2}(\bar{s}_ib_k)V_{-A} \sum_q e_q(\bar{q}_jq_i)V_{-A}, \quad O_{10} = \frac{3}{2}(\bar{s}_ib_k)V_{-A} \sum_q e_q(\bar{q}_jq_i)V_{-A},$$
\( O_{\gamma} = \frac{e}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) b_i F_{\mu\nu} \), \( O_{8G} = -\frac{g_s}{8\pi^2} m_b \bar{s}_i \sigma^{\mu\nu} (1 + \gamma_5) T_{ij}^a b_j G^a_{\mu\nu} \), \( 2.2 \)

where \( i \) and \( j \) are color indices, and \( q \) is taken to be \( u, d, s \) and \( c \). We use the leading logarithmic results of the Wilson coefficients in our calculations. We consider the chromo-magnetic penguin contribution for hadronic two-body decays. The non-local operator \( O'_{8G} \) generated by \( O_{8G} \) is given by

\[
O'_{8G} = -\frac{\alpha_s}{2\pi} m_b \bar{s}_i \sigma^{\mu\nu} T_{ij}^a (1 + \gamma_5) b_j \left[ q^\mu \frac{-i}{q^2} \left( \bar{s}_i \gamma^\nu T_{ij}^a s_j \right) \right] (2i) \\
+ 4\pi \alpha_s f^{abc} \frac{-i}{q^2} \left( \bar{q}_i \gamma^\mu T_{ij}^b q_j \right) \frac{-i}{q'^2} \left( \bar{q}_i' \gamma^\nu T_{ij'}^c q_{j'} \right) \\
+ 4\pi \alpha_s q^\mu f^{abc} \left[ g^{\nu}(q + k_1) + g^{\lambda}(q + k_2) + g^{\sigma}(q - k_2 - q) \right] \frac{-i}{q^2} \\
\times \frac{-i}{k_1^2} \left( \bar{q}_i \gamma_\lambda T_{ij}^b q_j \right) \frac{-i}{k_2^2} \left( \bar{q}_i' \gamma_\sigma T_{ij'}^c q_{j'} \right) (2i), \tag{2.3}
\]

where \( q \) is the momentum transferred by the gluon, which goes out from the \( \sigma^{\mu\nu} \) vertex. The second line in Eq. \( (2.3) \) is induced by the self-interaction of gluons in \( G^a_{\mu\nu} \), and the third and fourth lines contain a 3-point vertex of gluons. The quantities \( k_1 \) and \( k_2 \) are the momenta that go out from the 3-point vertex of gluons. These terms are needed to maintain gauge invariance. The first term is first order in \( \alpha_s \), and the others are of order \( \alpha_s^2 \). In the PQCD approach, all of the terms contribute to the same order in \( \alpha_s \), as we see later in the next section.

§3. Magnetic penguin amplitudes in the PQCD approach

In this section, we calculate the chromo-magnetic penguin amplitudes in \( B \rightarrow \phi K \) decays using the PQCD approach. The chromo-magnetic penguin operator for hadronic two-body decays is written as in Eq. \( (2.3) \), and there must be at least one hard gluon emitted by the spectator quark in PQCD. Therefore, the diagrams with the chromo-magnetic penguin operator are as shown in Fig. 1. The diagrams (a) – (h) come from the first line in Eq. \( (2.3) \), and the diagrams (i) and (j) come from the other terms. All of these diagrams contribute to the same order in \( \alpha_s \). The chromo-magnetic penguin contributions are of next-to-leading order in \( \alpha_s \) within the PQCD formalism. However, if we include the Wilson coefficient, they are of the same order in \( \alpha_s \) as the penguin contributions. If we consider the full diagrams of the penguin and the chromo-magnetic penguin, we see that they are of same order in \( \alpha_s \). Therefore, we cannot say that the magnetic penguin contributions are smaller than the penguin contributions. In order to obtain a meaningful result for the branching ratios, we must calculate other higher-order diagrams, which are the charm penguin, the vertex corrections, and so on. In this study, we consider only the chromo-magnetic penguin, as this operator is most sensitive to new physics.

