GEOPHYSICS, VOL. 47, NO. 7 (JULY 1982); P. 1100–1105, 9 FIGS., 1 TABLE.

Short Note

A simple method for calculating apparent resistivity from electromagnetic sounding data

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Frequency-domain electromagnetic (EM) sounding is becoming an increasingly useful deep-exploration tool with recent applications to crustal sounding and geothermal exploration (Tripp et al, 1978; Sternberg, 1979; Duncan et al, 1980; Stark et al, 1980). A major drawback of the method is that most field data are analyzed by computer after the data are returned to the laboratory; no intermediate parameters, such as apparent resistivity, are calculated to provide on-site information. The following example illustrates this problem.

Figure 1 shows EM sounding data taken with the EM-60 frequency-domain system in central Nevada (Morrison et al, 1978; Stark et al, 1980). The spectral plots in this figure give normalized vertical (H_z) and radial (H_R) magnetic field amplitudes and phases at a distance of 720 m from the loop transmitter. Also shown are spectral plots of the ellipse polarization parameters, ellipticity, and tilt angle of the ellipse traced by the magnetic field vector. We have fitted these combined spectra to a layered model at least-squares inversion; the calculated curves for the models are also shown on the figure. Inspection of the spectral data alone gives very little direct information on the earth resistivity structure. Meaningful estimates of layer resistivities and thicknesses are usually impossible to make even for experienced interpreters. This leads to several difficulties: (1) the quality of incoming field data is often difficult to evaluate, which could lead to an attempt to analyze "smooth-looking" noise; (2) a field supervisor cannot alter the survey on the basis of incoming results; (3) the interpreter must initiate a model inversion program by trial and error, a process which could give misleading results if a poor first guess were used; and (4) the spectra are virtually meaningless to a nonspecialist, thus providing no basis for a geologic understanding. These problems led us to seek an apparent resistivity transformation for EM data similar to those used with dc resistivity and magnetotelluric (MT) data.

The concept of obtaining apparent resistivity from EM data has been examined in the literature. Keller (1971) derived "early time" and "late time" apparent resistivity from asymptotic behavior of time-domain data. These simple formulas proved effective on field data but are only valid for asymptotic portions of field curves. Morrison et al (1969) computed apparent resistivity from time-domain field curves by matching field data to a curve calculation for a homogeneous half-space and determining the apparent resistivity from the normalized time axis of the half-space curve. Kauahikaua (1981) expanded this to include calculations for apparent depth based on two-layer asympotic approximations. Apparent resistivity was obtained from frequency-domain EM data by Sternberg (1979) using an iterative approach. Sternberg (1979) programmed the analytic solution for the magnetic field over a homogeneous half-space into a hand-held calculator and solved iteratively for the resistivity that matches the observed field. This method produces a smooth apparent resistivity curve and is suitable for in-field use. The method we introduce below calculates apparent resistivity directly from EM field data; it is sufficiently simple for in-field application and useful for partial field curves.

DESCRIPTION OF THE METHOD

Compared to those used in magnetotellurics and dc resistivity, equations describing the electric and magnetic fields caused by an oscillating dipole source over a half-space are very complicated (Ryu et al, 1970). Although these cannot be solved analytically, we can use theoretical fields over a half-space to obtain apparent resistivity for horizontal-loop induction sounding data.

Ryu et al (1970) showed that field equations can be written as a function of a dimensionless "induction number"

$$B = \sqrt{\frac{\mu\omega}{2\rho}}\,R, \tag{1}$$

where ω is the angular frequency, μ is the magnetic permeability, R is the transmitter-receiver separation, and ρ is the half-space resistivity. A sample plot of radial magnetic field over a 10 Ω-m half-space is given in Figure 2. If similar plots for all normally measured field quantities are generated, then this set of generalized field curves can be used to estimate apparent resistivity for each individually measured quantity.

