

# Stochastic multiobjective reservoir operation under imprecise objectives: multicriteria decision-making approach

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## ABSTRACT

Multiobjective reservoir operations are generally complex, as they are often associated with a large quantity of uncertain factors in combination with noncommensurable objectives. In this study, fuzzy-state stochastic dynamic programming (FSDP) and multicriteria decision-making (MCDM) are integrated to derive operating rules for a single multiobjective reservoir operation problem. The model addresses uncertainties due to randomness in inflows and imprecision in variables' discretization and objectives. The FSDP model takes into account uncertainties due to the random nature of inflows and imprecision due to variable discretization. Imprecise and noncommensurable objectives are quantified by a set of subjective criteria, the aggregation of which is performed through an MCDM model, by which possible decisions at every stage of the FSDP model are evaluated and compared. The proposed approach is then employed in deriving operating rules for the Karoon 1 reservoir in south-west Iran and the rules are tested and evaluated through simulation. Results show the model's capability in handling different kinds of uncertainties involved in real reservoir operation problems.

**Key words** | fuzzy-state stochastic dynamic programming, multicriteria, multiobjective, reservoir operation

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## ABBREVIATIONS AND NOTATION

$C$	Criteria set
$C_i$	$i$ th criterion
$D$	Alternatives decision set
$d_j$	$j$ th candidate alternative
$db_j$	Fuzzy distance from alternative $d_j$ to the weighted anti-ideal alternative
$dg_j$	Fuzzy distance from alternative $d_j$ to the weighted ideal alternative
$DOF_r(\cdot)$	Degree of fulfillment of $r$ th fuzzy rule
$\tilde{E}_t$	Energy generated in period $t$
$E_{\max}$	Maximum allowable production of power
$\tilde{e}_t(\cdot)$	Reservoir evaporation in period $t$
$f_n^*(\cdot)$	Optimum value of expected cumulative return from current period $t$ to the end of the planning horizon

FI	Fairly important
FSDP	Fuzzy-state stochastic dynamic programming
FST	Fuzzy set theory
$g$	The function gives $u_j$ with respect to considered criteria
$\tilde{h}_t$	Effective average head on turbines in period $t$
$I$	Important
$I_t$	Historical inflow in period $t$
$(I_{\max})_t$	Maximum historical inflow in period $t$
$(I_{\min})_t$	Minimum historical inflow in period $t$
$\tilde{I}_t(l)$	$l$ th fuzzy interval of inflow to reservoir in period $t$
$i$	Index of criterion
$j$	Index of decision alternative
$k$	Index of fuzzy interval of initial reservoir storage
$l$	Index of fuzzy interval of inflow
MCDM	Multicriteria decision-making

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MCM	Million cubic meters
$n$	Number of stages remaining until the end of the planning horizon
$N$	Maximum number of fuzzy rules for each period
NI	Number of inflow intervals
NS	Number of storage intervals
$p$	Number of criteria
$p_{i,ll}^t$	Transition probability value from $l$ th class of inflow in period $t$ to $ll$ th class of inflow in period $t + 1$
$q$	Number of decision alternatives
$\tilde{R}$	Fuzzy decision matrix considering of all criteria and alternatives
$R_{\max}$	Maximum allowable turbine release
$R_{\min}$	Minimum allowable turbine release
$R_r$	Defuzzified value of fuzzy release of the $r$ th rule
$\tilde{R}_t$	Turbine release during period $t$
$\tilde{r}_{ij}$	Dimensionless fuzzy value of alternative $d_j$ associated with criterion $c_i$
$\tilde{r}_j$	Vector of utility degree of the alternative $d_j$ with considering all criteria
$r$	Index of fuzzy rule
$S_{\max}$	Maximum storage volume of reservoir
$S_{\min}$	Minimum storage volume of reservoir
$S_{\text{safe}}$	Safe storage determined with respect to required flood control volumes of a predefined or design flood
$S_t$	Storage variable in period $t$
$\tilde{S}_t(k)$	$k$ th fuzzy interval of initial reservoir storage in period $t$
$t$	Index of period
$U$	Unimportant
$u_j$	Crisp global utility value for each alternative $j$ regarding all criteria
$u_j(t,k,l)$	Immediate return from system operation for state vector $(k,l)$ during period $t$
VI	Very important
VU	Very unimportant
$\tilde{v}_j$	Vector of weighted utility degree of the alternative $d_j$ with considering all criteria
$\tilde{W}_t$	Spill volume in period $t$
$\tilde{w}_i$	Weight of criterion $c_i$
$\tilde{x}_{ij}$	Fuzzy value of alternative $d_j$ associated with criterion $c_i$

$\mu(\cdot)$	Membership function of variable
$\mu_i(\cdot)$	Utility function of $i$ th quantitative criterion
$\eta_t$	Power plant efficiency in period $t$

## INTRODUCTION

The determination of multipurpose reservoir operating rules satisfying various operational needs is challenging mainly due to complex interactions between a mix of quantitative and qualitative objectives and the different kinds of uncertainties involved. A variety of optimization models have been developed so far to facilitate the operation of reservoir systems under the above difficulties; a summary can be found in [Ranie & Moreira \(2010\)](#). The efficiency of such modeling depends, to a large extent, on the appropriate handling of the multiobjective nature of the operational problem as well as treating uncertainties.