The decay width for \( B \rightarrow \phi K \) decays is written as

\[
\Gamma = \frac{G_F^2}{32\pi M_B} |A|^2, \tag{3.1}
\]
where $\mathcal{A}$ is the sum of the leading amplitude and the magnetic penguin amplitude. The decay amplitudes from the magnetic penguin are given by

$$\mathcal{A}^{MP} = V_{tb}^* V_{ts} (\mathcal{M}_d^{MP} + \mathcal{M}_b^{MP} + \mathcal{M}_c^{MP} + \mathcal{M}_d^{MP} + \mathcal{M}_e^{MP} + \mathcal{M}_f^{MP} + \mathcal{M}_g^{MP} + \mathcal{M}_h^{MP} + \mathcal{M}_i^{MP} + \mathcal{M}_j^{MP}),$$

(3.2)

where the indices denote the labels in Fig. 1. The charged modes and neutral modes in $B \rightarrow \phi K$ have the same contributions of the chromo-magnetic penguin. The amplitudes $\mathcal{M}$ are expressed as the convolutions of meson wave functions, a hard part, and the Wilson coefficients.\(^{24}\) The meson wave functions are non-perturbative, and the hard part, which includes the exchange of a hard gluon, is perturbative. We introduce small transverse momenta for quarks and anti-quarks in mesons. Large double logarithms are then generated by corrections in the meson wave functions. Their resummation leads to the Sudakov factor.\(^{25}\) The Sudakov factor guarantees a perturbative calculation of the hard part.\(^{26}\) The other double logarithms appear from the end-point region of the parton momenta. The resummation of these logarithms leads to the threshold factor in the hard part.\(^{27}\)

We consider the $B$ meson to be at rest. In the light-cone coordinates, the $B$ meson momentum $P_1$, the $K$ meson momentum $P_2$, and the $\phi$ meson momentum $P_3$
are taken to be

\[ P_1 = \frac{M_B}{\sqrt{2}}(1, 1, 0_T), \quad P_2 = \frac{M_B}{\sqrt{2}}(1 - r_0^2, 0, 0_T), \quad P_3 = \frac{M_B}{\sqrt{2}}(r_0^2, 1, 0_T), \]  

(3.3)

where \( r_0 = M_0/M_B \), and the \( K \) meson mass is ignored. The momentum of the spectator quark in the \( B \) meson is written as \( k_1 \). The light quark in the \( B \) meson has a minus component \( k_1^- \) given by \( k_1^- = x_1 P_1 \), where \( x_1 \) is the momentum fraction. The quarks in the \( K \) meson have the plus components \( x_2 P_2^+ \) and \( (1 - x_2) P_2^- \) and the small transverse components \( k_{2T} \) and \( -k_{2T} \), respectively. The quarks in the \( \phi \) meson have the minus components \( x_3 P_3^- \) and \( (1 - x_3) P_3^- \) and the small transverse components \( k_{3T} \) and \( -k_{3T} \), respectively. The \( \phi \) meson longitudinal polarization vector \( \epsilon_\phi \) and two transverse polarization vectors \( \epsilon_{\phi T} \) are given by \( \epsilon_\phi = (1/\sqrt{2}r_\phi)(-r_0^2, 1, 0_T) \) and \( \epsilon_{\phi T} = (0, 0, 1_T) \).

For example, the amplitude for Fig. 1(a) is written as

\[ \mathcal{M}_a^{MP} = -8M_B^0 C_F^2 \sqrt{2}N_c \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 b_3 db_3 \phi_B(x_1, b_1) \]
\[ \times \left[ -(1 - x_2) \{ 2\phi_K^A(x_2) + r_K (3\phi_K^P(x_2) + \phi_K^T(x_2)) \right] \]
\[ + r_K x_2 (\phi_K^P(x_2) - \phi_K^T(x_2)) \}
\[ \phi (x_3) - r_\phi (1 + x_2)_x_3 \phi_A^A(x_3) (3\phi_\phi^A(x_3) - \phi_\theta^A(x_3)) \]
\[ - r_\phi r_K (1 - x_2) (\phi_K^P(x_2) - \phi_K^T(x_2)) (3\phi_\phi^A(x_3) + \phi_\theta^A(x_3)) \]
\[ - r_\phi r_K x_3 (1 - 2x_2) (\phi_K^P(x_2) + \phi_K^T(x_2)) (3\phi_\phi^A(x_3) - \phi_\theta^A(x_3)) \]
\[ \times E_\phi(t) N_t \{ x_2(1 - x_2) \}^c h_c^{MP}(A, B, C, b_1, b_2, b_3), \]  