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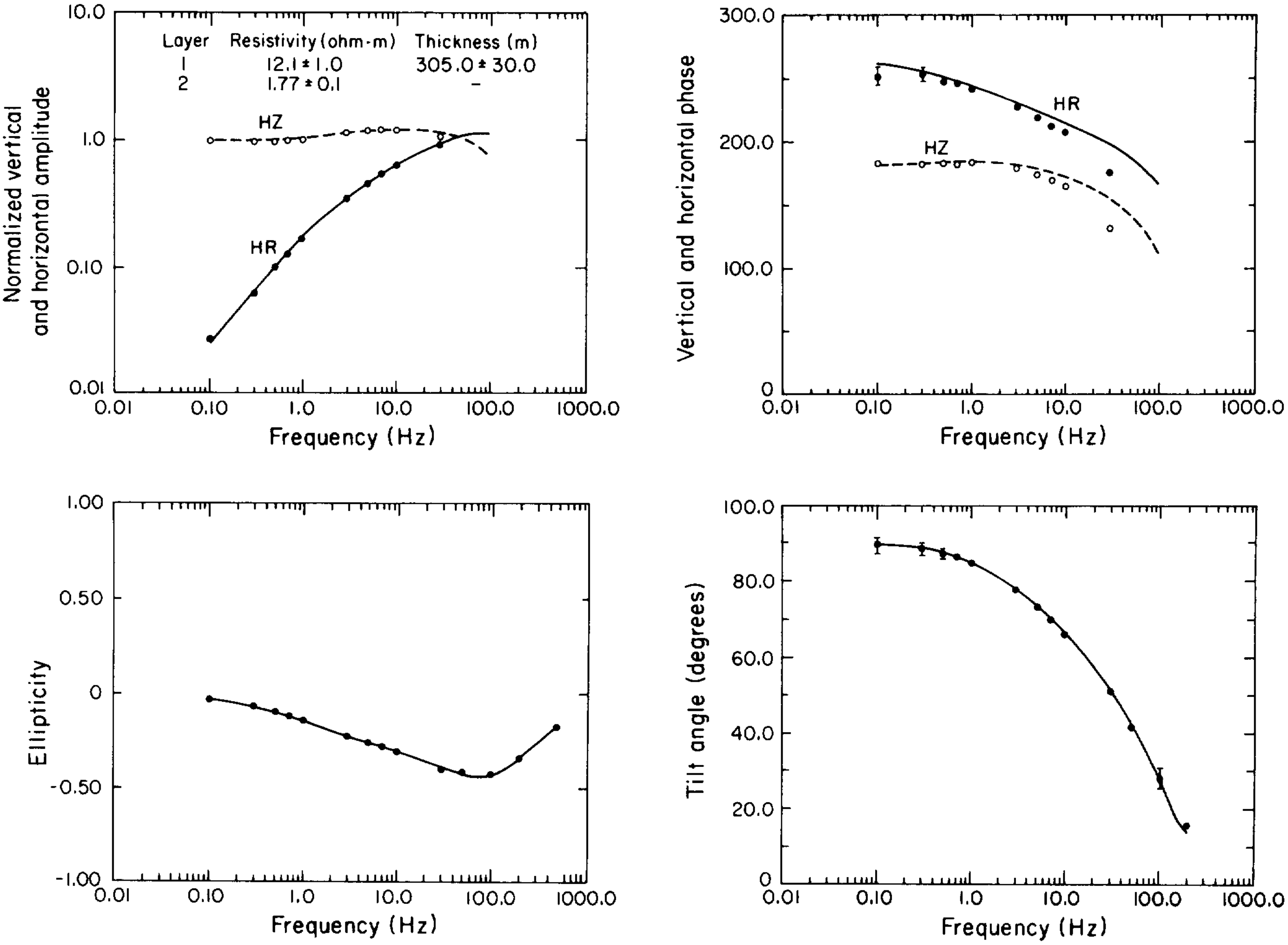


FIG. 1. Amplitude, phase, ellipticity, and tilt angle spectra for central Nevada EM-60 electromagnetic sounding.