The presence of conflicting and incommensurable objectives affecting different groups of people or interests forms the basic problem in multiobjective optimizations ([Chen \*et al.\* 2007](#)). The resolution of the problem is quite a challenge, particularly when system managers fail to perceive the tradeoffs among the several purposes, given the existing conditions relevant to system operation ([Yeh & Becker 1982](#)). Attempts to commensurate all objectives into monetary units have had limited success, partially due to the difficulty in placing economic values on objectives such as environmental quality, flood damage reduction, projected revenue and other intangible cost and benefit items. In addition, the transformation of objectives into monetary functions is often unrealistic because of the relative scarcity of the desirable data.

In reality, it may be possible for some or all of the planning objectives to be vague and so imprecise handling of which by precise mathematical frameworks may not seem to be rational. Fuzzy set theory (FST) as a tool for dealing with vague objects and approximate reasoning has been extensively considered ([Fontane \*et al.\* 1997](#); [Owen \*et al.\* 1997](#); [Tilmant \*et al.\* 2002a,b](#); [Chaves & Kojiri 2003](#)). The precise quantification of system performance criteria and parameter and decision variables may not always be possible, nor is it always necessary. When the values of variables cannot be precisely specified, they are said to be

uncertain or fuzzy. If the values are uncertain, probability distributions may be used to quantify them. Statistical uncertainties are related to the randomness nature of some variables, which can be estimated by statistical and probability theory over historical data. Alternatively, if they are best described by qualitative adjectives, fuzzy membership functions can be used to quantify them. The membership functions whose value ranges between 0 and 1 represent the degree to which the state of the system satisfies a specific criterion. The latter cases are subject to non-statistical uncertainties which cannot be estimated with statistically based methods (Loucks & Van Beek 2005).

Uncertainty may be classified as (1) knowledge uncertainty, (2) natural uncertainty and (3) decision uncertainty (Loucks & Van Beek 2005). Under natural uncertainty one may refer to temporal variability and spatial variability. Decision uncertainty, on the other hand, consists of uncertainties in goal objectives and value preferences. Therefore, both probability distributions and fuzzy membership functions of these uncertain or qualitative variables can be included in quantitative optimization models. This paper attempts to address uncertainty in random inflow to the reservoir and decision uncertainty regarding goal objective and value preferences.

Fontane *et al.* (1997) employed FST in a reservoir operation problem with imprecise objectives in which operational preferences were described linguistically. Tilmant *et al.* (2002b) explicitly considered preferences of decision-makers and hard-to-quantify objectives in the form of membership functions. In fact, economic objectives were replaced by some fuzzy criteria in which the economic value of objectives was taken into account, but under a fuzzified form by means of membership functions. Preference-based membership functions are a suitable tool for cases where the priority of objectives are not clear and the decision-making process relies mainly on the subjective preferences of the decision-maker (Karamouz *et al.* 2005).

Complexities due to the presence of intangible and incommensurable objectives can also be interpreted as economic uncertainties in the reservoir operation problem. But in reservoir systems, uncertainty due to stochasticity of inflows may be regarded as the most important source of uncertainty which has been extensively investigated employing implicit and explicit stochastic optimization

models (Stedinger *et al.* 1984; Kelman *et al.* 1990; Fontane *et al.* 1997; Kim & Heo 2000; Celeste *et al.* 2009). Implicit stochastic optimization models deal with the uncertain inflows by generating a large number of possible scenarios. On the other hand, explicit stochastic optimization models directly embed the probabilistic descriptions of random streamflows into the model structure (Labadie 2004). It is often required in real-world reservoir operation problems to forecast inflows to the reservoir and thus the system operation would be under forecast uncertainty. Most of the explicit stochastic optimization models, although they consider inflows as stochastic processes, assume perfect statistical information and uncertainties associated with inflow forecasts are disregarded (Karamouz & Mousavi 2003). Nevertheless some researchers consider the forecast uncertainty in inflow forecasting processes (Stedinger *et al.* 1984; Kelman *et al.* 1990; Karamouz & Vasiliadis 1992; Karamouz & Mousavi 2003).

Statistical and probability theory has successfully been applied in optimal operation of reservoirs to tackle the statistical uncertainties due to randomness. But very often uncertainties in models may not be described by random numbers with defined probability density functions (PDFs). Some features of this type of model uncertainties are addressed by Akter & Simonovic (2004). They used FST to model uncertainties involved in the expression of reservoir penalty functions and target releases.

Mousavi *et al.* (2004a,b), by introducing fuzzy-state stochastic dynamic programming (FSDP), showed how to capture imprecision caused in the model by discretization of state variables using fuzzy interval classes.