(3.4)

where \( r_K \equiv m_0K/M_B \) and \( m_0K \) is the chiral factor defined as \( m_0K \equiv M_K^2/(m_d + m_s) \). The meson distribution amplitudes, \( \phi_B, \phi_K^A \), and so on, are given in Appendix B. \( N_t \{ x_2(1 - x_2) \}^c \) is the threshold factor, where \( N_t = 1.775 \) and \( c = 0.3.28 \) The evolution factors \( E_\phi(t) \) are defined by \( E_\phi(t) = \{ \alpha_e(t) \}^2 C_{_8G}(t) \exp[-S_B(t) - S_K(t) - S_\phi(t)] \), in which \( S_B(t), S_K(t) \) and \( S_\phi(t) \) are the Sudakov factors given in Appendix A. The hard functions are given by

\[ h_c^{MP}(A, B, C, b_1, b_2, b_3) = -K_0 (Bb_1) K_0 (Cb_3) \]
\[ \times \int_0^{\frac{\pi}{2}} d\theta \tan \theta J_0 (Ab_1 \tan \theta) J_0 (Ab_2 \tan \theta) J_0 (Ab_3 \tan \theta), \]  

(3.5)

where \( A, B \) and \( C \) are given on Table II. The values \( A^2, B^2 \) and \( C^2 \) are the squares of the virtual quark and virtual gluons momenta. We assume that the hard scale \( t \) is defined as

\[ t = \max(\sqrt{|A^2|}, \sqrt{|B^2|}, 1/b_1, 1/b_2, 1/b_3). \]  

(3.6)

In the above definition of \( t \), we have chosen the same definition for \( t \) as we employed for computation of the leading diagrams. There is a slight dependence on \( t \) in the chromo-magnetic penguin amplitude, as we see in the next section. The expressions for other diagrams are summarized in Appendix C.
§ 4. Numerical result

The parameters that we used in this calculation are as follows: \(M_B = 5.28\) GeV, \(M_K = 0.49\) GeV, \(M_\phi = 1.02\) GeV, \(m_b = 4.8\) GeV, \(m_t = 174.3\) GeV, \(f_B = 190\) MeV, \(f_K = 160\) MeV, \(f_\phi = 237\) MeV, \(f_T^\tau = 220\) MeV, \(\tau_{B^0} = 1.54 \times 10^{-12}\) sec, \(\tau_{B^\pm} = 1.67 \times 10^{-12}\) sec, \(A_{QCD}^{(4)} = 0.250\) GeV. In addition, we used the value of the chiral factor \(m_{0K} = 1.70\) GeV, and the parameter values in wave functions are as given in Appendix B.

The numerical results are listed in Table I. The leading amplitude for \(B^0 \to \phi K^0\) is \(0.0944 - i 0.0383\). We find that the diagrams (a) and (b) are dominant, and the magnetic penguin amplitudes are about 30% of the leading amplitudes.

We study the hard scale dependence of chromo-magnetic amplitudes. We define the hard scale as \(t = \kappa \cdot \max(\sqrt{|A^2|}, \sqrt{|B^2|}, 1/b_1, 1/b_2, 1/b_3)\), where \(\kappa\) is a parameter, and vary \(\kappa\) between 1 and 1.5. Figure 2 shows the hard scale dependence of \(M_{a+}^{MP} + M_{b+}^{MP}\). We find that there is a slight dependence on \(t\) in the chromo-magnetic penguin. This is consistent with the fact that the Wilson coefficient for \(O_{8G}\) is most sensitive to new physics, and its contribution, by itself, is physical.

The value of \(q^2\) of the virtual gluon in \(O_{8G}'\) is written in terms of \(x_1, x_2\) and Table I. Magnetic penguin contributions for \(B \to \phi K\).

| \(M_{a+}^{MP} + M_{b+}^{MP}\) | \(-0.0308 - i 0.0169\) |
| \(M_{a+}^{MP} + M_{b+}^{MP}\) | \(0.00309 + i 0.00544\) |
| \(M_{a+}^{MP} + M_{b+}^{MP}\) + \(M_{c+}^{MP} + M_{d+}^{MP}\) | negligible |
| \(M_{a+}^{MP}\) | \(-0.00229 - i 0.000798\) |
| \(M_{b+}^{MP}\) | \(0.00623 - i 0.0147\) |
| \(M_{c+}^{MP}\) | \(-0.0238 - i 0.0270\) |

Fig. 2. \(t\) dependence of \(M_{a+}^{MP} + M_{b+}^{MP}\). The lower curve is the real part and the upper curve is the imaginary part. We see that there is a slight dependence on \(t\) in the chromo-magnetic penguin.