To obtain an apparent resistivity estimate from an observed field value, first it is necessary to match the observed field value to its corresponding point on the appropriate generalized field curve. Then the corresponding induction number can be read from the graph, and apparent resistivity can be obtained by solving equation (1) for ρ:

$$\rho = \rho_A = \frac{\omega}{2B^2} R^2. \tag{2}$$

Since R and ω are known experimental parameters, μ for most field situations is the constant $\mu_0 = 4\pi \times 10^{-7}$ mks, and B, read from the generalized curve ρ_A, can be readily calculated. For example, suppose we wish to know the apparent resistivity at 10 Hz of the radial magnetic field component in Figure 1. The field value (0.68) when mapped on Figure 2 corresponds to an induction number of 1.6. The radius is 720 m, $\mu_0 = 4\pi \times 10^{-7}$, and the apparent resistivity given by

$$\rho_A = \frac{(4\pi \times 10^{-7})(2\pi)(10)}{2(1.6)^2} (720)^2 \approx 8 \ \Omega\text{-m}$$

is a reasonable approximation to the top-layer resistivity found by inversion (Figure 1). By applying this scheme to field measurements at a wide range of frequencies, apparent resistivity spectra can readily be calculated from field data.

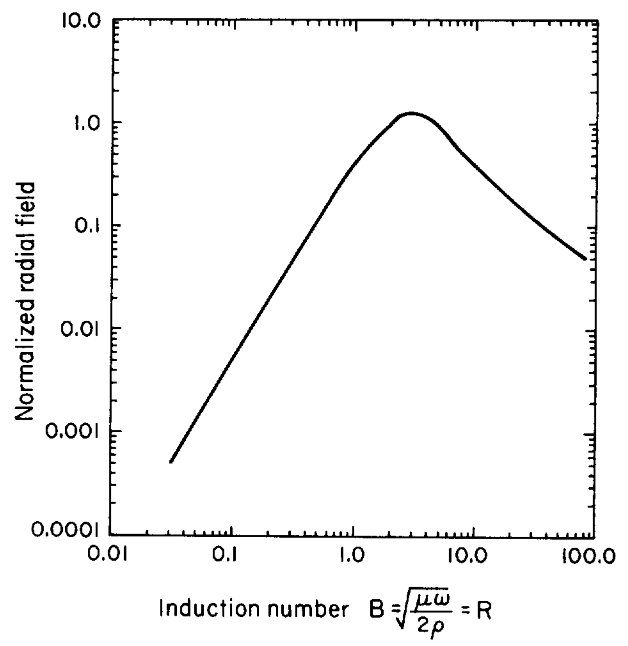


FIG. 2. Generalized curve for radial magnetic field amplitude for a half-space of 10 Ω-m.

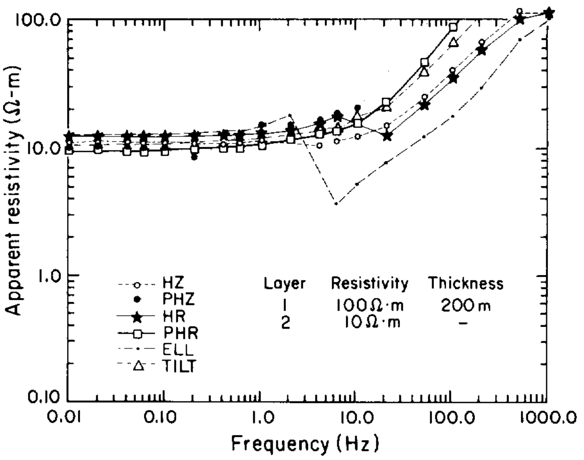


FIG. 3. Apparent resistivity spectra for six field components over a two-layer model.