In this paper fuzzy state stochastic dynamic programming and multicriteria decision-making models are integrated to derive operating rules for a multipurpose reservoir system addressing uncertainties due to random inflows, error of discretization and imprecise objectives. Uncertainties due to random inflows are captured using transition probabilities and impression caused by discretization of storage and inflow variables is controlled using fuzzy interval classes. Moreover, uncertainties in quantification of imprecise objectives are treated by preference-based membership functions. The membership functions are related to some subjective criteria to properly evaluate the imprecise objectives. In all previous studies

dealing with preference-based membership functions, an averaging operator was adopted for aggregating multiple and unequal membership functions to allow trade-off amongst different criteria (Fontane *et al.* 1997; Tilmant *et al.* 2002b; Chaves & Kojiri 2003). The model proposed in this study employs an MCDM model to aggregate all criteria and globally evaluate any decision alternatives within the FSDP model. In other words, the MCDM model is called for when deciding on the best decision at any stage and for every state of the FSDP model.

This work is different from previous studies done by Mousavi *et al.* (2004a,b) where they only considered uncertainties due to the random nature of hydrological variables and imprecision due to variable discretization in a single objective problem. The model presented is a multiobjective reservoir operation model which also considers uncertainties related to imprecise objectives and benefits from the MCDM approach.

This paper is organized as follows. The MCDM model is described first. The FSDP model formulation is then presented, after which the fuzzy simulation model is explained. The case study is also introduced and followed by the results of the proposed methodology. Finally the paper ends with the conclusions.

## MULTICRITERIA DECISION-MAKING (MCDM) MODEL

Multicriteria decision analysis is a methodology commonly used to aid the decision-makers to facilitate stakeholder participation and collaborative decision-making. The assignment of monetary values to environmental or social criteria is not mandatory; therefore, it allows the consideration of multiple criteria in incommensurable units (Hyde *et al.* 2004). During the past few decades, a significant increase in multi-criterion decision-making methods in water resources management has been seen (Bender & Simonovic 2000; Cheng & Chau 2001, 2002; Chen & Hou 2004; Srinivasa Raju & Duckstein 2004; Simonovic & Nirupama 2005; Fu 2008). In this study, different objectives of reservoir operations including water supply, flood control and hydropower are transformed into some operational criteria reflecting the benefits of related objectives.

There exist several multicriteria decision-making methods, the major part of which are derived from the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method, proposed by Hwang & Yoon (1981). TOPSIS is based on the concepts of ideal and anti-ideal points, in which the most satisfying alternative should be the one as close as possible to the ideal point and as far as possible from the anti-ideal point. Recently, Fu (2008) employed a fuzzy version of the TOPSIS method for alternatives ranking in operation of a flood control reservoir.

Let  $D = \{d_1, d_2, \dots, d_q\}$  presents the alternatives decision set for a multicriteria decision problem, where  $d_j$  is the  $j$ th candidate alternative. Each alternative is described by the criterion set  $C = \{c_1, c_2, \dots, c_p\}$  including  $p$  criteria. The value of alternative  $d_j$  associated with criterion  $c_i$  is represented by a triangular fuzzy number  $\tilde{x}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij})$ . The fuzziness may stem from uncertainties in modeling, forecasting, lack of data, error of measurement and the error caused by discretization of variables. Since different criteria may be assessed by different incommensurable units, the values of alternatives must be converted into a dimensionless unit for comparison. This study benefits from the concept of utility degree for each criterion as a common measurement unit. Therefore, the following fuzzy number is used for  $j$ th alternative evaluation with regard to criterion  $c_i$  as follows:

$$\tilde{r}_{ij} = (\mu_i(\alpha_{ij}), \mu_i(\beta_{ij}), \mu_i(\gamma_{ij})) \quad (1)$$

It should be noted that a fuzzy number is a specific fuzzy set that is a pair of  $x$  defined on a closed interval and its membership function  $\mu(x)$ . Membership functions  $\mu_i(\cdot)$  in (1) are evaluated using the utility function of the  $i$ th quantitative criterion reflecting a decision-maker's preferences. Consideration of all criteria and alternatives forms the following fuzzy decision matrix:

$$\tilde{R} = (\tilde{r}_{ij})_{p \times q} \quad (2)$$

The  $j$ th column of the matrix represents the utility degree of alternative  $d_j$  with respect to all criteria:

$$\tilde{r}_j = (\tilde{r}_{1j}, \tilde{r}_{2j}, \dots, \tilde{r}_{pj})^T \quad (3)$$

The decision-maker may consider different weights for different criteria in an MCDM problem.

In the proposed model, linguistic values are used as fuzzy weights of the criteria. Figure 1 shows the linguistic values used in the case study which are triangular fuzzy numbers in the range of [0,1]. The weighted utility degree of the alternative  $d_j$  can be represented as

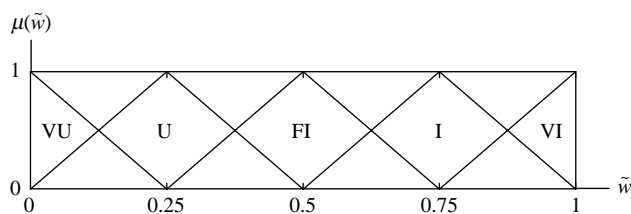
$$\tilde{v}_j = (\tilde{v}_{1j}, \tilde{v}_{2j}, \dots, \tilde{v}_{pj})^T, \quad \tilde{v}_{ij} = \tilde{w}_i \otimes \tilde{r}_{ij} \quad (4)$$

where  $\tilde{w}_i$  represents the weight of criterion  $c_i$  and the symbol  $\otimes$  stands for the fuzzy product.