Fig. 3. The distribution of \(q^2\) for \(\text{Re} M_{a+}^{MP}\). The expectation value of \(q^2\) is 6.3 GeV².
Calculation of Magnetic Penguin Amplitudes in $B \to \phi K$ Decays

$x_3$, which are the momentum fractions of partons. For example, in the diagram (a), $q^2 = (1 - x_2)x_3 M_B^2$. In Fig. 3, we show the distribution of $q^2$ for $M_{\phi K}$. The shape of this graph is not simple. The expectation value of $q^2$ is calculated as

$$\langle q^2 \rangle = 6.3 \text{ GeV}^2.$$  \hfill (4.1)

This is approximately equal to $M_B^2/4$.

Next, we analyze the FA using the value of $q^2$ given in Eq. (4.1). In the FA, the effective Hamiltonian for the chromo-magnetic penguin is

$$H_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \frac{\alpha_s}{8\pi} \left( -\frac{2m_b}{\langle q^2 \rangle} C_{8G} \left( O_4 + O_6 - \frac{1}{N_c} (O_3 + O_5) \right) \right).$$ \hfill (4.2)

We use the value 6.3 GeV$^2$ for $\langle q^2 \rangle$ and 1.5 GeV for the renormalization scale, which is a typical scale of the hard part in the PQCD approach. The result for the amplitude is $M = -0.029$, where we input the value $F_{BK} = 0.38$. This is to be compared with $M_{MP} = -0.0308 - 0.0169i$. The real part is in agreement with the above result.

§5. Summary

In this paper, we calculated the chromo-magnetic penguin amplitudes for $B \to \phi K$ decays in the PQCD approach. We find that the chromo-magnetic penguin contribution is about 30% of the leading contribution. Therefore, we find that the chromo-magnetic penguin is not negligible in hadronic two-body decays. However, for the Standard Model there are many other higher-order diagrams that must be considered simultaneously if we are to add the magnetic penguin term. If we consider the contribution of new physics, this diagram can be considered separately. It is noted that there is only weak energy scale dependence. In addition, we calculated the expectation value of $q^2$ of the gluon in the chromo-magnetic penguin, and the result is approximately $M_B^2/4$.

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Appendix A

The Sudakov Factor

The Sudakov factor is written as

\[ S_B(t) = s(x_1P_1^+, b_1) + \int_{1/b_1}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) , \] (A.1)

\[ S_K(t) = s(x_2P_2^+, b_2) + s((1 - x_2)P_2^+, b_2) + \int_{1/b_2}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) , \] (A.2)

\[ S_\phi(t) = s(x_3P_3^-, b_3) + s((1 - x_3)P_3^-, b_3) + \int_{1/b_3}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_\phi(\alpha_s(\bar{\mu})) . \] (A.3)

The exponent \( s \) is given by

\[ s(Q, b) = \int_{1/b}^{Q} \frac{d\mu}{\mu} \left[ \ln \left( \frac{Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right] , \] (A.4)

where the anomalous dimensions \( A \), to two loops, and \( B \), to one loop, are

\[ A = C_F \frac{\alpha_s}{\pi} + \left[ \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} f + \frac{2}{3} \beta_0 \ln \left( \frac{e^{2\gamma_E}}{2} \right) \right] \left( \frac{\alpha_s}{\pi} \right)^2 , \] (A.5)

\[ B = \frac{2}{3} \frac{\alpha_s}{\pi} \ln \left( \frac{e^{2\gamma_E - 1}}{2} \right) , \] (A.6)

with \( C_F = 4/3 \) being a color factor, \( f \) being the number of active flavors, \( \gamma_E \) being the Euler constant, and \( \beta_0 = (33 - 2f)/3 \). The anomalous dimension of mesons is given by

\[ \gamma_\phi(\alpha_s(\mu)) = 2\gamma_q(\alpha_s(\mu)) = -2\frac{\alpha_s(\mu)}{\pi} . \] (A.7)