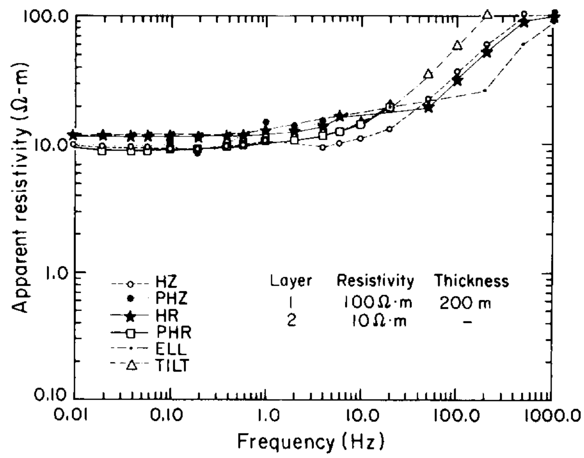


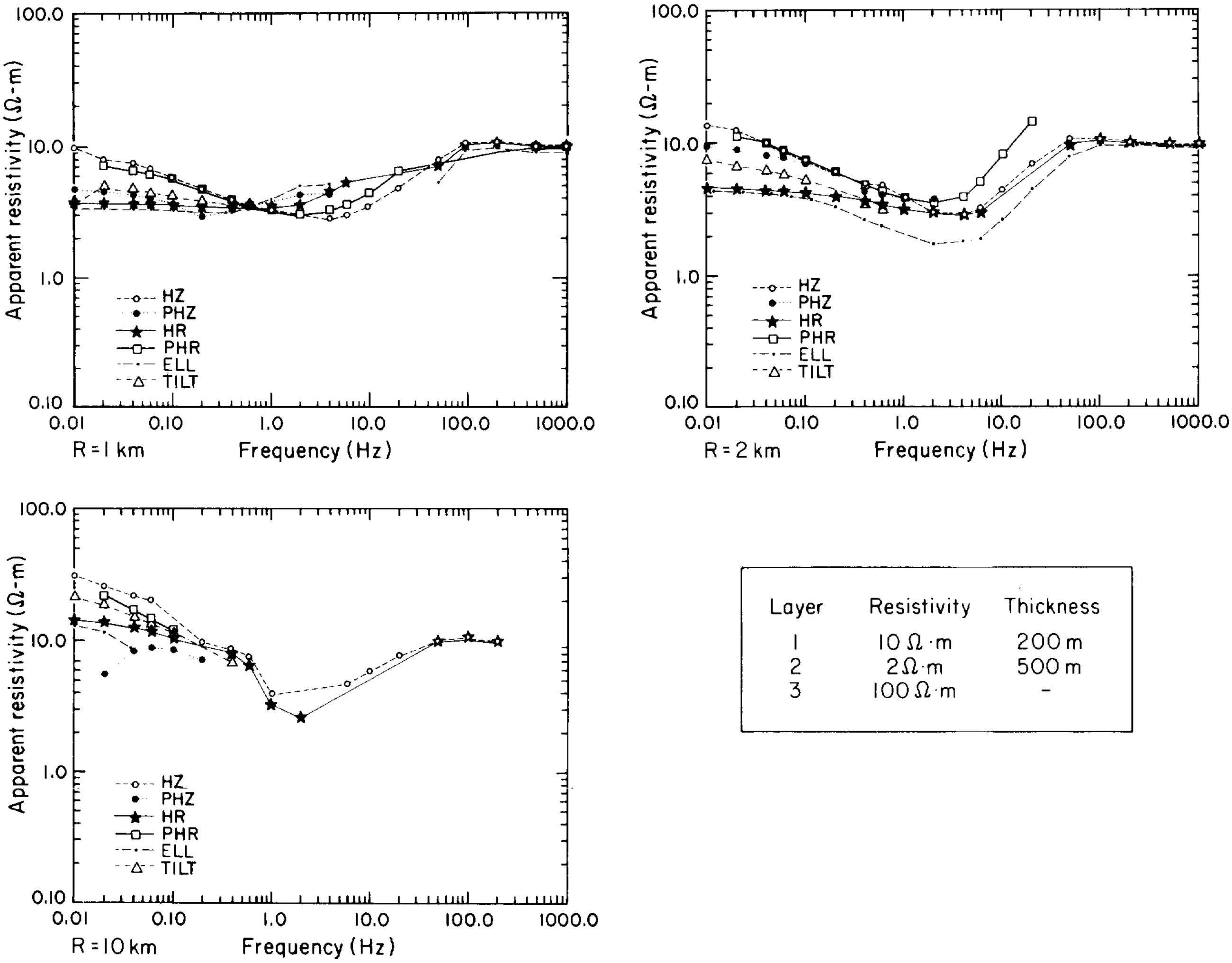
FIG. 4. Apparent resistivity spectra for six field components over a two-layer model with spurious points deleted.