In order to aggregate the multiple criteria and ranking of alternatives, one may adopt different operators and different ranking methods, respectively. Tilmant *et al.* (2002a) discussed the relation between operator type and the decision-makers' index of optimism and its effect on the performance of the system. Also different comparison methods of fuzzy numbers may lead to different ranking of alternatives. This study uses the concept of ideal and anti-ideal points for determination of the best alternative in the decision set. Use of this concept eliminates difficulties inherent in using different operators and different comparison methods of fuzzy numbers for aggregating and ranking processes. Accordingly, the following relation is used for evaluation of alternatives (Fu 2008):

$$u_j = \frac{db_j^2}{db_j^2 + dg_j^2} \quad u_j \in [0, 1] \quad (5)$$

where  $dg_j$  and  $db_j$  are, respectively, fuzzy distances from alternative  $d_j$  to the weighted ideal and anti-ideal alternatives.  $u_j$  is a crisp number providing the global evaluation for each alternative regarding all criteria. In fact,  $u_j$  can be interpreted as the membership degree of the  $j$ th alternative to the ideal alternative. The best decision corresponds to the largest  $u$  among the alternatives (Cheng & Chau 2001, 2002; Fu 2008).



**Figure 1** | Membership function values for evaluation of fuzzy weights (VI: very important, I: important, FI: fairly important, U: unimportant, VU: very unimportant).

The MCDM model described above is used in determination of the immediate return function of the FSDP model. In fact, for each element of the state vector, there is a finite set of candidate alternatives,  $j(s)$ , whose immediate benefits are  $u_j(s)$ . In other words, the alternatives are, in fact, different decisions being evaluated for any state at any stage of the FSDP model and the immediate benefit of each alternative is the aggregated degree of satisfaction corresponding to all operational criteria. In this sense, the MCDM and FSDP models are fully coupled in the proposed methodology.

The presented model differs from that of Fu (2008) in one major aspect wherein marginal evaluation is used in Equation (1) instead of utility degree for each criterion. Marginal evaluation is based on a comparison of each alternative to the best and worst available alternatives. Although the determined values of  $u_j(s)$  may be efficiently used for alternatives ranking, they may not truly represent the decision-makers' real benefits. Therefore, this study employs utility functions to determine the values of  $u_j(s)$  to be used in the decision-making process. Utility function-based values of  $u_j(s)$  may now be used both for addressing the best alternative and the real benefit of decision at each stage.

## FUZZY-STATE STOCHASTIC DYNAMIC PROGRAMMING (FSDP) MODEL

Deterministic and stochastic dynamic programming (SDP) models have been widely used in reservoir operation optimization (Kelman *et al.* 1990; Karamouz & Vasiliadis 1992; Fontane *et al.* 1997; Chaves & Kojiri 2003; Karamouz & Mousavi 2003; Labadie 2004). SDP was designed to capture a probabilistic description of random streamflow processes rather than deterministic hydrologic sequences. In addition, optimal steady state operating policies are determined directly from results of the optimization, whereas methods such as linear or fuzzy regression may be used for inferring operational rules from the results of a deterministic dynamic programming model (Mousavi *et al.* 2007).

A main drawback of discrete dynamic programming models is that the continuous space of state and decision variables is discretized into finite single-valued points while they are continuous in nature. To model the error



associated with discretizing the variables in an SDP model, Mousavi et al. (2004a,b) developed two different fuzzy-state stochastic dynamic programming (FSDP) models in which the storage and inflow volumes are partitioned into fuzzy intervals, as shown in Figures 2 and 3. This approach allows smooth transitions between storage and inflow intervals.

The FSDP model presented in this paper first takes both the storages and inflows as fuzzy numbers and, second, uses a fuzzy MCDM approach for every state in comparing different decisions to choose the best (optimal) decision from. In other words, the immediate benefit assigned to each state vector  $(k,l)$  for the  $j$ th decision alternative in period  $t$ ,  $u_j(t,k,l)$ , is evaluated based on the MCDM model presented in the previous section. It should be noted that fuzziness in the fuzzy decision matrix  $(\tilde{R})$  for the MCDM model is related to fuzzy states and consequently fuzzy decision variables of the FSDP model. In this approach, different fuzzy final storages, as decision variables, at each state are evaluated by their membership degrees to the ideal alternative. Therefore, the proposed FSDP formulation is as follows:

$$f_n^*(\tilde{S}_t(k), \tilde{I}_t(l)) = \max_{\tilde{S}_{t+1}(j)} \left\{ u_j(t,k,l) + \sum_{l=1}^{NI} p_{l,ll}^t f_{n-1}^*(\tilde{S}_{t+1}(j), \tilde{I}_{t+1}(ll)) \right\} \quad (6)$$

subject to:

$$u_j(t,k,l) = g(\tilde{S}_t(k), \tilde{S}_{t+1}(j), \tilde{I}_t(l), \tilde{R}(t), \tilde{E}_t) \quad (7)$$

$$\tilde{S}_{t+1}(j) = \tilde{S}_t(k) \oplus \tilde{I}_t(l) \ominus \tilde{R}_t \ominus \tilde{W}_t \ominus \tilde{e}_t(\tilde{S}_t(k), \tilde{S}_{t+1}(j)) \quad (8)$$

$$\tilde{E}_t = 2.725 \times \tilde{R}_t \otimes \tilde{h}_t \otimes \tilde{\eta}_t \quad (9)$$