Appendix B

Wave Functions

The \( B \) meson wave function is defined by

\[ \Phi^{(\text{in})}_{B,\alpha\beta,ij} \equiv \langle 0 | \bar{b}_{\beta j}(0) d_{\alpha i}(z) | B(P) \rangle = \frac{i\delta_{ij}}{\sqrt{2N_c}} \int dx d^2k_T e^{-i(xP^+ - z^+ - k_T z_T)} [(xP + MB)\gamma_5 \phi_B(x, k_T)]_{\alpha\beta} , \] (B.1)

where the indices \( \alpha \) and \( \beta \) are spin indices, \( i \) and \( j \) are color indices, and \( N_c \) is the color factor. The distribution amplitude \( \phi_B \) is normalized as

\[ \int_0^1 dx_1 \phi_B(x_1, b_1 = 0) = \frac{f_B}{2\sqrt{2N_c}} , \] (B.2)

where \( b_1 \) is the conjugate space of \( k_1 \), and \( f_B \) is the decay constant of the \( B \) meson. In this study, we used the model functions

\[ \phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{xM_B}{\omega_B} \right)^2 - \frac{\omega_B^2 b^2}{2} \right] , \] (B.3)
where \( N_B \) is the normalization constant and \( \omega_B \) is the shape parameter. We used \( \omega_B = 0.4 \) GeV, which was determined by the calculation of form factors.\(^{28}\)

The \( K \) meson wave function is given by

\[
\Phi_{K,\alpha\beta,ij}(z) = \langle K(P)|\bar{s}(z)\gamma_i s_{\alpha i}(0)|0\rangle
\]

The function \( \phi_A^0(x) \) is the leading-twist distribution amplitude, and the other terms are the twist-3 amplitudes. They were also calculated using the light-cone QCD sum rule:\(^{30}\)

\[
\phi_A^0(x) = \frac{f_K}{2\sqrt{2N_c}} 6x(1-x) \left[ 1 + a_1 C_1^3(1-2x) + a_2 C_2^3(1-2x) \right],
\]

\[
\phi_P^0(x) = \frac{f_K}{2\sqrt{2N_c}} \left[ 1 + \left( 30\eta_3 - \frac{5}{2} \rho_K^2 \right) \frac{C_2^1}{2}(1-2x) \right.
- 3 \left\{ \eta_3 \omega_3 + \frac{9}{20} \rho_K^2 (1 + 6a_2) \right\} \left[ C_4^1(1-2x) \right],
\]

\[
\phi_T^0(x) = \frac{f_K}{2\sqrt{2N_c}} (1-2x)
\times \left[ 1 + 6 \left( 5\eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_K^2 - \frac{3}{5} \rho_K^2 a_2 \right) \right] (1-10x + 10x^2). \tag{B.7}
\]

Here, \( \rho_K = (m_d + m_s)/M_K \) and \( C_\mu^n(x) \) is the Gegenbauer polynomial. The parameter values are \( a_1 = 0.17, a_2 = 0.20, \eta_3 = 0.015 \) and \( \omega_3 = -3.0 \).

The \( \phi \) meson wave function of the longitudinal parts is given by

\[
\phi_{\phi,\alpha\beta,ij}^{(out)}(z) = \langle \phi(P_3)|\bar{s}(z)\gamma_i s_{\alpha i}(0)|0\rangle
\]

The function \( \phi_A^0(x) \) is the leading-twist distribution amplitude, and the other terms are the twist-3 amplitudes. They were also calculated using the light-cone QCD sum rule:\(^{31}\)

\[
\phi_A^T(x) = \frac{f_T}{2\sqrt{2N_c}} \left[ 3(1-2x)^2 + \frac{35}{4} \zeta_3^T \{3 - 30(1-2x)^2 + 35(1-2x)^4\} \right.
+ \frac{3}{2} \delta_+ \left\{ 1 - (1-2x) \log \frac{1-x}{x} \right\} \left\{ 1 - (1-2x) \log \frac{1-x}{x} \right\},
\]