The process is easily adaptable to field situations. A set of generalized field curves and a small programmable calculator are all that one needs to calculate apparent resistivities quickly from incoming field data. Generalized curves can also be digitized, and linear or logarithmic interpolation can provide more precise matching of field data. Table 1 displays digitized field curves for six field quantities for this purpose.

An example of computed apparent resistivity spectra of some theoretical layered-model data illustrates some of the advantages and disadvantages inherent in this scheme. Figure 3 is an apparent resistivity spectral plot calculated from theoretical data corresponding to the two-layer model section shown in the figure. The plots clearly reflect the general character of the two-layer model where higher frequency segments show sensitivity to the upper layer and lower frequency segments are more sensitive to the bottom layer. However, there is some scatter and some of the calculated values give unreasonable estimates for apparent resistivity. The unreasonable estimates normally come from flat portions of

Table 1. Theoretical magnetic field data over a uniform half-space.

B	H_R	Phase H_R (degrees)	H_z	Phase H_z (degrees)	Ellipticity	Tilt (degrees)
0.1000	0.0049	269.19	1.0004	180.25	−0.00492	90.00
1.1414	0.0098	268.64	1.0013	180.48	−0.00979	89.98
0.1732	0.0147	268.08	1.0024	180.70	−0.015	89.96
0.2000	0.0195	267.54	1.0036	180.90	−0.019	89.94
0.2449	0.0291	266.54	1.0064	181.26	−0.029	89.86
0.3162	0.0475	264.85	1.0128	181.88	−0.047	89.67
0.4472	0.0927	261.57	1.0314	183.00	−0.088	88.97
0.6325	0.1759	256.39	1.0718	184.06	−0.156	87.08
0.7746	0.2516	252.13	1.1104	184.19	−0.208	84.91
1.0000	0.3847	245.22	1.1761	183.02	−0.282	80.57
1.4142	0.6420	232.35	1.2768	177.12	−0.376	71.25
1.7321	0.8253	222.62	1.3160	170.38	−0.418	64.15
2.0000	0.9585	214.62	1.3192	163.84	−0.440	58.60
2.4495	1.1261	201.80	1.2689	152.01	−0.459	50.25
3.1623	1.2421	183.32	1.0886	132.87	−0.465	39.13
4.4722	1.1090	156.82	0.6628	102.07	−0.424	23.51
5.4773	0.8927	144.23	0.4002	86.42	−0.356	15.43
6.3246	0.7294	138.94	0.2559	81.21	−0.286	11.57
7.7461	0.5546	137.23	0.1444	86.35	−0.196	9.71
10.0002	0.4270	137.31	0.0914	91.00	−0.151	8.61
14.1424	0.3024	136.09	0.0459	89.97	−0.108	6.07
17.3208	0.2468	135.73	0.0306	90.02	−0.088	4.99
20.0003	0.2138	135.55	0.0230	90.02	−0.076	4.33
24.4953	0.1745	135.37	0.0153	90.02	−0.062	3.54
31.6232	0.1352	135.22	0.0092	90.03	−0.048	2.75



Layer	Resistivity	Thickness
1	10 Ω·m	200 m
2	2 Ω·m	500 m
3	100 Ω·m	-

FIG. 5. Apparent resistivity spectra for a three-layer model with transmitter-receiver separations of 1, 2, and 10 km.

the field curves, since for these points a unique induction number cannot be found from the generalized curve. In Figure 1, for example, vertical magnetic field amplitudes could not be used for frequencies less than 0.5 Hz, and radial phase data are unusable below 1 Hz. If points from these portions of the curves are rejected, a smoother, more reasonable spectrum results (Figure 4). Recording a complete spectrum of orthogonal field components guarantees that no gaps will exist on apparent resistivity spectral curves.