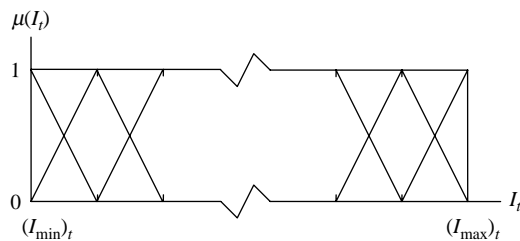


Figure 2 | Fuzzy discretization of inflow variables in each month and their membership functions (adopted from Mousavi et al. 2004a,b).

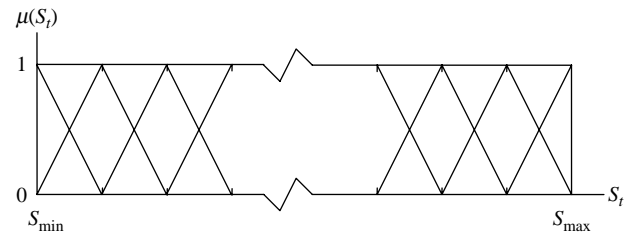


Figure 3 | Fuzzy discretization of storage variables and their membership functions (adopted from Mousavi et al. 2004a,b).

$$S_{\min} \leq \tilde{S}_t(k) \leq S_{\max} \quad (10)$$

$$R_{\min} \leq \tilde{R}_t \leq R_{\max} \quad (11)$$

where symbols  $\oplus$  and  $\ominus$  stand for fuzzy summation and fuzzy subtraction, respectively. In this formulation,  $g$  is the function resulted from Equation (5) with respect to considered criteria,  $n$  = number of stages remaining until the end of the planning horizon,  $t$  = index of period ( $t = 1, \dots, T$ ),  $T$  = the total number of time periods within a cycle (e.g.  $T = 12$  in this study because a cycle is 1 year and a period is 1 month),  $\tilde{S}_t(k) = k$ th fuzzy interval of initial reservoir storage ( $k = 1, \dots, NS$ ) in period  $t$ ,  $\tilde{I}_t(l) = l$ th fuzzy interval of inflow to reservoir ( $l = 1, \dots, NI$ ),  $j = j$ th fuzzy interval of final reservoir storage ( $j = 1, \dots, NS$ ),  $p_{l,ll}^t$  = transition probability value from  $l$ th class of inflow in period  $t$  to  $ll$ th class of inflow in period  $t + 1$ ,  $u_j(t,k,l)$  = immediate return from system operation during period  $t$ ,  $f_n^*(\tilde{S}_t(k), \tilde{I}_t(l))$  = optimum value of expected cumulative return from current period  $t$  to the end of the planning horizon for the vector of state variables  $(k,l)$ ,  $\tilde{R}_t$  = turbine release during period  $t$ ,  $\tilde{W}_t$  = spill,  $\tilde{e}_t(\tilde{S}_t(k), \tilde{S}_{t+1}(j))$  = reservoir evaporation,  $\tilde{E}_t$  = energy generated,  $\tilde{h}_t$  = effective average head on turbines,  $\tilde{\eta}_t$  = power plant efficiency,  $R_{\min}/R_{\max}$  = minimum/maximum allowable turbine release and  $S_{\min}/S_{\max}$  = minimum/maximum storage volume of reservoir.

As storage volume and inflow to the reservoir are fuzzy, the following dependant variables  $\tilde{R}_t$ ,  $\tilde{W}_t$ ,  $\tilde{e}_t(\tilde{S}_t(k), \tilde{S}_{t+1}(j))$ ,  $\tilde{E}_t$ ,  $\tilde{h}_t$  and  $\tilde{\eta}_t$  are fuzzy numbers too. It should be noted that the algebraic operations at recursive function are performed with crisp numbers, while the constraints are treated as fuzzy numbers. The storage variable is discretized between  $S_{\min}$  and  $S_{\max}$  and thus constraint (10) is satisfied automatically. Constraint (11) is satisfied after defuzzifying

release volume  $\tilde{R}_t$ . The defuzzified value is selected as the center of area under triangular fuzzy release. The transition probabilities  $p_{i,ll}^t$  are estimated by using the fuzzy Markov chain model proposed by Mousavi et al. (2004b) in which the inflow state variable is taken as a fuzzy state and the frequency of an inflow value located in a fuzzy interval class is equal to the membership degree of that value. Consequently, the concepts of fuzzy probabilities (fuzzy Markov chain), fuzzy storage classes with fuzzy algebraic operations and fuzzy MCDM are all integrated in the proposed FSDP model.

Another point which should be noted in the extended FSDP model developed in this study is full coupling of the MCDM and optimization models, because the MCDM model is directly called for online in the FSDP model while comparing candidate solutions. What is used traditionally is to employ an offline MCDM model out of optimization or other scenario generating models. A pseudo-code presented in Figure 4 illustrates the interactions between these models.