\[
\phi_P^T(x) = \frac{f_T}{4\sqrt{2N_c}} \left[ (1-2x) \left\{ 6 + 9\delta_+ + 140\zeta_3^T(1-10x + 10x^2) \right\} \right.
\]
Below, we present the decay amplitudes for Fig. 1 (b) – (j):

\[ \mathcal{M}_b^{MP} = 8M_B^6C_F^2 \frac{\sqrt{2N_c}}{2N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 db_3 db_3 \phi_B(x_1, b_1) \]
\[ \times \left[ x_1 \phi_A^K(x_2) \phi(x_3) - 2r_K(-2 + x_1) \phi_P^K(x_2) \phi(x_3) 
+ r_\phi x_1 \phi_T^K(x_2) (3 \phi^s(x_3) + \phi^t(x_3)) 
- 12r_\phi r_K x_1 \phi_P^K(x_2) \phi(x_3) + 2r_\phi r_K x_3 \phi_P^K(x_2) (3 \phi^s(x_3) - \phi^t(x_3)) \right] 
\times E_g(t)N_t\{x_1(1 - x_1)\}^c h^{MP}_c (A, B, C, b_1, b_3), \quad (C.1) \]

\[ \mathcal{M}_c^{MP} = 8M_B^6C_F \left( C_F - \frac{N_c}{2} \right) \frac{\sqrt{2N_c}}{2N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 db_3 db_3 \phi_B(x_1, b_1) \]
\[ \times \left[ 2(1 - x_2)(1 - x_1 - x_3) \phi_A^K(x_2) \phi(x_3) 
+ r_K(1 - x_2)(1 - x_1 - x_2 - x_3) (\phi_P^K(x_2) - \phi_T^K(x_2)) \phi(x_3) 
+ r_\phi x_3 (1 - x_1 - x_2 - x_3) \phi_A^T(x_2) (\phi^s(x_3) + \phi^t(x_3)) 
+ r_K r_\phi \{ (1 - x_1)(1 - x_2 - x_3) (\phi_P^K(x_2) - \phi_T^K(x_2)) (3 \phi^s(x_3) + \phi^t(x_3)) 
+ 2r_K r_\phi x_2 x_3 (3 \phi_P^K(x_2) \phi^s(x_3) - \phi_T^K(x_2) \phi^t(x_3)) \} \right] 
\times E_g(t)h^{MP}_n (A, B, C, b_1, b_3), \quad (C.2) \]

\[ \mathcal{M}_d^{MP} = -8M_B^6C_F \left( C_F - \frac{N_c}{2} \right) \frac{\sqrt{2N_c}}{2N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 db_3 db_3 \phi_B(x_1, b_1) \]
\[ \times (x_1 - x_2) \left[ -(2 + x_2) \phi_A^K(x_2) \phi(x_3) 
- r_K \{ (\phi_P^K(x_2) - \phi_T^K(x_2)) - x_2 (\phi_P^K(x_2) + \phi_T^K(x_2)) \} \phi(x_3) 
+ r_\phi (2x_1 - x_2 - 2x_3) \phi_A^T(x_2) (\phi^s(x_3) - \phi^t(x_3)) 
+ 2r_K r_\phi \{ (\phi_P^K(x_2) - \phi_T^K(x_2)) - x_2 (\phi_P^K(x_2) + \phi_T^K(x_2)) \phi^s(x_3) \} \right] 
\times E_g(t)h^{MP}_n (A, B, C, b_2, b_3), \quad (C.3) \]

\[ \mathcal{M}_e^{MP} = 8M_B^6C_F^2 \frac{\sqrt{2N_c}}{2N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 db_3 db_3 \phi_B(x_1, b_1) \]
\[ \times \left[ 3x_3 \phi_A^K(x_2) \phi(x_3) + 2r_K (2 - x_1 + x_3) \phi_P^K(x_2) \phi(x_3) 
+ r_\phi x_3 \{ 1 - 2(x_1 - x_3) \} \phi_A^T(x_2) (\phi^s(x_3) - \phi^t(x_3)) 
- 6r_K r_\phi x_1 \phi_P^K(x_2) (3 \phi^s(x_3) - \phi^t(x_3)) + 12r_K r_\phi x_3 \phi_P^K(x_2) \phi(x_3) \right] 
\times E_g(t)N_t\{x_3(1 - x_3)\} c h^{MP}_e (A, B, C, b_2, b_3, b_1), \quad (C.4) \]