It is not unusual for field data to exceed the range of values on the half-space curves. For example, ellipticities for some layered-model sections may exceed −0.50, which is outside the range of half-space calculations (Table 1). When this occurs, the section of the curve exceeding the half-space curve, as well as the data adjacent to this section, should be deleted before apparent resistivity calculations are made. Normally, the affected region is only a small portion of one of the field curves; apparent resistivities calculated from the remainder of the field curves seem to be unaffected.

THEORETICAL EXAMPLES AND INTERPRETATION

Examples of apparent resistivity spectra for a three-layer model with a conductive middle layer are given in Figure 5. These spectra are from the same model with transmitter-receiver separations of 1, 2, and 10 km. The predominant effect of increasing separation is greater sensitivity to the bottom-layer resistivity and lesser sensitivity to the top-layer resistivity; however, the three curves show a remarkable degree of similarity, especially for the higher frequency segments. A possible reason for the similarity is that the observed fields for these separations and frequencies may approximate a plane-wave field. Hoversten et al (1982) show that two-dimensional (2-D) plots of observed magnetic fields from a dipolar source appear to converge to plane-wave conditions as the separation and/or source frequency is increased. Thus the EM induction apparent-resistivity curve may be equivalent to the MT apparent-resistivity curve under these conditions. In Figure 6 the MT apparent-resistivity curve corresponding to the three-layer model is plotted with the EM apparent-resistivity spectra at a

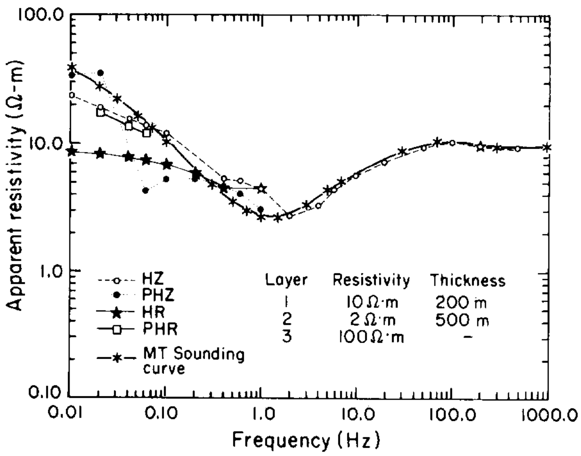


FIG. 6. Apparent resistivity spectra for a three-layer model with a transmitter-receiver separation of 5 km; MT apparent resistivity curve is also shown.

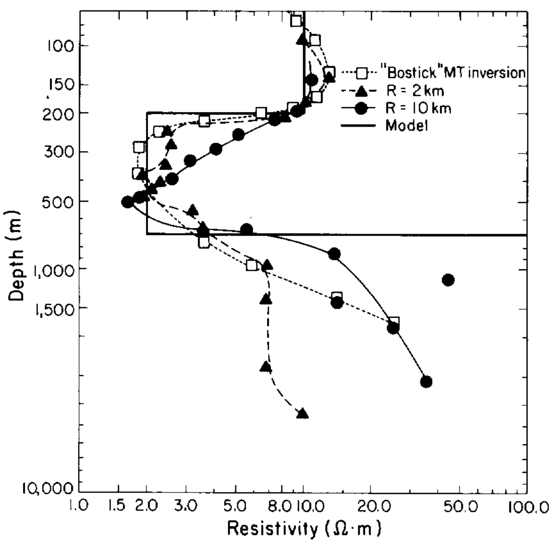


FIG. 7. Bostick inversion interpretation of EM apparent resistivity curves for transmitter-receiver separations of 2 and 10 km. Three-layer model section is also shown.