## FUZZY-RULE-BASED SIMULATION MODEL

Optimal steady state policies derived from the FSDP model form a fuzzy rule base including fuzzy IF-THEN rules. The "IF" part contains a vector of fuzzy state variables, i.e. initial storage and inflow variables while the "THEN" part is the fuzzy final storage or fuzzy release variable. At most,  $N = NS * NI$  fuzzy rules for each period  $t(t = 1 \dots 12)$  may

```

BEGIN
  Calculate the transition probabilities for each period t
END
BEGIN ("Optimization process for n = 1 and t = T")
  For k = 1 to NS
    For l = 1 to NI
      For j = 1 to NS
        Determine the degree of satisfaction of each criterion using the corresponding utility function
        Call MCDM model to calculate  $u_j(t,k,l)$ 
      End j
      Determine  $f_n^*(\tilde{S}_t(k), \tilde{I}_t(l))$  and optimum decision for the state vector (k,l)
    End l
  End k
END ("End of optimization process for n = 1 and t = T")
Set n = n+1 and t = t-1 (if t = 1 then t-1 = T)
Repeat until steady state conditions are met

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Figure 4 | Pseudo-code for the presented model.

be developed, where NS and NI are the number of storage and inflow intervals, respectively.

To evaluate the optimal policies obtained from the FSDP model, a fuzzy simulation model is used. In this model, the actual release from the reservoir is calculated by aggregating the consequence parts of the fuzzy rules that can be adopted partially to the initial state of the reservoir system. In other words, once the initial storage  $S_t$  and inflow  $I_t$  as premise variables of fuzzy rules become known at the beginning of any time period  $t$ , the release  $R_t$  is determined by aggregating the consequences of the adopted fuzzy rules to that premise value. In fact, depending on  $S_t$  and  $I_t$  values, the relevant rules are fired. The degree of fulfillment (DOF) or firing strength of the state vector  $(S_t, I_t)$  with respect to the  $i$ th rule is defined as

$$\text{DOF}_r(S_t, I_t) = \mu_{\tilde{S}_t(k)}(S_t) \cdot \mu_{\tilde{I}_t(l)}(I_t) \quad (12)$$

Then release  $R_t$  for a given state vector is determined as

$$R_t = \frac{\sum_r R_r \cdot \text{DOF}_r(S_t, I_t)}{\sum_r \text{DOF}_r(S_t, I_t)} \quad (13)$$

where  $R_r$  is the defuzzified value of the fuzzy release of the  $r$ th rule. Having the reservoir release computed from Equation (13), one can calculate the final storage volume through a balance equation and the procedure is repeated until the end of the simulation horizon whose performance criteria such as vulnerability, reliability and resiliency can be estimated based on the results.

## APPLICATION

The case study on which the model is evaluated is the Karoon 1 reservoir in the south-west of Iran. The Karoon 1 reservoir with an effective storage capacity of 2464 million cubic meters (MCM) is one of the major storage reservoirs in the region. The reservoir is a multipurpose reservoir for flood control, hydroelectric power generation and water supply. The maximum possible empty space is desirable for flood control, while water storage is required for the conservative objectives of water supply and hydropower generation.

The maximum and minimum reservoir operating heads for the Karoon 1 power plant with 1,000 MW installed capacity are 192.5 m and 150 m, respectively. Table 1 presents the mean monthly inflows and water demands. In this region, the flooding season consists of months 9, 10, 11, 12, 1 and 2 in accordance with the Iranian calendar, which correspond to December–May.

The FSDP model is applied to derive the operating rules in order to maximize the benefits resulted from hydro-power generation while minimizing water supply shortages as well as flood damage risk. To evaluate the optimal policies obtained from the FSDP model, a fuzzy simulation model is used. Thirty-nine years of available historical inflow data in monthly periods are available and used in this study.

## UTILITY FUNCTIONS AND FUZZY WEIGHTS

This paper introduces a set of subjective criteria in order to treat the imprecise objectives. To do so, the membership functions are defined in the form of preference-based functions. The preference-based membership functions or utility functions are constructed by eliciting the preference information from the decision-makers. The utility function enables decision-makers to incorporate the priority of different objectives in developing optimal operating policies.