\[ \mathcal{M}_f^{MP} = -8M_B^6C_F^2 \frac{\sqrt{2N_c}}{2N_c} \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 db_2 db_3 db_3 \phi_B(x_1, b_1) \]
\[ \times \left[ -3x_1(1 - x_2) \phi_A^K(x_2) \phi(x_3) 
+ r_K (1 - x_2)(2x_1 - x_2) (\phi_P^K(x_2) + \phi_T^K(x_2)) \phi(x_3) \right] \]
Here, $b$ is defined in Eq. (3.6), and the hard function $h_{eMP}$ is defined in Eq. (3.5). The other hard functions are defined as follows:

$$h_{nMP}(A, B, C, b_1, b_2, b_3) = - \int_0^\infty b_1 db_1 b_2 db_2 b_3 db_3 \frac{1}{(2\pi)^3} \int_0^{2\pi} d\theta'_1 d\theta'_2 d\theta'_3 \times K_0 \left( C \sqrt{b_1^2 + b_2^2 + 2b_1b_2 \cos(\theta'_1 - \theta'_2)} \right) K_0 \left( B \sqrt{b_2^2 + b_3^2 + 2b_2b_3 \cos(\theta'_2 - \theta'_3)} \right) \times \int_0^\infty dX \frac{X \cos(X|D|) + iX \sin(XD)}{X^2 + A^2},$$

$$h_{iMP}(B, C, b_1, b_2, b_3) = K_0(Bb_1) K_0(Cb_3) \left\{ \begin{array}{ll} 1/(2\pi\Delta) & \text{for } b_1 + b_2 > b_3 > |b_1 - b_2| \\ 0 & \text{otherwise} \end{array} \right.,$$

$$h_{jMP}(A, B, C, b_1, b_2, b_3) = - \int_0^{2\pi} d\theta'_1 d\theta'_2 \frac{f(b_1, b_2, \sqrt{b_1^2 + b_2^2 + 2b_1b_2 \cos(\theta'_1 - \theta'_2)})}{(2\pi)^2 C^2} \times K_0(Ab_1) K_0 \left( B \sqrt{b_2^2 + b_3^2 + 2b_2b_3 \cos(\theta'_1 - \theta'_2)} \right).$$

Here, $\Delta = \sqrt{2b_1^2 b_2^2 + 2b_1^2 b_3^2 + 2b_2^2 b_3^2 - b_1^4 - b_2^4 - b_3^4}/4$ and $D \equiv b_1 \cos\theta'_1 + b_2 \cos\theta'_2 + b_3 \cos\theta'_3$. The quantities $A$, $B$ and $C$, which are functions of $x_1$, $x_2$ and $x_3$, are given in Table II. For simplicity, we ignored $|k_{1T} - k_{3T}|$ in the calculation of $h_{jMP}$.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\sqrt{x_2}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
</tr>
<tr>
<td>(b)</td>
<td>$\sqrt{x_1}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
<td>$\sqrt{x_1 - x_3}MB$</td>
</tr>
<tr>
<td>(c)</td>
<td>$i\sqrt{x_2(1 - x_1 - x_3)}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
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<tr>
<td>(d)</td>
<td>$\sqrt{x_2(x_1 - x_3)}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
<td>$\sqrt{x_1 - x_3}MB$</td>
</tr>
<tr>
<td>(e)</td>
<td>$i\sqrt{x_3}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1 - x_3}MB$</td>
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<tr>
<td>(f)</td>
<td>$i\sqrt{x_1 - x_2}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
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<tr>
<td>(g)</td>
<td>$\sqrt{(x_1 - x_3)(1 - x_2)}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1 - x_3}MB$</td>
</tr>
<tr>
<td>(h)</td>
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<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
</tr>
<tr>
<td>(i)</td>
<td>$\sqrt{x_1x_2}MB$</td>
<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1 - x_3}MB$</td>
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<tr>
<td>(j)</td>
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<td>$i\sqrt{(1 - x_2)x_3}MB$</td>
<td>$\sqrt{x_1x_2}MB$</td>
</tr>
</tbody>
</table>
References

1) B. Aubert et al. [BABAR Collaboration], hep-ex/0207070.
2) K. Abe et al. [Belle Collaboration], hep-ex/0207098.