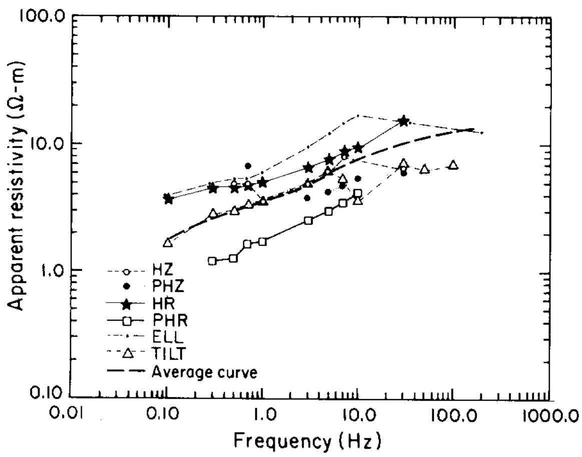


FIG. 8. Apparent resistivity spectra calculated from data in Figure 1. Heavy broken line indicates average curve.

5-km separation. The curves match fairly closely except at the lower frequencies, suggesting that standard techniques for interpreting MT data may be successful in interpreting EM apparent-resistivity spectra where the plane-wave approximation holds. A useful rule of thumb is that the plane-wave approximation will hold if the transmitter-receiver separation is greater than about three skin depths. In Figure 6, for example, the approximation is valid for frequencies above about 0.1 Hz. To test this assertion, we have interpreted the apparent resistivity curves (Figure 5) with the Bostick continuous inversion algorithm (Bostick, 1977). This particular algorithm was selected because it is amenable to a small programmable calculator and useful on partial field curves; it is therefore suitable for in-field application.

Figure 7 shows the results of the Bostick inversion of the two EM apparent-resistivity curves. Agreement of the inverted data with the three-layer model is good for both separations, although the 10-km separation curve provides a superior fit with the bottom-layer resistivity, and the 2-km separation curve fits the top-layer parameters better.

To illustrate the application of this scheme to field data, we calculated apparent resistivity spectra from field data shown in Figure 1. In Figure 8 the calculated apparent resistivities from these EM data are shown. This plot is more typical of field apparent-resistivity data because the spectrum is not complete, and there is sufficient scatter to make quantitative interpretation somewhat ambiguous. Nevertheless, a Bostick inversion was performed on an average curve determined from the arithmetic mean of the calculated points (Figure 8). The results of this inversion are shown in Figure 9, along with the model obtained by direct least-squares inversion. The Bostick interpretation of the apparent-resistivity data shows good agreement with the layered-model inversion, although there is some scatter to the fit.

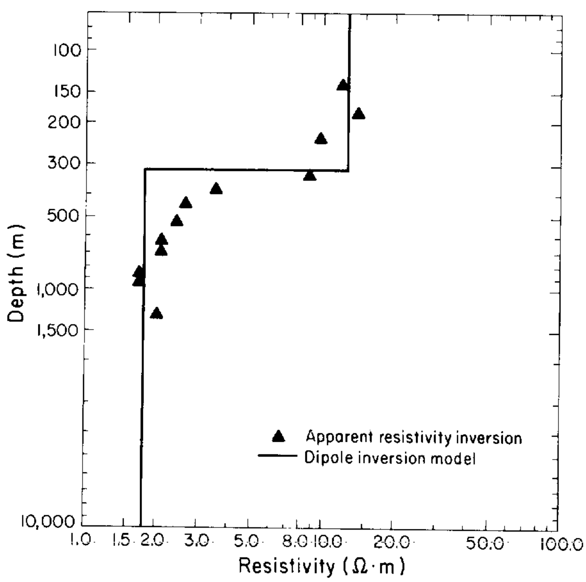


FIG. 9. Bostick inversion interpretation of average apparent resistivity curve in Figure 8 as compared to a two-layer model obtained by direct inversion of EM spectra in Figure 1.

CONCLUSIONS

In this note we have introduced a simple method for calculating apparent resistivity from frequency-domain EM sounding data. The method is sufficiently simple for in-field use, provides valuable feedback of data quality, and gives qualitative evaluation of incoming results. The apparent resistivity spectra may be interpreted with existing MT software, but since curves do not closely approximate plane-wave conditions for all frequencies, this procedure should be used with caution.

The scheme may be applied easily to frequency-domain configurations other than the loop-loop setup shown here, and the application of the method to time-domain EM sounding should simply require a Fourier transform of the appropriate generalized curves.

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