When eliciting utility functions for a specific application, the determination of the necessary lower and upper bounds pertaining to membership degrees of 0 and 1 is very important. Besides, a suitable curve representing the rate of utility variation of decision-makers from 0 to 1 should be prepared.

In this study different objectives of reservoir operations are satisfied through applying the following criteria. Higher relative degrees of satisfaction are indicated by values closer to 1. A quantitative criterion which implicitly accounts for the flood risk is introduced for flood control objective (Figure 5). The membership function associated with this criterion is constructed such that the lower flood level in the reservoir corresponds to the higher preference and utility ( $c_1$ ). For this criterion, safe storages are determined with respect to the required flood control volumes of a predefined or designed flood. This criterion is activated only in flooding seasons.

The second criterion ( $c_2$ ) denotes the amount of release/demand ratio during each period (Figure 6). Generally the larger the criterion value, the more satisfaction in the water supply objective is achieved.

Also a quantitative criterion is defined for the hydro-power objective (Figure 7). The membership function related to this criterion is based on the assumption that satisfaction is proportional to the ratio between current production and maximum allowable production ( $c_3$ ).

Other criteria may be considered for each objective, depending upon the case study (Cheng & Chau 2001, 2002; Chaves & Kojiri 2003; Chen & Hou 2004; Fu 2008).

The shape of each function is constructed based on the general and expert's knowledge about each criterion. Although we do not intend to detail the formulation process of the membership functions here, it is important to mention that a better formulation of the functions will increase the chance of operational success. The functions must represent the desire of stakeholders and should be

**Table 1** | The monthly mean inflows to Karoon 1 reservoir as well as monthly downstream water demands

Month	1	2	3	4	5	6	7	8	9	10	11	12
Mean inflow (MCM)	2,082	1,904	1,252	856	633	531	434	492	704	885	1,148	1,495
Water demand (MCM)	812	857	999	1,208	1,229	849	607	511	389	389	389	641



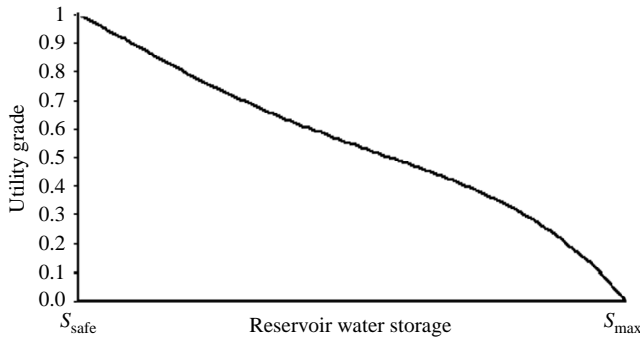


Figure 5 | Fuzzy membership function for flood control during flood seasons.

consistently evaluated and updated to properly represent operational objectives.

To obtain more appropriate membership functions, in addition to the experts, corresponding users and reservoir managers may be interviewed (Fontane *et al.* 1997). In such cases, the interviewers' opinions may be relatively diverse. The variability in opinions among the interviewers was investigated through distributions of the membership functions proposed by Owen *et al.* (1997).

Also, of particular importance is the determination of the fuzzy weights corresponding to the subjective criteria. The weights express a decision-maker's experience, knowledge and subjective judgment, and they concern the decision-making environment as well. In this study, the linguistic values shown in Figure 2 are used to evaluate the relative importance of the criteria. For the Karoon system, power production receives the highest priority. Flood control is important only in flood seasons while water supply for irrigation is the secondary objective. Thus, linguistic variables of VI, FI and VI are assigned to the

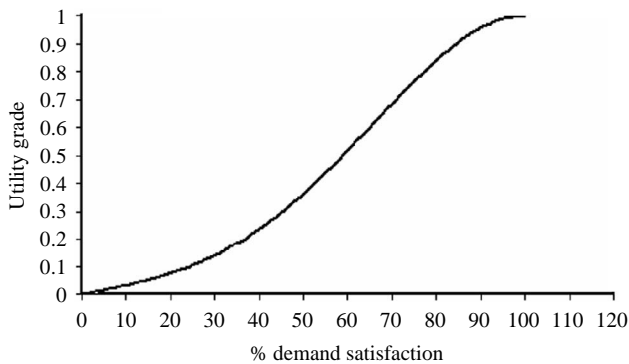


Figure 6 | Fuzzy membership function for water supply as a percentage of water demand.

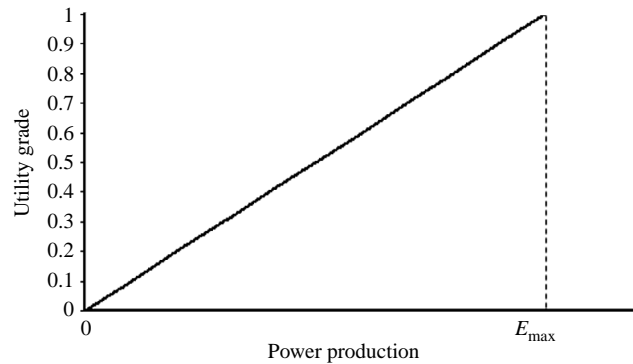


Figure 7 | Fuzzy membership function for power production.

weights of criteria  $c_1$ ,  $c_2$  and  $c_3$ , respectively. The results of the model are highly dependent on the selected weights. Therefore, sensitivity analysis of the model performance and robustness of the results with respect to the weights is always recommended.

## RESULTS AND DISCUSSION

Operational policies derived from the FSDP model are examined in the fuzzy simulation model. Figure 8 presents the maximum, mean and minimum values of monthly storage volumes resulting from the simulation in which historical inflow data are used. As expected, storage volumes are low in flood seasons while dry periods demand higher storage volumes.

Figure 9 presents monthly evolution of the average satisfaction associated with water supply, flood control and hydropower objectives. Overall, flood control and hydropower objectives are satisfied better than water supply demand. This issue is mainly due to assigning a smaller

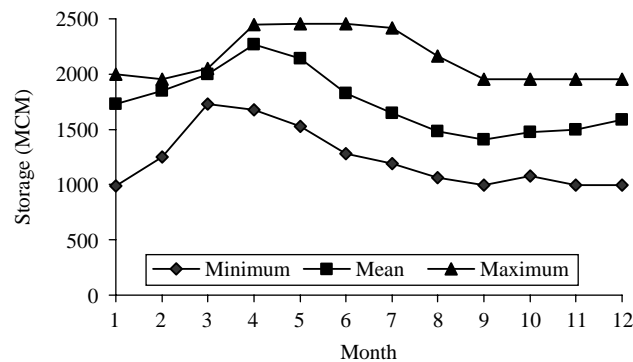
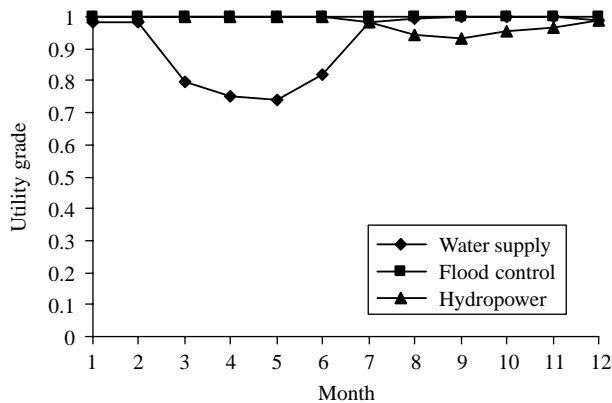


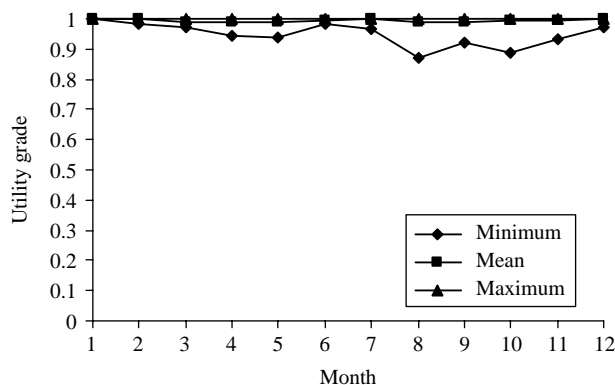
Figure 8 | The range of monthly storage volumes.



**Figure 9** | Simulated average satisfaction with respect to different objectives.

weight to the water supply objective compared to other objectives. Furthermore, the utility grade of the water supply objective is fairly low in the summer season. The relatively poor grade is due to low inflows and high demands in summer. Also, the utility grade of the hydropower objective in the flood season is a little less than that in other seasons. This can be due to low storage levels in flood seasons.

Figure 10 shows the range of simulated global evaluation with respect to all objectives. Results reveal that average global satisfaction of objectives is quite satisfactory, although its minimum looks a little poor, especially, in drier and wetter months. This is due to the fact that extreme inflows can result in weak utility for some objectives. Extremely low inflows may reduce the utility degree of the water supply and hydropower objectives while high extreme inflows may weaken the utility degree of the flood control objective.



**Figure 10** | The range of simulated overall satisfaction with respect to all objectives.

It should be noted that the efficiency of fuzzy operating rules is evaluated with respect to the results of the simulation model. If the results do not match the desired purposes, one may modify the rules by either changing the weights or introducing new criteria. The relative importance of the criteria is changed by changing the weights and the degree of satisfaction of objectives can be adjusted. As the corresponding weights are linguistic variables, rules modification may be facilitated. New criteria can be replaced or added to the FSDP model such that different objectives should be satisfied properly.

## CONCLUSION

Imprecise objectives and different uncertainties for a reservoir operation problem were addressed through an integrated model coupling the FSDP and MCDM models. The FSDP model treats uncertainties caused by stochastic inflows and error of variables' discretization while the MCDM model deals with imprecise objectives. In this study, different objectives of reservoir operations such as water supply, flood control and hydropower are transformed into some operational criteria which should properly reflect the benefits of the related objectives.

In this sense a multiobjective problem was converted into a multicriteria decision-making problem where decision alternatives for the MCDM model were different decisions being evaluated for any state at any stage of the FSDP model. To evaluate the economic value of each decision alternative, fuzzy membership functions were used in determining the degree of satisfaction of criteria by each alternative. The model takes the experience and knowledge of the decision-maker in determining subjective criteria and fuzzy membership functions as well as fuzzy weights. This study has benefited from the concept of ideal and anti-ideal points in determination of the best alternative without the need to compare fuzzy numbers.

The model was employed in the Karoon 1 reservoir system in Iran and optimal policies were then tested in a fuzzy simulation model. Overall results showed that flood control and hydropower objectives were found to have a more reasonable expectation of success compared to water supply demand. This issue is mainly due to assigning a smaller weight to the water supply objective. But the

proposed model is flexible in the sense that the relative importance of criteria can be changed and so the degree of satisfaction of objectives can be adjusted as well.

It is believed that the proposed approach may facilitate the operation of multiobjective reservoirs where the information about economic values of the objectives is not available.

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